On Exploring Sparsity in Widely Separated MIMO Radar

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***Abstract*—The scenario of widely separated multi-input multi-output (MIMO) radar is considered. For a small number of targets, the target returns are sparse in the target space. First, a decoupled Lasso approach is proposed, which by exploiting the structure of the basis matrix decomposes the large size problem into a number of smaller size problems, thus reducing computational complexity. Second, it is shown that by reordering the columns of the basis matrix, group sparsity can be introduced to the returns. This structure can be exploited by a group Lasso approach to achieve significant performance gains.**

Keywords: compressive sampling, MIMO Radar, target localization, sparsity, group sparsity

I. Introduction

Multiple-input multiple-output (MIMO) radar have received considerable recent attention [1]-[4]. A MIMO radar consists of multiple transmit and receive antennas and is advantageous in two different scenarios, namely, *widely separated antennas* [3][4], and *collocated antennas* [1][2]. In the former scenario, the transmit antennas are located far apart from each other relative to their distance to the target. The MIMO radar system transmits independent probing signals from its antennas that follow independent paths, and thus each target return carries independent information about the target. Joint processing of the target returns results in diversity gain, which enables the MIMO radar to achieve high target resolution. Widely distributed MIMO radar systems are shown to offer considerable advantages for targets' parameter estimation, such as location [5] and velocity [6]. In the collocated scenario [1], the transmit and receive antennas are located close to each other relative to the target, so that all antennas view the same aspect of the target. In this scenario, the phase differences induced by transmit and receive antennas can be exploited to form a long virtual array with number of elements equal to the product of the numbers of transmit and receive nodes. This enables the MIMO radar system to achieve superior resolution in terms of direction of arrival (DOA) estimation and parameter identification [1].

Compressive sensing (CS) theory has drawn recent attention in diverse fields [7]-[10]. According to CS theory, a signal that is sparse with respect to some basis, can be recovered from much fewer samples than required by Nyquist theory. The application of CS to colocated MIMO radar was explored in [12]-[19], where the transmit and receive antennas can be placed on the nodes of a network. Following the observation that the target is sparse in the angle-Doppler-range space, CS recovery can enable target estimation based on a small number of measurements obtained by each receive node. Combining a step-frequency approach during transmission with compressive sampling during reception and CS recovery, significant gains in range resolution can be achieved with a small number of step frequencies [16]-[18]. Compressive sensing was also applied to MIMO radar with widely separately radar in [20] and [21]. In [20], the authors applied CS for coherent processing of a known number of point targets which uniformly reflect in all directions. The CS approach was utilized to produce initial estimates on velocity and reflectivity for the likelihood ratio detector. In [21], extended targets were considered, which reflect non-uniformly in different directions, and it was shown that the CS approach enables one to achieve a certain performance level with fewer samples as compared to conventional methods.

In this paper, we consider the same scenario as in [21], in which targets are already sparse in the target space and CS theory can be applied for target recovery. In that scenario, we explore the special sparsity structure of radar signal to reduce the computational cost involved, or improve the performance of target detection. By using the orthogonality between the submatrices of the basis matrix, we can decouple a large-size Lasso problem into several small-size ones. In this way, the complexity of Lasso is significantly reduced. We also propose that one can significantly improve estimation by inducing a group-sparse structure to the targets in the target space. This is achieved by appropriately re-arranging the columns of the sparsifying basis matrix during the CS recovery.

Notation: Lower case and capital letters in bold denote respectively vectors and matrices. The expectation of a random variable is denoted by $E\{\cdot\}$. Superscripts $(\cdot)^H$ and $\mathrm{Tr}(\cdot)$ denote respectively the Hermitian transpose and trace of a matrix. $A(m,n)$ represents the (m,n)-th entry of the matrix $\mathbf{A}$. $\mathbf{0}_{L\times M}$ and $\mathbf{1}_{L\times M}$ respectively denote an $L\times M$ matrix with"0" and "1" entries.

II. Signal Model

Let us assume that there are K extended targets in the cell under test and the targets are located in the same plane as the transmit and receive antennas. We also assume that

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the transmit waveform is sufficiently narrowbanded such that the scatters of a target cannot be resolved. Therefore, all the scatters of the k-th target can be represented by the gravity center of this target, which is denoted by (x_k, y_k) (in cartesian coordinates) at the initial time of sampling. The k-th target moves at velocity $[v_x^k, v_y^k]$. Let (x_i^t, y_i^t) and (x_i^r, y_i^r) denote the locations of the i-th transmit and receive antenna, respectively. The parameters to be estimated are $x_k, y_k, v_x^k, v_y^k, k = 1, \ldots, K$. Therefore, we define the target state vector as $[x, y, v_x, v_y]$.

The i-th antenna transmits the narrowband signal $x_i(t)$. Then the baseband signal at the l-th receive node arising due to the transmission of the i-th node equals [4]

$$z_{il}(t) = \sum_{k=1}^{K} \beta_k^{il} x_i(t - \frac{d_{ki}^t(t) + d_{kl}^r(t)}{c}) e^{j2\pi f_k^{il} t} + n_{il}(t),$$
$$l = 1, \ldots, M_r \quad (1)$$

where β_k^{il} and f_k^{il} represents the attenuation coefficient and the Doppler shift associated with the k-th target and the transmit-receive antenna pair (i, l), respectively; $n_{il}(t)$ denotes interference; $d_{ki}^{t/r}(t)$ denotes the Euclidean distance between the i-th transmit/receive antenna and the k-th target at time t, i.e.,

$$d_{ki}^{t/r}(t) = \sqrt{(x_i^{t/r} - x_k + v_x^k t)^2 + (y_i^{t/r} - y_k + v_y^k t)^2}$$
$$\text{and} \quad f_k^{il} = \frac{v_x^k(x_i^t - x_k) + v_y^k(y_i^t - y_k)}{\lambda\sqrt{(x_i^t - x_k)^2 + (y_i^t - y_k)^2}} + \frac{v_x^k(x_l^r - x_k) + v_y^k(y_l^r - y_k)}{\lambda\sqrt{(x_l^r - x_k)^2 + (y_l^r - y_k)^2}} \quad (2)$$

The Doppler shift in (2) is based on the first order term of the Taylor expansion of $d_{ki}^{t/r}(t)$

Let us discretize the target state space into N grid points, i.e, $[(x_n, y_n, v_x^n, v_y^n)], n = 1, \ldots, N$ and let s_n^{il} denote the coefficient associated with the n-th grid point for the transmit-receive antenna pair (i, l). The received signal $z_{il}(t)$ can be rewritten as a linear combination of target returns reflected from all grid points, i.e.,

$$z_{il}(t) = \sum_{n=1}^{N} s_n^{il} x_i(t - \tau_n^{il}) e^{j2\pi f_n^{il} t} + n_{il}(t)$$
$$= \mathbf{p}_{il}^T(t)\mathbf{s}^{il} + n_{il}(t) \quad (3)$$

where $\tau_n^{il} = \frac{d_{ni}^t(t) + d_{nl}^r(t)}{c}$, f_n^{il} is the Doppler shift corresponding to the n-th grid point, $\mathbf{p}_{il}(t) = [x(t - \tau_1^{il})e^{j2\pi f_1^{il} t}, \ldots, x(t - \tau_N^{il})e^{j2\pi f_N^{il} t}]^T$ and $\mathbf{s}^{il} = [s_1^{il}, \ldots, s_N^{il}]^T$. If the k-th target is located at (x_n, y_n, v_x^n, v_y^n), the coefficient s_n^{il} equals β_k^{il}; otherwise, it equals zero.

On letting L denote the number of T_s-spacing samples and T denote pulse repetition interval, we stack L samples collected during the m-th pulse for the transmit-receive antenna pair (i, l) into a vector $\mathbf{z}_{il}^m$ as

$$\mathbf{z}_{il}^m = [z_{il}(0T_s + (m-1)T), \ldots, z_{il}((L-1)T_s + (m-1)T)]^T$$
$$= \mathbf{\Psi}_{il}^m \mathbf{s}^{il} + \mathbf{n}_{il}^m \quad (4)$$

where $\mathbf{\Psi}_{il}^m = e^{j2\pi f_n^{il}(m-1)T}[\mathbf{p}_{il}(0T_s), \ldots, \mathbf{p}_{il}((L-1)T_s)]^T$ and $\mathbf{n}_{il}^m = [n_{il}(0T_s + (m-1)T), \ldots, n_{il}((L-1)T_s + (m-1)T)]^T$.

Let M_t and N_r denote respectively the number of transmit and receive antennas. Let us define

$$\mathbf{\Psi}_{il} \triangleq [(\mathbf{\Psi}_{il}^1)^T, \ldots, (\mathbf{\Psi}_{il}^{N_p})^T]^T,$$
$$\mathbf{\Psi} \triangleq diag\{[\mathbf{\Psi}_{11}, \ldots, \mathbf{\Psi}_{M_t N_r}]\}. \quad (5)$$

We can stack the received samples during N_p pulses from all the pairs of transmit and receive antenna into a vector $\mathbf{z}$ of length $M_t N_r L N_p$, i.e.,

$$\mathbf{z} = \left[(\mathbf{z}_{11}^1)^T, \ldots, (\mathbf{z}_{11}^{N_p})^T, (\mathbf{z}_{12}^1)^T \ldots, (\mathbf{z}_{M_t N_r}^{N_p})^T\right]^T$$
$$= \mathbf{\Psi}\mathbf{s} + \mathbf{n} \quad (6)$$

where $\mathbf{s}$ is a concatenation of $\mathbf{s}^{11}, ..., \mathbf{s}^{1N_r}, \mathbf{s}^{21}, ...$ Note that for all i, l, the vector $\mathbf{s}^{il}$ contains all zeros except at locations corresponding to grid points occupied by targets. The locations of the non-zero values in $\mathbf{s}^{il}$ are the same for all paths (i, l); the actual non-zero values are different, depending on the path (i, l). Thus, $\mathbf{s}$ is a sparse vector. In the next section, we will explore the group sparse structure of $\mathbf{s}$.

III. Sparse recovery for widely separated MIMO radar

Based on the sparse model for the received signal of widely separated MIMO radar as shown in (6), $\mathbf{s}$, the non-zero elements of which indicate the locations of targets in the target state space, can be recovered by the Lasso approach, i.e.,

$$\arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{\Phi}(\mathbf{z} - \mathbf{\Psi}\mathbf{s})\|_2^2 + \lambda\|\mathbf{s}\|_1. \quad (7)$$

where $\mathbf{\Phi}$ is the measurement matrix, λ is the tradeoff coefficient. The Lasso approach above enables to recover a signal with any sparsity structure. However, the structure of the basis matrix can be further explored to either reduce complexity of improve performance, as discussed in the subsequent sections.

A. Decoupled Lasso

It can be seen from (5) that the basis matrix has $M_t N_r$ blocks on its diagonal, each block corresponding to a pair of TX/RX antenna. This enables us to divide $\mathbf{\Psi}$ into $M_t N_r$ submatrices that are orthogonal to each other, i.e., $\mathbf{\Psi}_i, \ i = 1, \ldots, M_t N_r$. By successively multiplying the received signal by these submatrices, we can decompose the signal model of (6) into $M_t N_r$ smaller-size problems. i.e.,

$$\mathbf{\Psi}_i^H \mathbf{y} = \mathbf{\Psi}_i^H \mathbf{\Psi}\mathbf{s} + \mathbf{\Psi}_i^H \mathbf{n}$$
$$= \mathbf{\Psi}_i^H \mathbf{\Psi}_i \mathbf{s}_i + \mathbf{\Psi}_i^H \mathbf{n}, \ i.e., i = 1, \ldots, M_t N_r \quad (8)$$

where $\mathbf{s}_i$ contains the elements of $\mathbf{s}$ indexed as $(i-1)N + 1$ to iN, and can be solved by applying the Lasso approach to (8). We can extract target information by adding up the square amplitudes of $\mathbf{s}_i, \ i = 1, \ldots, M_t N_r$. The size of the signal model in (8) is N, which can be significantly smaller than that of (6). Therefore, solving the small-size problems via Lasso can significantly reduce overall computational load.

B. Group Lasso

Recall that if there is a target at the n-th grid point, the n-th entries of $\mathbf{s}_{il}$ for all the TX-RX antenna pairs, i.e., $s_n^{il},\ i = 1,\dots,M_t,\ l = 1,\dots,N_r$, are all non-zero. Therefore, by appropriately rearranging the columns of $\mathbf{\Psi}$, the non-zero elements of $\mathbf{s}$ corresponding to different paths and the same target can be clustered together (as seen in Fig. 1). In this way, we can form groups with non-zero elements of cardinality M_tN_r, in other words, we enforce *group sparsity*. Let $\mathbf{u}_{il}^n$ denote the n-th column of $\mathbf{\Psi}_{il}$. To create group sparsity, let us arrange the columns of $\mathbf{\Psi}$ to form $\mathbf{\Psi}_g$, defined as $\mathbf{\Psi}_g = \left[\tilde{\mathbf{\Psi}}_1,\dots,\tilde{\mathbf{\Psi}}_N\right]$, where $\tilde{\mathbf{\Psi}}_n = \left[\mathbf{u}_{11}^n,\dots,\mathbf{u}_{1N_r}^n,\mathbf{u}_{21}^n,\dots,\mathbf{u}_{M_tN_r}^n\right]$.

The sparse vector $\mathbf{s}$ associated with $\mathbf{\Psi}_g$ contains K groups of non-zeros entries and each of group is of length M_tN_r. The group sparsity of $\mathbf{s}$ can be exploited using a group Lasso approach [22]-[24], i.e.,

$$\arg\min_{\mathbf{s}} \underbrace{\frac{1}{2}\|\mathbf{\Phi}(\mathbf{z}-\mathbf{\Psi}_g\mathbf{s})\|_2^2}_{f_1(\mathbf{s})} + \underbrace{\lambda\sum_{n=1}^{N}\|\mathbf{s}_n\|_2}_{f_2(\mathbf{s})} \qquad (9)$$

where $\mathbf{s}_n$ has been defined in Section III-A but its element locations have been changed. $f_2(\mathbf{s})$ can be recast the ℓ_1 norm of the vector $\lambda[\|\mathbf{s}_1\|_2,\dots,\|\mathbf{s}_N\|_2]^T$. Minimization of $f_2(\mathbf{s})$ produces a group-sparse solution [22]-[24]. Due to the nature of ℓ_2 norm, all entries of the nth group $\mathbf{s}_n$ will be zero if $\|\mathbf{s}_n\|_2$ is zero, and would be non-zero otherwise. Since $f_2(s)$ is non-smooth, it is not trivial to directly solve (9). Instead of minimizing $f_1(s)$ and $f_2(s)$ simultaneously, the proximal gradient algorithm [34][35] proceeds by dealing with $f_1(s)$ and $f_s(s)$ individually in an iterative way. An accelerated version of the above algorithm above can be found in [35].

Let $\hat{\mathbf{s}}$ denote the solution to (9). We can formulate the target indicator vector, $\mathbf{h}$, so that its the nth entry equals to $\|\tilde{\mathbf{s}}_n\|_2^2$. The peaks of $\mathbf{h}$ will give provide the target information.

IV. Simulation results

In this section, we demonstrate the performance of Lasso, decoupled Lasso and group Lasso in the context of MIMO radar. We consider a MIMO radar system with transmit/receive radars that are uniformly located on a circle of radius $1,000$m. The carrier frequency is $f = 5GHz$. Each transmit radar uses orthogonal quadrature phase shift keying (QPSK) waveform sequences of length $L = 45$, and unit power. The received signal is corrupted by zero-mean Gaussian noise and SNR is set to 20dB. Each radar receive 45 measurements during each pulse. Three targets are assumed to be present in the search space $[8000, 8080,\dots,8320]m \times [8000, 8080,\dots,8320]m$, and in each run, the targets are randomly located on grid points. The target reflectivity is a Gaussian random variable with unit variance. We use the term probability of detection (PD) to denotes the percentage of cases in which all the targets are detected. The percentage of cases in which false targets are detected is denoted by the probability of false alarm (PFA). For simplicity, the case of stationary targets is considered. Figures 2 and 3 demonstrate the detection performance of Lasso, decoupled Lasso and group Lasso. The performance of the matched filter method (MFM) for MIMO radar is also shown for comparison. Figure 2 shows the square amplitude of estimate $\mathbf{h}$ for all grid points in one realization. It can seen that group Lasso produces smaller ripples than the other three methods and decoupled Lasso performs similarly to Lasso. Figure 3 shows the receiver operating characteristic (ROC) curves over 400 random and independent runs. It is clear that the group Lasso outperforms the other three methods in terms of ROC. Figure 3 also demonstrates that increasing the number of transmit/receive radars improves the detection performance.

V. Conclusion

We have considered sparse recovery of target information in the scenario of widely separated MIMO radar. By exploring the sparsity structure of radar signal, we proposed to apply decoupled Lasso, or group Lasso to target detection. The decoupled Lasso approach significantly reduces computational cost as compared to Lasso but offers no performance gain. On the other hand, the group Lasso approach produces significant performance gains in terms of the detection accuracy over the other two methods.

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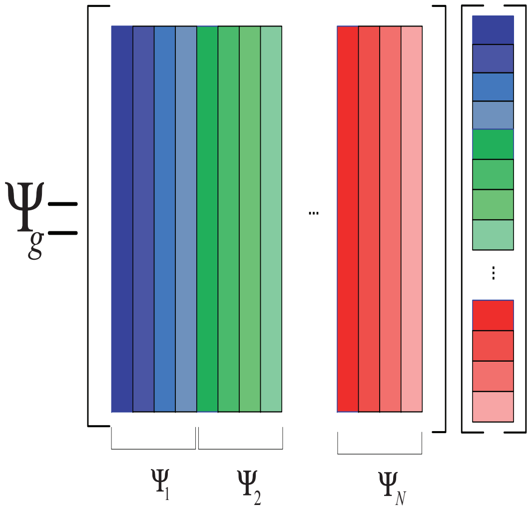

Fig. 1. The group sparse structure of the basis matrix.

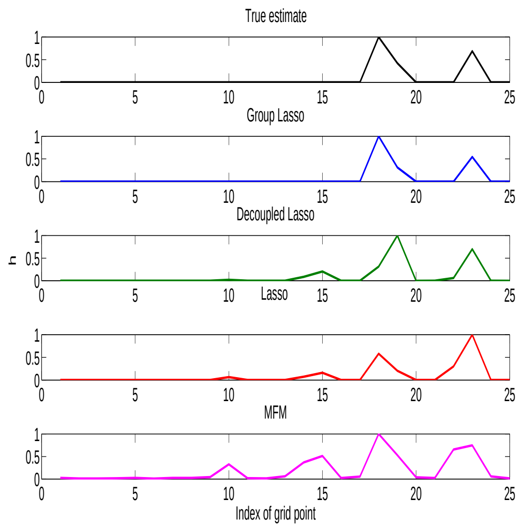

Fig. 2. Amplitude square of estimate $\mathbf{h}$ for all grid points in one realization.

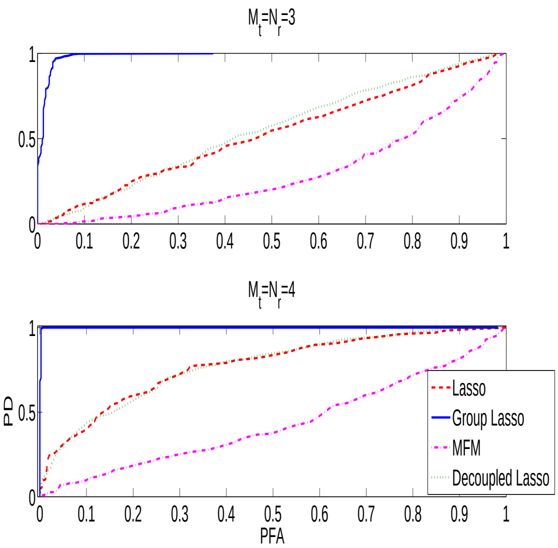

Fig. 3. ROC curves corresponding to GSR, SR and MFM for the case of stationary targets.