Accelerated Parallel Magnetic Resonance Imaging with Combined Gradient and Wavelet Sparsity

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Abstract. Parallel Magnetic Resonance Imaging (pMRI) is a fast developing technique to reduce MR scanning time. In pMRI, multi-channel coils simultaneously receive a fraction of k-space data and the field of view (FOV) is then reconstructed with the coil profiles. The techniques for pMRI can be mainly divided in two groups: image domain techniques such as PILS, SENSE and Fourier domain techniques like SMASH and GRAPPA. In this paper, we propose a new method based on SENSE framework to reconstruct MR image from multi-coil data. The proposed method combines compressive sensing (CS) to further improve the acceleration rate and utilizes total variation and wavelet sparsity regularization to remove artifacts. Both reconstruction problems can be solved by a recent fastest algorithm. Experiments show that the proposed method outperforms all other previous methods under SENSE framework.

1 Introduction

Parallel Magnetic Resonance Imaging (pMRI) with multiple coils is one of the most successful techniques to improve MR scanning speed in real applications. When scanning, each coil only acquires a fraction of data in k-space instead of all the data. Then the sensitivity profiles of all coils help to solve the nonlinear reconstruction problem. Generally, the pMRI techniques can be divided in two groups. Ones in k-space domain such as SMASH [1], GRAPPA [2] need interpolation weights to recover the unsampled Fourier frequency data and then the field of view (FOV) is obtained by the inverse Fourier transform. Techniques in the second group directly transfer the undersampled data to aliased images and then unfold these images in image domain. PILS [3] and SENSE [4] are two classical methods in the second group. In addition, SENSE, GRAPPA are auto-calibration methods and may be easier used in real application as they do not depend on the coil configuration.

Recently, by applying compressive sensing (CS) [5] on the image domain techniques one can gain significant benefit for further pMRI acceleration. Considering the Fourier coefficients of the aliased images as full data, CS-SENSE

[6] reconstructs the aliased images with further reduced samples. Then in the next step, it unfolds these aliased images to the FOV with standard SENSE encoding. It is based on a fact that if the image to be reconstruction is sparse in some domain, the aliased images obtained by multi coils should also be sparse in the same domain. On the other side, SparseSENSE [7] and similar ones [8] [9] introduce sparsity regularization to improve SENSE encoding. Actually, their models are better to describe as regularization but not CS. Because the problem is overdeterminated when the number of coils is larger than the reduction factor. The regularization has shown much more robust especially when the problem is ill-conditioned but none of them applies CS in the first step to further reduce the measurements. Briefly speaking, none of previous methods combines CS or regularization in both steps for pMRI reconstruction.

In this paper, we propose a new approach to improve pMRI based on SENSE framework. In the first step, we introduce a recent fast algorithm FCSA [10] for aliased images reconstruction, which has much better performance than CG [11] used in CS-SENSE [6]. For the second step, both total variation and wavelet sparsity are considered for regularization. This problem also can be solved by FCSA. Experiments show that the proposed method outperforms all tested methods at the same reduction factor, and is more robust to noise. Our work makes CS pMRI more feasible in real applications.

2 Related Work

2.1 SENSE

As mentioned above, SENSE [4] can be seen as a two-step reconstruction. In the original paper of SENSE, the fist step is to reconstruct aliased images directly by the inverse Fourier transform. Then these aliased images are unfolded to FOV in the second step. All later improvement in literature is taken on one of these two steps.

Figure 1 demonstrates the whole procedure of SENSE encoding with a reduction factor $R = 2$. In this figure, every other row of full k-space data is acquired for SENSE reconstruction. In the first step, aliased images are obtained by the inverse Fourier transform. Therefore, they are only half size of FOV. These aliased images are also called reduced FOVs due to their sizes. With the sensitivity profile of each coil, a series of equations can be setup to solve the FOV.

One pixel in a half sized reduced FOV, can be seen as a linear combination of two pixels in the FOV. It can be written as:

$$d_l(x, y) = S_l(x, y_1)f(x, y_1) + S_l(x, y_2)f(x, y_2) \tag{1}$$

where l denotes the l-th coil; $d_l(x, y)$ denotes the pixel in the aliased image at (x, y). S_l and f are the sensitivity map of the l-th coil and the FOV to be reconstructed. Each coil can contribute an equation at one position and the problem becomes solving linear equations. Note that each coil has a different sensitivity profile due to its unique position in MR scanner. So unique solution

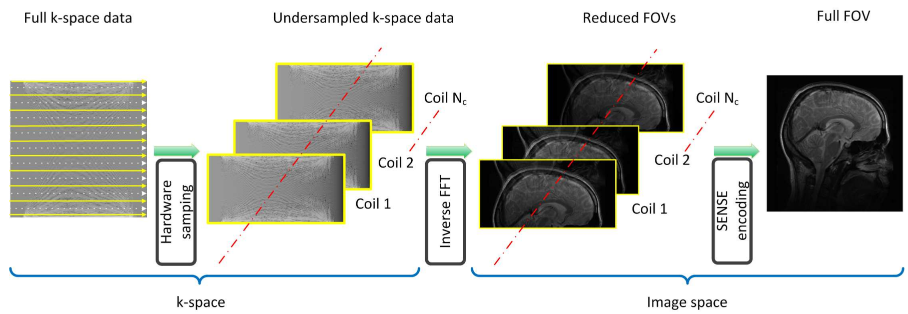

Fig. 1: Illustration of basic Cartesian SENSE procedure with N_c coils. The reduction factor is 2 in this example. The solid lines indicates the acquired k-space data and dashed lines denotes the nonacquired data. In the first step, reduced FOVs are obtained by the inverse Fourier transform. The second step is to unfold these reduced FOVs by SENSE encoding.

can be obtained when the number of coils N_c is no less than the reduction factor R. We can rewrite the equations in matrix form:

$$d = Sf \tag{2}$$

where d is all the aliased images in vector form, S is the sensitivity matrix and f is the image to be reconstructed. The well-conditioned solution without noise given in SENSE [4] is:

$$f = (S^H S)^{-1} S^H d \tag{3}$$

where H indicates the transposed complex conjugate.

SENSE assumes the sensitivity map is known or can be estimated with enough accuracy. When the sensitivity map can not be estimated precisely, least square fitting is used to approximate the solution [12]. In this paper, we only consider Cartesian type sampling. Non-Cartesian sampling cases (et. radial) would be much more complex.

2.2 SparseSENSE

SparseSENSE [7] is a method combining wavelet sparsity or total variation (TV) regularization with least square fitting to unfold the aliased images. It is based on a fact that most MR images are sparse in wavelet and gradient domains. In real applications, the sensitivity map may be not well estimated or undersampled data may contain noise. These kinds of error both could be amplified significantly in the final reconstructed image and resulting visible artifacts, especially in the case with a large reduction factor. Wavelet or TV regularization has shown good property to reduce the artifacts and makes the edges in the FOV more sharp. The formulation of SparseSENSE can be written as:

$$\min ||\Psi(f)||_1 \quad s.t. \quad Sf = d \tag{4}$$

where $\Psi(f)$ can be wavelet sparsity or total variation. This model is solved by a Bregman Iterative method. It utilizes the sparse prior of MR images when reconstructing and shows better performance than CG-SENSE [12] with just least square fitting. However, it is unknown how to solve the linear combination of both wavelet sparsity and total variation regularization.

2.3 CS-SENSE

Applying compressive sensing in SENSE, CS-SENSE [6] could significantly accelerate the scanning speed. In the first step of SENSE framework, it only collects partial k-space data to recover the reduced FOVs, while SENSE or SparseSENSE needs the full k-space data. The total reduction factor of it is the multiplication of that in both two steps $R = R_{CS} \times R_c$. R_{CS} denotes the reduction factor with CS and R_c denotes the reduction factor with multi coils. It assumes that if the original image is sparse in wavelet and gradient domains, the reduced FOVs also should be sparse in the same domains. The reconstruction of the reduced FOV d_l in l-th coil can be formulated as:

$$\min \frac{1}{2}||R_l d_l - b_l||_2^2 + \alpha||d_l||_{TV} + \beta||\Phi d_l||_1 \quad l = 1, 2, ..., N_c \tag{5}$$

where R_l is a partial Fourier transform and Φ denotes the wavelet transform. b_l is the corresponding k-space measurements. This classical model can reconstruct d_l with high accuracy if the parameters α and β are selected properly.

This approach could produce better result than the method only with regularization like SparseSENSE at the same reduction factor R. It is because only $1/R_{CS}$ reduction factor of that in SparseSENSE is needed for the second step reconstruction. However, it applies CG [11] to solve (5), which has been proved with much slower convergence than recent fast algorithms [10].

3 Algorithm

Few of previous work combines TV and wavelet sparsity regularization to improve both steps of SENSE reconstruction. In this paper, we introduce CS to reconstruct reduced FOVs with undersampled k-space data. The problem also can be formulated as (5). The difference is we apply FCSA [10] to solve it, with much better performance than the classical CG used in CS-SENSE. In our observation, the artifacts in the reduced FOVs can be easily amplified in the final image. So good recovery in the first step always plays a crucial role to the final result.

For the second step, our model can be written as:

$$\min \frac{1}{2}||Ef - d||_2^2 + \alpha'||f||_{TV} + \beta'||\Phi f||_1 \tag{6}$$

where α', β' are two parameters different from α and β. d is the vector denoting of all the reduced FOVs obtained from step 1, with $d = [d_1, d_2, ..., d_{Nc}]'$. f is the FOV to be reconstructed. E is the corresponding image-wise form of S. This problem also can be solved by FCSA efficiently.

Our method is much different from previous ones in three points: 1) Although we use the same model in the first step as that in CS-SENSE, we apply FCSA to solve it while CS-SENSE applies the classical CG algorithm. The reduced FOVs recovered by FCSA would much better than CG and the FOV should be with much higher quality. 2) We introduce both TV and wavelet sparsity regularization in the second step and solve it with FCSA. But few of previous work [7] [8] [9] can handle this problem. 3) None of previous work improves SENSE in both steps to make it more feasible in real applications. The experiments in the next section will demonstrate the benefit of the proposed method.

4 Experiments

We conduct numerous experiments comparing the proposed method with standard SENSE, SparseSENSE, and CS-SENSE. In all experiments, we assume the sensitivity information is already know or has been estimated for fair comparison. All acquired k-space data is added by Gaussian white noise with 0.01 variance. To test the robustness of the proposed method, we consider the situation that sensitivity matrix is with some noise. Signal-to-Noise Ratio (SNR) and mean Structural Similarity (MSSIM) [13] are used for result evaluation. If two images are the same, their MSSIM should be 100%.

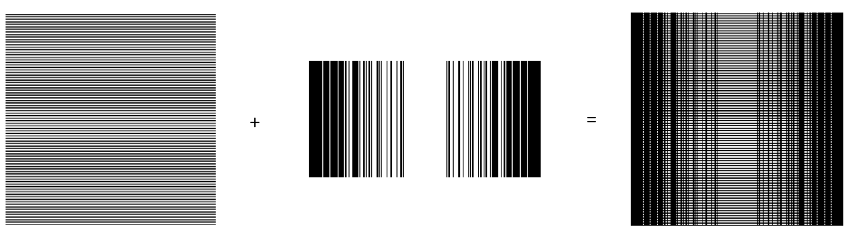

Fig. 2: Sampling mask of the proposed method. The left one is 1/2 sampling in Cartesian SENSE. The middle one is 1/2 sampling in CS. The right one is the final sampling mask with 1/4 sampling.

We tune parameters in all methods to achieve their best performance. All tested images are resized to 256×256 for convenience. The reduction factor for all methods is set as 4. CS-SENSE and the proposed method reconstruct reduced FOVs with 1/2 sampling. Then the final image is unfolded with these half size reduced FOVs. For the first step sampling, we follow the vertical lines strategy in SparseMRI [11]. More lines are selected in low frequency and less in high frequency. For the second step, every other row of k-space data is chosen. The final sampling pattern is shown in Figure 2. We use it in our experiments to help reader distinguish the two steps of the proposed method. Other combinations also can be considered such as vertical and vertical, horizon and horizon for the two steps.

Figure 3 shows the visual results on a real brain MR image. The original image is in Figure 1. We add 0.05% white Gaussian noise in each sensitivity map. This noise is amplified in standard SENSE encoding and result visible artifacts in the reconstructed image. SparseSENSE is much less sensitive to the noise due to the regularization in their model. CS-SENSE's result is better than the previous two because the reduction factor in the second step is just half of that in previous ones. The proposed method outperforms all of them. It is reasonable as we combine TV and wavelet regularization in both reconstruction steps and apply one of the recent most efficient algorithms to solve the problems.

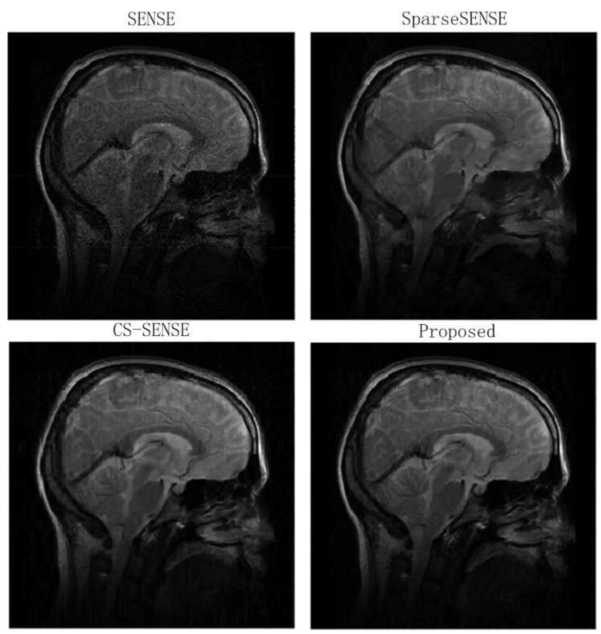

Fig. 3: Visual reconstruction results of different pMRI methods. SNR of the image recovered by SENSE, SparseSENSE, CS-SENSE and the proposed are 13.68, 16.22, 17.95 and 21.09 respectively.

Each method is executed with 1 iteration for inner TV denoise subproblem. The total execution time on a 2.5GHz CPU laptop for SENSE, SparseSENSE, CS-SENSE and proposed method is 4.27s, 14.20s, 79.31s and 11.45s respectively. The proposed method takes only very short time because we apply the recent fastest algorithm to solve our problems. Note that the first step CS reconstruction in each coil is totally independent and can be easily implemented with multi-core or GPU programming to get further acceleration.

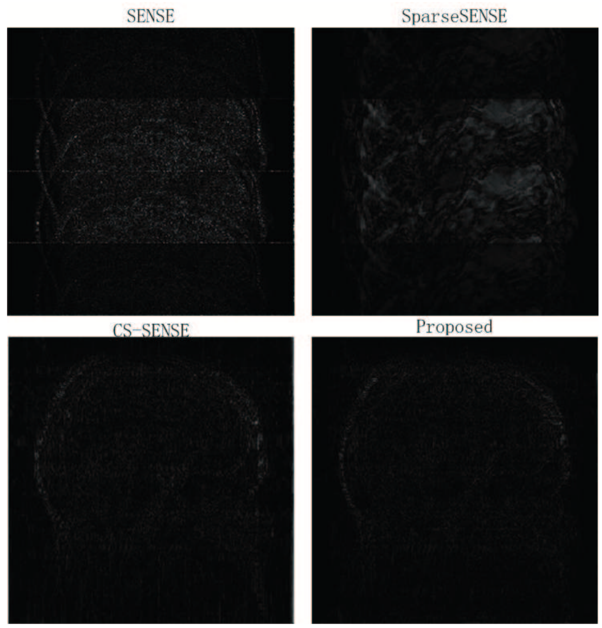

Fig. 4: Error images for Figure 3.

Table 1: Varying noise variance in the sensitivity map

	SNR (DB)				MSSIM (%)			
Noise variance (10^{-4})	1	5	10	50	1	5	10	50
SENSE	27.68	13.68	6.95	-1.3	98.65	80.25	59.03	29.13
SparseSENSE	16.42	16.22	15.82	10.87	91.71	91.46	90.79	76.69
CS-SENSE	17.95	17.95	17.94	17.84	91.69	91.69	91.68	91.40
Proposed	21.09	21.09	21.08	20.90	95.26	95.26	95.25	94.96

Table 1 shows the comparisons on the brain MR image under different noise variance in the sensitivity map. When there is no noise, the problem can be described as linear inverse problem and SENSE obtains the optimal solution. As noise increasing, SparseSENSE becomes much better than standard SENSE because of the regularization. CS-SENSE and the proposed method are much more robust to the noise, since the reduction factor in the SENSE encoding step is just half of that in the previous two methods. The proposed method always achieves the best reconstruction in terms of SNR and MSSIM when noise variance is larger than 10^{-4}.

5 Conclusion and Future Work

In this paper, we propose a two-step approach for pMRI based on SENSE framework. In the first step, aliasing images are reconstructed by compressive sensing techniques. For the second step, joint gradient and wavelet sparsity is used as regularization to improve standard SENSE encoding. Problems in both steps are solved by recently proposed algorithm FCSA. we conduct experiments to show the superiority of our method to those only exploiting sparsity in one step. The proposed method is also very robust when the sensitivity profile is not estimated precisely enough. In future work, we will try to improve the result of the proposed method at a high acceleration rate with more priors.

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