1

Symbolize the following argument: *Every ambassador speaks only to diplomats. Some ambassador speaks to someone. Therefore, there is a diplomat.*

Solution: Let $A(x)$ denote "x is an ambassador", $D(x)$ denote "x is a diplomat" and $T(x, y)$ denote "x talks to y."

Accordingly, the argument can be symbolized as:

$$(((\forall x)(\forall y)[(A(x) \wedge T(x,y)) \rightarrow D(y)]) \wedge (\exists x)(\exists y)[A(x) \wedge T(x,y)]) \rightarrow (\exists x)D(x)$$

□

2

Consider the following verbal argument: *If John took the necklace or the janitor lied, then a crime was committed. Robert was not in town. If a crime was committed, then Robert must have been in town. Therefore, John did not take the necklace.* Should a jury buy this argument?

Solution: Let $J \equiv$ "John took the necklace," $L \equiv$ "The janitor lied," $C \equiv$ "A crime was committed and $R \equiv$ "Robert was in town". Accordingly, the verbal argument can be symbolized as follows:

$$[((J \vee L) \rightarrow C) \wedge R' \wedge (C \rightarrow R)] \rightarrow J'$$

Consider the following proof sequence:

(i) R' hypothesis.
(ii) $C \rightarrow R$ hypothesis.
(iii) C' (i), (ii), Modus Tollens.
(iv) $(J \vee L) \rightarrow C$, hypothesis.
(v) $(J \vee L)'$ (iii), (iv), Modus Tollens.
(vi) $J' \wedge L'$ (v), De Morgan's Law.
(vii) J' (vi), Simplification.

□

3

Consider the following verbal argument: *If the program is efficient, it executes quickly. Either the program is efficient, or it has a bug. However, the program does not execute quickly. Hence it has a bug.*

Is the argument valid?

Solution: We use the following symbols:

(a) E - the program is efficient;
(b) Q - the program executes quickly;
(c) B - the program has a bug.

Accordingly, the argument can be expressed as:

$$[(E \rightarrow Q) \wedge (E \vee B) \wedge Q'] \rightarrow B$$

Consider the following proof sequence:

(i) $(E \rightarrow Q)$ hypothesis.
(ii) Q' hypothesis.
(iii) E' (i), (ii), Modus Tollens.
(iv) $(E \vee B)$ hypothesis.
(v) $(E' \rightarrow B)$ (iv), implication rule.
(vi) B, (iii), (v), Modus Ponens.

In other words, the argument is valid. □

4.

(a)

We first apply the Deduction Method to rewrite the argument as:

$$[(A' \vee B) \wedge (B \rightarrow C) \wedge A] \rightarrow C.$$

Consider the following proof sequence:

(i) $(A' \vee B)$ hypothesis
(ii) $A \rightarrow B$ (i), implication.
(iii) $B \rightarrow C$ hypothesis.
(iv) A hypothesis.
(v) B (ii), (iv), Modus Ponens.
(vi) C (iii), (v), Modus Ponens.

(b)

$[A \rightarrow (A \rightarrow B)] \rightarrow (A \rightarrow B)$.

Solution: Using the Deduction Rule, we can rephrase the above argument as:

$$[A \rightarrow (A \rightarrow B) \wedge A] \rightarrow B$$

Now consider the following proof sequence:

(i) A hypothesis.
(ii) $A \rightarrow (A \rightarrow B)$ hypothesis.
(iii) $A \rightarrow B$ (i), (ii), Modus Ponens.
(iv) B (i), (iii), Modus Ponens.

(c)

$[((A \vee B') \rightarrow C) \wedge (C \rightarrow D) \wedge A] \rightarrow D$.

Solution: Consider the following proof sequence:

(i) A hypothesis.
(ii) $A \vee B'$ (i), Addition.
(iii) $(A \vee B') \rightarrow C$ hypothesis.
(iv) C (ii), (iii), Modus Ponens.
(v) $C \rightarrow D$ hypothesis.
(vi) D (iv),(v), Modus Ponens.

(d)

$[A' \wedge (A \vee B)] \rightarrow B$.

Solution: The key observation is that $(A \vee B)$ can be written as: $(A' \rightarrow B)$. At this point, the argument is valid, by Modus Ponens. □

(e)

We first use implication rules to rewrite the argument as:

$$[(Q \wedge R)' \rightarrow P] \rightarrow (Q' \rightarrow P)$$

Using the Deduction Method, the above argument can be rewritten as:

$$([(Q \wedge R)' \rightarrow P] \wedge Q') \rightarrow P$$

Using De Morgan's Laws, the above argument can be revised to:

$$([(Q' \vee R') \rightarrow P] \wedge Q') \rightarrow P$$

Consider the following proof sequence:

(i) Q' hypothesis.
(ii) $Q' \vee R'$ (i), Addition.
(iii) $(Q' \vee R') \rightarrow P$ hypothesis.
(iv) P (ii), (iii), Modus Ponens.

(f)

Using the Deduction Method, we rewrite the given argument as:

$$[[P \to (Q \to R)] \wedge (P \vee S') \wedge Q \wedge S] \to R)$$

Consider the following proof sequence:

(i) S hypothesis.
(ii) $(P \vee S')$ hypothesis.
(iii) $(S \to P)$ (ii), implication.
(iv) P (i), (iii), Modus Ponens.
(v) $[P \to (Q \to R)]$ hypothesis.
(vi) $Q \to R$ (iv), (v), Modus Ponens.
(vii) Q hypothesis.
(viii) R (vi), (vii), hypothesis.

5

Solution:

(i) Let the domain be the set of integers and $A(x) \equiv$ "x is an odd number," and $B(x) \equiv$ "x is an even number." In this interpretation, we have a number which is odd and another which is even, but there is no number, which is both odd and even. Hence the wff is not **true** in this interpretation, and hence not valid.

(ii) Once again, we use the set of integers as our domain and assign $P(x, y)$ to mean $x > y$. Accordingly, the hypothesis states that, "For every integer x, there exists another integer y, such that $x > y$." This is **true**, because in this domain $x > (x - 1)$ is **true**. The consequence statest that "there exists an integer x, such that for all integers y, $x > y$." The consequence is clearly **false**, since $x \not> (x + 1)$ in this domain. Hence, the wff is not **true** in this interpretation, and hence not valid.

6

Is the following argument valid: $[(\forall x)P(x) \wedge (\exists x)Q(x)] \to (\exists x)[P(x) \wedge Q(x)]$.

Solution: Consider the following proof sequence:

(i) $(\exists x)Q(x)$ hypothesis.
(ii) $Q(a)$ (i), ei.
(iii) $(\forall x)P(x)$ hypothesis.
(iv) $P(a)$ hypothesis.
(v) $P(a) \wedge Q(a)$ (ii), (iv), Conjunction.
(vi) $(\exists x)[P(x) \wedge Q(x)]$ (v), eg.

□

7

Prove that the following argument is valid: $(\forall x)P(x) \to (\forall x)[P(x) \vee Q(x)]$.

Solution: Consider the following proof sequence:

(i) $(\forall x)P(x)$ hypothesis.
(ii) $P(x)$ (i), ui.
(iii) $P(x) \vee Q(x)$ (ii), Addition.
(iv) $(\forall x)[P(x) \vee Q(x)]$ (iii), ug.

We are justified in using Universal Generalization in Step (iv), since Existential Instantiation was not used in the proof sequence and $(P(x) \vee Q(x))$ was deduced from the hypotheis $(\forall x)P(x)$, in which x is bound. □

8

Consider the following assertion: *There is an astronomer who is not nearsighted. Everyone who wears glasses is nearsighted. Furthermore, everyone either wears glasses or wears contact lenses. Therefore, some astronomer wears contact lenses.* Is the assertion a valid argument?

Solution: Let $A(x) \equiv x$ is an astronomer, $N(x) \equiv x$ is nearsighted, $G(x) \equiv x$ wears glasses and $L(x) \equiv x$ wears contact lenses.

The given argument can be symbolized as follows:

$$[(\exists x)(A(x) \wedge N(x)') \wedge (\forall x)(G(x) \to N(x)) \wedge (\forall x)(G(x) \vee L(x))] \to (\exists x)(A(x) \wedge L(x))$$

The argument is valid. Consider the following proof sequence:

(i) $(\exists x)(A(x) \wedge N(x)')$ hypothesis.

(ii) $A(a) \wedge N(a)'$ (i), existential instantiation.

(iii) $(\forall x)(G(x) \vee L(x))$ hypothesis.

(iv) $G(a) \vee L(a)$ (iii), universal instantiation.

(v) $G(a)' \to L(a)$ (iv), implication equivalence.

(vi) $N(a)'$ (ii), simplification.

(vii) $(\forall x)(G(x) \to N(x)$ hypothesis.

(viii) $G(a) \to N(a)$ (vii), universal instantiation.

(ix) $G(a)'$ (vi), (viii), Modus Tollens.

(x) $L(a)$ (v), (ix) Modus Ponens.

(xi) $A(a)$ (ii), simplification.

(xii) $A(a) \wedge L(a)$ (x), (xi), conjunction.

(xiii) $(\exists x)(A(x) \wedge L(x))$ (xii), existential generalization.

□