1.

(a)

It's definitely not reflexive, as no integer is coprime with itself except -1 and 1. It is symmetric because $\gcd(a,b) = \gcd(b,a)$, so $\gcd(a,b) = 1$ iff $\gcd(b,a) = 1$. Not antisymmetric — *every* coprime pair, such as (5,7) and (7,5), will show this. Not transitive — $\gcd(5,7) = 1$, $\gcd(7,10) = 1$, but $\gcd(5,10) \neq 1$.

(b)

Not reflexive, since 0|0 is undefined. Not symmetric, for example $2 \mid 4$ but $4 \nmid 2$. It not antisymmetric on $\mathbb{Z}$, since $a \mid -a$ and $-a \mid a$, although it would be antisymmetric if restricted to $\mathbb{N}$. It is transitive — if $a \mid b$ then $b = ka$ for some $k \in \mathbb{Z}$, and if $b \mid c$ then $c = lb$ for some $l \in \mathbb{Z}$, thus $c = (lk)a$ and $(lk) \in \mathbb{Z}$ so $a \mid c$.

(c)

Not reflexive, for example $\sqrt[4]{2}\sqrt[4]{2} = \sqrt{2}$ which is definitely not in $\mathbb{Q}$. Definitely symmetric since multiplication is commutative, $ab = ba$ always. Not antisymmetric, since $\sqrt{2}\sqrt{8} = \sqrt{8}\sqrt{2} = 4$ but $\sqrt{2} \neq \sqrt{8}$. Also not transitive — consider $a = \pi$, $b = \frac{1}{\pi}$, and $c = \pi$. $ab, bc \in \mathbb{Q}$ but $ac = \pi^2 \notin \mathbb{Q}$.

2.

(a)

Longest chain is powers of 2 as high as they can go, length is $log_2 n + 1$. There is one chain of this length, except for $n = 3$ where there are two chains of length two.

(b)

Each set in the chain must have distinct cardinality, so the longest chains are $n + 1$. The number of chains is the product of all binomial coefficients for n as they correspond to the number of sets of each cardinality.

3.

RU{(1,4), (3,4)} or

{(1, 2), (3, 1), (3, 2), (2, 4), (1,4), (3,4)}

4.

Note that the LHS represents the number of distinct ways of selecting 2 objects from n distinct objects. We break up the set of n objects into two disjoint sets, with one set containing r objects and the other set containing $n - r$ objects. Observe that 2 objects can be selected in one of the following distinct ways:

(a) Both objects from the set of r objects; this can be done in $C(r, 2)$ ways.

(b) Both objects from the set of $n - r$ objects; this can be done in $C(n - r, 2)$ ways.

(c) One object from the set of r objects and the other object from the set of $n - r$ objects; this can be done in $r \cdot (n - r)$ ways.

Thus, the total number of ways to select 2 objects out of n is $C(r, 2) + C(n - r, 2) + r \cdot (n - r)$, which proves the identity.

You are of course welcome to attempt a proof using induction or first principles!