CSE2315: Homework 5
Out: April 10
Due: April 17.

1. (24 points) For each of the following relations, state whether they fulfill each of the 4 main properties - reflexive, symmetric, antisymmetric, transitive. Briefly substantiate each of your answers.

   (a) The coprime relation on $\mathbb{Z}$. ($a, b \in \mathbb{Z}$ are coprime if and only if $gcd(a, b) = 1$.)

   (b) Divisibility on $\mathbb{Z}$.

   (c) The relation T on $\mathbb{R}$ such that $a\ T\ b$ if and only if $ab \in \mathbb{Q}$.

2. (16 points) In a partially ordered set, a chain is a totally ordered subset. For example, in the set 1, 2, 3, 4, 5, 6, the divisibility relation is a partial order and 1, 2, 4 and 1, 3, 6 are chains.

   (a) What is the longest chain on the set $\{1, 2, \cdots, n\}$ using the divisibility relation? How many distinct chains have this length? For the second part, make sure to consider all positive values of n.

   (b) What is the longest chain on the powerset of a set A with $|A| = n$ with the $\subseteq$ relation? How many distinct chains have this length?

3. (10 points) Find the transitive closure of $R = \{(1, 2), (3, 1), (3, 2), (2, 4)\}$.

4. (10 points) Show that for any n and r, with $0 \leq r \leq n$,

$$C(n, 2) = C(r, 2) + C(n - r, 2) + r \cdot (n - r).$$