# Discrete Structures

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Lecture 10 Proof

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# Proof Techniques

- Applicable to all domains
  - Exhaustive proof.
  - Direct proof.
  - Proof by contraposition.
  - Proof by contradiction.
  - Serendipity.

- Applicable to Structured Domains
  - Mathematical Induction.
  - Diagonalization.

# Serendipity

- Prayer! Good Luck.

- Example

  Number of games in a tennis tournament.

# Induction

- Motivation: reaching arbitrary rungs of a ladder.

  Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g. integers.

- Assume that the domain is the set of positive integers
  - P(1) is **true**.
  - $(\forall k)[P(k) \rightarrow P(k + 1)]$
  - Then, P(n) is **true**, for all positive integers n.

- The first principle of mathematical induction
  - Showing that P(1) is **true** is called the basis step.
  - Assuming that P(k) is **true**, in order to show that P(k + 1) is **true** is called the inductive hypothesis.

# Induction Examples

- Show that the sum of the first n integers is $n(n+1)/2$
- Main ideas
  - Mathematicize the conjecture.
  - Prove the basis (usually $P(1)$ and usually easy.)
  - Assume $P(k)$.
  - Show $P(k + 1)$. (The hard part. Use mathematical manipulation.)
- Show that the sum of the squares of the first n integers is $n(n+1)(2n+1)/6$
- Show that the sum of the first n odd integers is $n^2$
- Show that $7^n - 5^n$ is always an even number for $n >= 0$, i.e., show that $2 \mid (7^n - 5^n), \forall n>=0$.

# Second Principle of Induction

- Also called Strong Induction. Is necessary, when the first principle does not help us.

- Assume that the domain is the set of integers.
  - $P(1)$ is **true**.
  - $(\forall k)[P(r)$ true for all $r$ , $1 <= r <= k \rightarrow P(k + 1)]$
  - Then, $P(n)$ is **true** for all n.

- Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

# Induction Example

- Proof

  The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r , 8<=r <=k. Consider the integer k + 1. Without loss of generality, we assume that (k +1)>=11. Observe that (k +1)−3 = k −2 is at least 8 and less than k. As per the inductive hypothesis, k − 2 can be expressed in the form 3a + 5b, for suitably chosen a and b. It follows that (k + 1) = 3(a + 1) + 5b, can also be so expressed. Applying the second principle of mathematical induction, we conclude that the conjecture is true.