Discrete Structures

CSE 2315 (Spring 2014)

Lecture 14 Binomial Theorem

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Motivation of The Binomial Theorem

(i) $(a+b)^1 = a+b$.

(ii) $(a+b)^2 = a^2 + 2ab + b^2$.

(iii) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

(iv) $(a+b)^4 = ???$

We want a general formula that permits us to write down the terms of $(a+b)^n$ without actual multiplication.

Pascal's Triangle

Row 0:					1				
Row 1:				1		1			
Row 2:			1		2		1		
Row 3:		1		3		3		1	
$\vdots$									
Row n:	1		n		$\ldots$		n		1

Row 0:					$C(0, 0)$				
Row 1:				$C(1, 0)$		$C(1, 1)$			
Row 2:			$C(2, 0)$		$C(2, 1)$		$C(2, 2)$		
Row 3:		$C(3, 0)$		$C(3, 1)$		$C(3, 2)$		$C(3, 3)$	
$\vdots$									
Row n:	$C(n, 0)$		$C(n, 1)$	$\ldots$	$\ldots$		$C(n, n-1)$		$C(n, n)$

The Binomial Theorem

$$(a+b)^n = \sum_{i=0}^{n} C(n,i)\; a^{n-i} \cdot b^i, \quad \forall n \geq 0.$$

- Expand $(x - 3)^4$.
- Show that $$\sum_{i=0}^{n} C(n,i) = 2^n$$

Pascal's Formula

- Theorem

  C(n, k) = C(n – 1, k - 1) + C(n – 1, k), 1 <= k <= n - 1.

  Proof:

$$
\begin{aligned}
C(n-1,k-1)+C(n-1,k) &= \frac{(n-1)!}{(k-1)![(n-1-(k-1)!]} + \frac{(n-1)!}{k!(n-1-k)!} \\
&= \frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-1-k)!} \\
&= \frac{k(n-1)!}{k!(n-k)!} + \frac{(n-k)(n-1)!}{k!(n-k)!} \\
&= \frac{(n-1)!}{k!(n-k)!}[k+(n-k)] \\
&= \frac{n(n-1)!}{k!(n-k)!} \\
&= \frac{n!}{k!(n-k)!}
\end{aligned}
$$