Discrete Structures

CSE 2315 (Spring 2014)

Lecture 4 Argument

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Recall: Tautologies

- Definition

  A wff which is always **true** is called a tautology, while a wff which is always **false** is called a contradiction.

- Example

  $A \vee A'$

- Tautology checking

  How do you check if a wff is a tautology? Truth-tables!

Common Tautological Equivalences

- De Morgan's Laws

  $(A \vee B)' \Leftrightarrow (A' \wedge B')$ $\qquad$ $(A \wedge B)' \Leftrightarrow (A' \vee B')$

- Commutativity

  $(A \vee B) \Leftrightarrow (B \vee A)$ $\qquad$ $(A \wedge B) \Leftrightarrow (B \wedge A)$

- Associativity

  $(A \vee B) \vee C \Leftrightarrow A \vee (B \vee C)$ $\qquad$ $(A \wedge B) \wedge C \Leftrightarrow A \wedge (B \wedge C)$

- Distributivity

  $A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge (A \vee C)$ $\qquad$ $A \wedge (B \vee C) \Leftrightarrow (A \wedge B) \vee (A \wedge C)$

- Exercise

  Try to prove all the above tautologies.

Arguments

- An argument is a statement of the form:

$$(P_1 \wedge P_2 \wedge \ldots P_n) \rightarrow Q$$

  where each of the P_i s and Q are propositions. The P_i s are called the hypotheses and Q is called the conclusion.

- When can Q be logically deduced from P_1, P_2 ,…, P_n?

- 2 + 2 = 4 and 7 + 3 = 10. Therefore, a minute has 60 seconds. Is this valid?

Valid Arguments

- The argument

$$(P_1 \wedge P_2 \wedge \ldots P_n) \rightarrow Q$$

  is said to be valid, if it is a tautology.

- The validity of an argument is based purely on its intrinsic structure and not on the specific meanings attached to its constituent propositions.

- If John is hungry, he will eat. John is hungry. Therefore, he will eat.

  Symbolically, $(H \rightarrow E) \wedge H \rightarrow E$.

Checking Validity

- Truth-table Method

  Simply check if all rows of the truth-table are **true**. Horribly expensive!

- Derivation Rules

  We will use a set of derivation rules and manipulate the hypotheses to arrive at the desired conclusion.

- Proof Sequence

  A proof sequence is a sequence of wffs in which each wff is either a hypothesis or the result of applying one of the formal system's derivation rules to earlier wffs in the sequence.

Derivation Rules

- Rule Types
  - Equivalence Rules.
  - Inference Rules.
- Equivalence Rules

Expression	Equivalent to	Name of Rule
$P \vee Q$ $P \wedge Q$	$Q \vee P$ $Q \wedge P$	Commutative - comm
$P \vee (Q \vee R)$ $P \wedge (Q \wedge R)$	$(P \vee Q) \vee R$ $(P \wedge Q) \wedge R$	Associative -ass
$(P \vee Q)'$ $(P \wedge Q)'$	$P' \wedge Q'$ $P' \vee Q'$	De Morgan
$P \rightarrow Q$	$P' \vee Q$	Implication - imp
P	$(P')'$	Double negation - dn
$P \leftrightarrow Q$	$(P \leftarrow Q) \wedge (Q \leftarrow P)$	Definition of equivalence

Derivation Rules

- Inference Rules

From	Can Derive	Name of Rule
P, $P \to Q$	Q	*Modus Ponens* (mp)
$P \to Q$, Q'	P'	*Modus Tollens* (mt)
P, Q	$P \wedge Q$	Conjunction
$P \wedge Q$	P, Q	Simplification
P	$P \vee Q$	Addition