Discrete Structures

CSE 2315 (Spring 2014)

Lecture 5 Argument and Proof

Heng Huang, Ph.D.
Department of Computer Science and Engineering

Recall: Arguments

- An argument is a statement of the form:

$$(P_1 \wedge P_2 \wedge \ldots P_n) \rightarrow Q$$

where each of the P_i s and Q are propositions. The P_i s are called the hypotheses and Q is called the conclusion.

- When can Q be logically deduced from $P_1, P_2, \ldots, P_n$?

- 2 + 2 = 4 and 7 + 3 = 10. Therefore, a minute has 60 seconds. Is this valid?

Recall: Valid Arguments

- The argument

$$(P_1 \wedge P_2 \wedge \ldots P_n) \to Q$$

  is said to be valid, if it is a tautology.

- The validity of an argument is based purely on its intrinsic structure and not on the specific meanings attached to its constituent propositions.

- If John is hungry, he will eat. John is hungry. Therefore, he will eat.

  Symbolically, $(H \to E) \wedge H \to E$.

Recall: Checking Validity

- Truth-table Method

  Simply check if all rows of the truth-table are **true**. Horribly expensive!

- Derivation Rules

  We will use a set of derivation rules and manipulate the hypotheses to arrive at the desired conclusion.

- Proof Sequence

  A proof sequence is a sequence of wffs in which each wff is either a hypothesis or the result of applying one of the formal system's derivation rules to earlier wffs in the sequence.

Derivation Rules

- Rule Types
  - Equivalence Rules.
  - Inference Rules.
- Equivalence Rules

Expression	Equivalent to	Name of Rule
$P \vee Q$ $P \wedge Q$	$Q \vee P$ $Q \wedge P$	Commutative - comm
$P \vee (Q \vee R)$ $P \wedge (Q \wedge R)$	$(P \vee Q) \vee R$ $(P \wedge Q) \wedge R$	Associative -ass
$(P \vee Q)'$ $(P \wedge Q)'$	$P' \wedge Q'$ $P' \vee Q'$	De Morgan
$P \rightarrow Q$	$P' \vee Q$	Implication - imp
P	$(P')'$	Double negation - dn
$P \leftrightarrow Q$	$(P \leftarrow Q) \wedge (Q \leftarrow P)$	Definition of equivalence

Derivation Rules

- Inference Rules

From	Can Derive	Name of Rule
P, $P \rightarrow Q$	Q	*Modus Ponens* (mp)
$P \rightarrow Q$, Q'	P'	*Modus Tollens* (mt)
P, Q	$P \wedge Q$	Conjunction
$P \wedge Q$	P, Q	Simplification
P	$P \vee Q$	Addition

A proof derivation

- Argue that

$$[A \wedge (B \to C) \wedge [(A \wedge B) \to (D \vee C')] \wedge B] \to D$$

is a valid argument.

(i)	A	hypothesis.
(ii)	B	hypothesis.
(iii)	$B \to C$	hypothesis.
(iv)	C	(ii), (iii), Modus Ponens.
(v)	$A \wedge B$	(i), (ii), Conjunction.
(vi)	$(A \wedge B) \to (D \vee C')$	hypothesis.
(vii)	$(D \vee C')$	(v), (vi), Modus Ponens.
(viii)	$(C \to D)$	(vii), Implication.
(ix)	D	(iv), (viii), Modus Ponens.

Two more rules

- Deduction Method

  The argument $P_1 \wedge P_2 ,\ldots, P_n \to (R \to S)$ is tautologically equivalent to the argument $P_1 \wedge P_2 ,\ldots, P_n \wedge R \to S$.

- Example

  Prove that $[(A \to B) \wedge (B \to C)] \to (A \to C)$

- Proof

  Using the Deduction Method, the above argument can be rewritten as: $[(A \to B) \wedge (B \to C) \wedge A] \to C$. Now apply Modus Ponens twice!

- The rule $[(A \to B) \wedge (B \to C)] \to (A \to C)$ is called hypothesis syllogism and can be used directly.

Proving validity of Verbal Arguments

- Methodology
  - Symbolize the argument.
  - Construct a proof sequence for the symbolic argument
- If interest rates drop, the housing market will improve. Either the federal discount rate will drop or the housing market will not improve. Interest rates will drop. Therefore, the federal discount rate will drop. Is this argument valid?
- Proof.

  Let I denote the event that interest rates will drop. Let H denote the event that the housing market will improve. Let F denote the event that the federal discount rate will drop. The symbolic argument is

  $$[(I \rightarrow H) \wedge (F \vee H') \wedge I] \rightarrow F$$

  From I and $(I \rightarrow H)$, we can derive H using Modus Ponens. From H and $(F \vee H')$ to $(H \rightarrow F)$, we can derive F using Modus Ponens!

One More Example

- Show that the argument $A' \wedge (B \to A) \to B'$ is valid.
- Is the following argument valid? "My client is left-handed, but if the diary is not missing, then my client is not left-handed; therefore, the diary is missing."
- Caveat

  Lots of additional rules are provided on Page 33; you are not permitted to use any of them in your homework assignments, quizzes and exams! Derive them if you must use them.