Discrete Structures

CSE 2315 (Spring 2014)

Lecture 8   Predicate Logic Proof Sequence

Heng Huang, Ph.D.

Department of Computer Science and Engineering

Universal Instantiation

- From $(\forall x)P(x)$, you can conclude $P(t)$, where t is any constant or variable.
- Rule is abbreviated as u.i.
- If t is a variable, it must not fall within the scope of a quantifier for t. For instance, from $(\forall x)(\exists y)P(x, y)$, you cannot conclude $(\exists y)P(y, y)$. (Domain of integers).

- Let us prove that the following argument is valid, using ui.

$$[(\forall x)[H(x) \rightarrow M(x)] \wedge H(s)] \rightarrow M(s)$$

Existential Instantiation

- From $(\exists x)P(x)$, you can conclude $P(a)$, where a is a constant symbol not used previously in the proof sequence.
- Rule is abbreviated as e.i.
- Must be the first rule that introduces a.

- Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists y)P(y)] \rightarrow (\exists y)Q(y)$ is valid.

Universal Generalization

- From P(x), you can conclude $(\forall x)P(x)$.
- Rule is abbreviated as u.g.
- P(x) has not been deduced from a hypothesis in which x is a free variable. Also, P(x) has not been deduced using e.i.

- Example

  Show that the following argument is valid.

$$[(\forall x)[P(x) \to Q(x)] \land (\forall x)P(x)] \to (\forall x)Q(x)$$

Existential Generalization

- From P(x) or P(a), you can conclude $(\exists x)P(x)$.
- Rule is abbreviated as e.g.
- To go from P(a) to $(\exists x)P(x)$, x must not appear in P(a). Otherwise, we could derive $(\exists y)P(y, y)$ from P(a, y)! The argument $P(a, y) \rightarrow (\exists y)P(y, y)$ is simply not valid. (Why?)

- Some more examples

$$(\forall x)[P(x) \wedge Q(x)] \rightarrow (\forall x)P(x) \wedge (\forall x)Q(x).$$
$$[(\forall y)[P(x) \rightarrow Q(x, y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x, y)].$$
$$[P(x) \rightarrow (\forall y)Q(x, y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x, y)].$$

Two Laws

- Negation
  - $[(\exists x)A(x)]' \Leftrightarrow (\forall x)[A(x)]'$
  - $[(\forall x)A(x)]' \Leftrightarrow (\exists x)[A(x)]'$

- The proof of the negation rules are tedious and I do not expect you to know them. Feel free to use the above two rules as and when you need them, without proof.

- One more example

  Establish the validity of the following statement:

$$(\forall x)[P(x) \vee Q(x)] \rightarrow [(\exists x)P(x) \vee (\forall x)Q(x)]$$

A Verbal Argument

- Every microcomputer has a serial interface port. Some microcomputers have a parallel port. Therefore, some microcomputers have both a serial and a parallel port. Symbolically,

$$[(\forall x)[M(x) \to S(x)] \wedge (\exists x)[M(x) \wedge P(x)]] \to (\exists x)[M(x) \wedge S(x) \wedge P(x)]$$