

1.

- (1) A' hypothesis
- (2) $B' \rightarrow (C \rightarrow A)$ hypothesis
- (3) $B \rightarrow A$ hypothesis
- (4) B' 1,3,mt
- (5) $C \rightarrow A$ 2,4,mp

You can also use the deduction method to prove this problem.

2.

Using the deduction method, we can rewrite the given argument as:

$$[(\forall x)[P(x) \vee Q(x)] \wedge [(\exists x)P(x)]' \rightarrow (\forall x)Q(x)$$

Consider the following proof sequence:

- (i) $[(\exists x)P(x)]'$ hypothesis.
- (ii) $P(a)'$ (i), e.i.
- (iii) $(\forall x)[P(x) \vee Q(x)]$ hypothesis.
- (iv) $P(a) \vee Q(a)$ (iii), u.i.
- (v) $P(a)' \vee Q(a)$ (iv), implication.
- (vi) $Q(a)$ (ii), (v), Modus Ponens.
- (vii) $(\forall x)Q(x)$ u.g.

The last step is justified, since $Q(a)$ was not deduced from a hypothesis in which a is a free variable nor has $Q(a)$ been deduced by existential instantiation from a formula in which a is a free variable.

3.

Use the same proof from Question 8 of Homework 2. The solution is posted in course website.

4.

- 1. $(\exists x)[W(x) \wedge P(x)]$
- 2. $(\exists x)[W(x) \wedge P(x)']$
- 3. $(\forall n)[W(n) \rightarrow (\exists m)[W(m) \wedge G(m, n)]]$
- 4. $(\exists x)[P(x) \wedge (\forall y)[P(y) \rightarrow G(x, y)]]$