1.

Let a_n = the number of decimal strings of length n with no occurrences of "000".

Case 1: $\frac{\text{9 possibilities}}{\text{can be 1 - 9}} \underbrace{\qquad\qquad}_{\text{length } n-1}$ $9a_{n-1}$ possibilities

Case 2: $\frac{1}{\text{"0"}} \frac{9}{\text{can be 1 - 9}} \underbrace{\qquad\qquad}_{\text{length } n-2}$ $9a_{n-2}$ possibilities

Case 3: $\frac{1}{\text{"0"}} \frac{1}{\text{"0"}} \frac{9}{\text{can be 1 - 9}} \underbrace{\qquad\qquad}_{\text{length } n-3}$ $9a_{n-3}$ possibilities

Answer: $a_n = 9a_{n-1} + 9a_{n-2} + 9a_{n-3}$

2.

(a) Let $X \in \mathcal{P}(A) \cup \mathcal{P}(B)$. It follows that $X \in \mathcal{P}(A)$ or $X \in \mathcal{P}(B)$ or both. Assume that $X \in \mathcal{P}(A)$. It follows that $X \subseteq A$ and hence, $X \subseteq (A \cup B)$. Hence, $X \in \mathcal{P}(A \cup B)$. In similar fashion, we can show that if $X \in \mathcal{P}(B)$, then $X \in \mathcal{P}(A \cup B)$. Thus, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

(b) Observe that,

$$
\begin{aligned}
&(A \cup C) \cap [(A \cap B) \cup (C' \cap B)] \\
&= (A \cup C) \cap [(B \cap A) \cup (B \cap C')] && \text{(commutativity)} \\
&= (A \cup C) \cap [B \cap (A \cup C')] && \text{(distributitivity)} \\
&= (A \cup C) \cap [(A \cup C') \cap B] && \text{(commutativity)} \\
&= [(A \cup C) \cap (A \cup C')] \cap B && \text{(associativity)} \\
&= [A \cap (C \cup C')] \cap B && \text{(distributivity)} \\
&= (A \cap S) \cap B && \text{(complement properties)} \\
&= (A \cap B) && \text{(identity properties)}
\end{aligned}
$$

□

3.

Solution: Let

(a) A denote the set of people with checking accounts,

(b) B denote the set of people with regular savings accounts, and

(c) C denote the set of people with money market savings accounts.

We have that,

(a) $|A \cup B \cup C| = 214$,

(b) $|A| = 189$,

(c) $|B| = 73$,

(d) $|C| = 114$,

(e) $|A \cap B| = 69$, and

(f) $|B \cap C| = 0$

We are required to calculate $|A \cap C|$. From the principle of inclusion and exclusion, we know that,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Since $|B \cap C| = 0$, it trivially follows that $|A \cap B \cap C| = 0$ and hence, $|A \cap C| = 93$. □

4.

Notation: "E" = "even". "-" means "any digit".
Let A be the set of decimal strings of the form "EEE--".
Let B be the set of decimal strings of the form "0----".
Let C be the set of decimal strings of the form "--EEE".

$$|A|: \quad \frac{5}{E}\frac{5}{E}\frac{5}{E}\frac{10}{\text{any}}\frac{10}{\text{any}} \qquad \text{Product} = 12{,}500$$

$$|B|: \quad \frac{1}{\text{"0"}}\frac{10}{\text{any}}\frac{10}{\text{any}}\frac{10}{\text{any}}\frac{10}{\text{any}} \qquad \text{Product} = 10{,}000$$

$$|C|: \quad \frac{10}{\text{any}}\frac{10}{\text{any}}\frac{5}{E}\frac{5}{E}\frac{5}{E} \qquad \text{Product} = 12{,}500 \text{ (symmetric to } |A| \text{ case)}$$

$$|A \cap B|: \quad \frac{1}{\text{"0"}}\frac{5}{E}\frac{5}{E}\frac{10}{\text{any}}\frac{10}{\text{any}} \qquad \text{Product} = 2{,}500$$

$$|B \cap C|: \quad \frac{1}{\text{"0"}}\frac{10}{\text{any}}\frac{5}{E}\frac{5}{E}\frac{5}{E} \qquad \text{Product} = 1{,}250$$

$$|A \cap C|: \quad \frac{5}{E}\frac{5}{E}\frac{5}{E}\frac{5}{E}\frac{5}{E} \qquad \text{Product} = 3{,}125$$

$$|A \cap B \cap C|: \quad \frac{1}{\text{"0"}}\frac{5}{E}\frac{5}{E}\frac{5}{E}\frac{5}{E} \qquad \text{Product} = 625$$

So, $|A \cup B \cup C|$ = 12,500 + 10,000 + 12,500 - 2,500 - 1,250 - 3,125 + 625
Answer: 28,750.