1.

Solution: This is a circular permutation problem, as opposed to a linear permutation problem. Note that n people can be seated at a regular table in P(n, n) = n! distinct ways. However, the situation is somewhat different when the seating is around a circular table. This is because circular shifts of a straight line permutation are indistinguishable from each other, in a circular table. Observe that corresponding to each circular permutation of n people, there are precisely n straight line permutations, since we can shift a permutation n times, without altering its circular structure. In other words the n linear shifts of a given linear permutation give rise to the same circular permutation. It follows that the total number of distinct circular permutations of n people is $\frac{n!}{n} = (n-1)!$.

The answer to our specific question is therefore 5! = 120. \Box

2

Solution: First note, that absent any restriction, there are 19! distinct linear arrangements of the men and women.

- (a) We have two choices, viz,, the men follow the women or the women follow the men. In each case, the men can be permuted in 11! ways among themselves, while the women can be permuted in 8! ways among themselves. So the total number of possible linear arrangements is 2 · (11! · 8!).
- (b) We need to break up our task into three sub-tasks, viz., seating the men, selecting the slots for the women, and seating the women. Observe that the first sub-task can be carried out in 11! ways. Having accomplished the first sub-task, the second sub-task can be accomplished in C(12, 8) ways, since there are 12 possible slots that are created by the seating of 11 men in a row and we need to select 8 of these. Finally, the women can be permuted in the chosen slots in 8! ways. Using the multiplication principle, the total number of ways of seating the men and women so that no two women are seated together is $11! \cdot C(12, 8) \cdot 8!$.

3.

Reflexive:

R is not reflexive, because (4,4) does not belong to R, but 4 belongs to A

Symmetric:

R is not symmetric, because (3,5) does not belong to R, but (5,3) belongs to R.

Anti-symmetric:

R is not anti-symmetric, because (1,3) and (3,1) belong to R, but 1 is not equal to 3

Transitive:

R is not transitive, because (3,1) and (1,5) belong to R, but (3,5) does not belong to R.

4.

R U {(3,2), (2,2),(2,1),(3,3)}