CSE 4308 / CSE 5360 - *Artificial Intelligence I*

Homework 5- Fall 2013

Sample Solution

Problems marked with a * are required only for students in the graduate section (CSE 5360). They will be graded for extra credit for students of CSE 4308.

Bayes Rule

1. Disease D affects 1% of the population ($P(D) = 0.01$) and shows symptom S in 40% of all cases ($P(S \mid D) = 0.4$) while the same symptom can be observed only in 2% of people who do not have the disease. There also is a test T for the disease that gives the correct diagnosis (A or $\neg A$) in 95% of all cases. Given that you do experience symptoms and your test comes back positive, how high is the probability that you have the disease ?

   $P(D \mid S, A) = \frac{P(S \wedge A \mid D)P(D)}{P(S \wedge A)} = \frac{P(A \mid D)P(S \mid A, D)P(D)}{P(S \wedge A)}$

   Assuming the test does not look for the symptoms we can assume conditional independence of symptoms from the test results we can drop the test result from the test condition:

   $= \frac{P(A \mid D)P(S \mid D)P(D)}{P(S \wedge A)} = \frac{0.95 * 0.4 * 0.01}{P(S \wedge A)} = \frac{0.0038}{P(S \wedge A)}$

   Doing the same for not being sick we get:

   $P(\neg D \mid S, A) = \frac{P(S \wedge A \mid \neg D)P(\neg D)}{P(S \wedge A)} = \frac{P(A \mid \neg D)P(S \mid A, \neg D)P(\neg D)}{P(S \wedge A)} = \frac{P(A \mid \neg D)P(S \mid \neg D)P(\neg D)}{P(S \wedge A)}$

   $= \frac{0.05 * 0.02 * 0.99}{P(S \wedge A)} = \frac{0.00099}{P(S \wedge A)}$

   For Normalization:

   $P(D \mid S, A) + P(\neg D \mid S, A) = \frac{0.0038}{P(S \wedge A)} + \frac{0.00099}{P(S \wedge A)} = \frac{0.00479}{P(S \wedge A)} = 1$

   Thus:

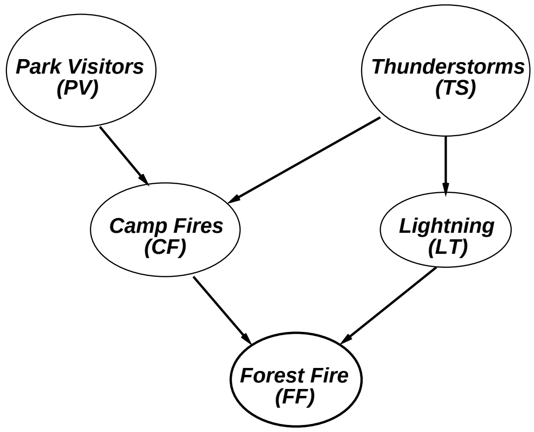
   $P(S \wedge A) = 0.00479$

   Thus the final answer is:

   $P(D \mid S, A) = \frac{0.0038}{0.00479} = 0.793$

Bayesian Networks

2. Below is a probabilistic network to assess the probability of forest fires in a state park. The goal is to use the available information about the presence of visitors in the park and about thunderstorms to predict the probability of a forest fire. According to this model, the probability of a forest fire (FF) depends on camp fires (CF) and on lightning (LT). The probability of camp fires, in turn, depends on the presence of park visitors (PV) and thunderstorms (TS). Similarly, lightning is influenced by the presence of thunderstorms.



a) Use the following conditional probabilities to determine the conditional probabilities of a forest fire for all possible, observable scenarios (i.e. $PV \wedge TS$, $PV \wedge \neg TS$, $\neg PV \wedge TS$, and $\neg PV \wedge \neg TS$). Make sure to show your calculations.

$$
\begin{aligned}
CF: \quad & P(CF \mid PV \wedge TS) = 0.3 \\
& P(CF \mid \neg PV \wedge TS) = 0.01 \\
& P(CF \mid PV \wedge \neg TS) = 0.7 \\
& P(CF \mid \neg PV \wedge \neg TS) = 0.01 \\
LT: \quad & P(LT \mid TS) = 0.25 \\
& P(LT \mid \neg TS) = 0.02 \\
FF: \quad & P(FF \mid CF \wedge LT) = 0.3 \\
& P(FF \mid \neg CF \wedge LT) = 0.2 \\
& P(FF \mid CF \wedge \neg LT) = 0.2 \\
& P(FF \mid \neg CF \wedge \neg LT) = 0.01
\end{aligned}
$$

$$
\begin{aligned}
& P(FF \mid PV \wedge TS) \\
& \quad = P(FF \mid CF \wedge LT) * P(CF \wedge LT \mid PV \wedge TS) \\
& \quad + P(FF \mid CF \wedge \neg LT) * P(CF \wedge \neg LT \mid PV \wedge TS) \\
& \quad + P(FF \mid \neg CF \wedge LT) * P(\neg CF \wedge LT \mid PV \wedge TS) \\
& \quad + P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \wedge \neg LT \mid PV \wedge TS) \\
& \quad = P(FF \mid CF \wedge LT) * P(CF \mid PV \wedge TS) * P(LT \mid TS) \\
& \quad + P(FF \mid CF \wedge \neg LT) * P(CF \mid PV \wedge TS) * P(\neg LT \mid TS) \\
& \quad + P(FF \mid \neg CF \wedge LT) * P(\neg CF \mid PV \wedge TS) * P(LT \mid TS) \\
& \quad + P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \mid PV \wedge TS) * P(\neg LT \mid TS) \\
& \quad = 0.3 * 0.3 * 0.25 + 0.2 * 0.3 * 0.75 + 0.2 * 0.7 * 0.25 + 0.01 * 0.7 * 0.75 = 0.10775
\end{aligned}
$$

$$
\begin{aligned}
& P(FF \mid PV \wedge \neg TS) \\
& \quad = P(FF \mid CF \wedge LT) * P(CF \wedge LT \mid PV \wedge \neg TS) \\
& \quad + P(FF \mid CF \wedge \neg LT) * P(CF \wedge \neg LT \mid PV \wedge \neg TS) \\
& \quad + P(FF \mid \neg CF \wedge LT) * P(\neg CF \wedge LT \mid PV \wedge \neg TS) \\
& \quad + P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \wedge \neg LT \mid PV \wedge \neg TS) \\
& \quad = P(FF \mid CF \wedge LT) * P(CF \mid PV \wedge \neg TS) * P(LT \mid \neg TS)
\end{aligned}
$$

$$
\begin{aligned}
&+ P(FF \mid CF \wedge \neg LT) * P(CF \mid PV \wedge \neg TS) * P(\neg LT \mid \neg TS) \\
&+ P(FF \mid \neg CF \wedge LT) * P(\neg CF \mid PV \wedge \neg TS) * P(LT \mid \neg TS) \\
&+ P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \mid PV \wedge \neg TS) * P(\neg LT \mid \neg TS) \\
&= 0.3 * 0.7 * 0.02 + 0.2 * 0.7 * 0.98 + 0.2 * 0.3 * 0.02 + 0.01 * 0.3 * 0.98 = 0.14554
\end{aligned}
$$

$$
\begin{aligned}
&P(FF \mid \neg PV \wedge TS) \\
&= P(FF \mid CF \wedge LT) * P(CF \wedge LT \mid \neg PV \wedge TS) \\
&+ P(FF \mid CF \wedge \neg LT) * P(CF \wedge \neg LT \mid \neg PV \wedge TS) \\
&+ P(FF \mid \neg CF \wedge LT) * P(\neg CF \wedge LT \mid \neg PV \wedge TS) \\
&+ P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \wedge \neg LT \mid \neg PV \wedge TS) \\
&= P(FF \mid CF \wedge LT) * P(CF \mid \neg PV \wedge TS) * P(LT \mid TS) \\
&+ P(FF \mid CF \wedge \neg LT) * P(CF \mid \neg PV \wedge TS) * P(\neg LT \mid TS) \\
&+ P(FF \mid \neg CF \wedge LT) * P(\neg CF \mid \neg PV \wedge TS) * P(LT \mid TS) \\
&+ P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \mid \neg PV \wedge TS) * P(\neg LT \mid TS) \\
&= 0.3 * 0.01 * 0.25 + 0.2 * 0.01 * 0.75 + 0.2 * 0.99 * 0.25 + 0.01 * 0.99 * 0.75 = 0.059175
\end{aligned}
$$

$$
\begin{aligned}
&P(FF \mid \neg PV \wedge \neg TS) \\
&= P(FF \mid CF \wedge LT) * P(CF \wedge LT \mid \neg PV \wedge \neg TS) \\
&+ P(FF \mid CF \wedge \neg LT) * P(CF \wedge \neg LT \mid \neg PV \wedge \neg TS) \\
&+ P(FF \mid \neg CF \wedge LT) * P(\neg CF \wedge LT \mid \neg PV \wedge \neg TS) \\
&+ P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \wedge \neg LT \mid \neg PV \wedge \neg TS) \\
&= P(FF \mid CF \wedge LT) * P(CF \mid \neg PV \wedge \neg TS) * P(LT \mid \neg TS) \\
&+ P(FF \mid CF \wedge \neg LT) * P(CF \mid \neg PV \wedge \neg TS) * P(\neg LT \mid \neg TS) \\
&+ P(FF \mid \neg CF \wedge LT) * P(\neg CF \mid \neg PV \wedge \neg TS) * P(LT \mid \neg TS) \\
&+ P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \mid \neg PV \wedge \neg TS) * P(\neg LT \mid \neg TS) \\
&= 0.3 * 0.01 * 0.02 + 0.2 * 0.01 * 0.98 + 0.2 * 0.99 * 0.02 + 0.01 * 0.99 * 0.98 = 0.015682
\end{aligned}
$$

b) The same model is used in a different park with different weather patterns. Over time, the park service has determined that the conditional probabilities for forest fire, and campfire in this model are the same as above. Through observations, it has also been determined that forest fires occur with the following conditional probabilities:

$P(FF \mid \neg PV \wedge TS) = 0.05$

$P(FF \mid \neg PV \wedge \neg TS) = 0.015$

Calculate the conditional probabilities for lightning in the probabilistic model (i.e. $P(LT \mid TS)$ and $P(LT \mid \neg TS)$). Show your calculations.

$$
\begin{aligned}
&P(FF \mid \neg PV \wedge TS) \\
&= P(FF \mid CF \wedge LT) * P(CF \mid \neg PV \wedge TS) * P(LT \mid TS) \\
&+ P(FF \mid CF \wedge \neg LT) * P(CF \mid \neg PV \wedge TS) * P(\neg LT \mid TS) \\
&+ P(FF \mid \neg CF \wedge LT) * P(\neg CF \mid \neg PV \wedge TS) * P(LT \mid TS) \\
&+ P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \mid \neg PV \wedge TS) * P(\neg LT \mid TS) \\
&= 0.3 * 0.01 * P(LT \mid TS) + 0.2 * 0.01 * (1 - P(LT \mid TS)) \\
&+ 0.2 * 0.99 * P(LT \mid TS) + 0.01 * 0.99 * (1 - P(LT \mid TS)) \\
&= P(LT \mid TS)(0.003 - 0.002 + 0.198 - 0.0099) + (0.002 + 0.0099) \\
&= P(LT \mid TS) * 0.1891 + 0.0119 = 0.05
\end{aligned}
$$

$$P(LT \mid TS) = \frac{0.05 - 0.0119}{0.1891} = 0.20148$$

$$
\begin{aligned}
&P(FF \mid \neg PV \wedge \neg TS) \\
&\quad = P(FF \mid CF \wedge LT) * P(CF \mid \neg PV \wedge \neg TS) * P(LT \mid \neg TS) \\
&\quad + P(FF \mid CF \wedge \neg LT) * P(CF \mid \neg PV \wedge \neg TS) * P(\neg LT \mid \neg TS) \\
&\quad + P(FF \mid \neg CF \wedge LT) * P(\neg CF \mid \neg PV \wedge \neg TS) * P(LT \mid \neg TS) \\
&\quad + P(FF \mid \neg CF \wedge \neg LT) * P(\neg CF \mid \neg PV \wedge \neg TS) * P(\neg LT \mid \neg TS) \\
&\quad = 0.3 * 0.01 * P(LT \mid \neg TS) + 0.2 * 0.01 * (1 - P(LT \mid \neg TS)) \\
&\quad + 0.2 * 0.99 * P(LT \mid \neg TS) + 0.01 * 0.99 * (1 - P(LT \mid \neg TS)) \\
&\quad = P(LT \mid \neg TS)(0.003 - 0.002 + 0.198 - 0.0099) + (0.002 + 0.0099) \\
&\quad = P(LT \mid \neg TS) * 0.1891 + 0.0119 = 0.015 \\
&P(LT \mid \neg TS) = \frac{0.015 - 0.0119}{0.1891} = 0.0163934
\end{aligned}
$$

3.* Use the AIspace Belief and Decision network solver (you can download it from http://www.aispace.org/downloads.shtml or run it over the web from there) to implement the Bayesian network from Problem 2 a) - with the addition of the prior probabilities for the two root nodes $P(PV) = 0.75$, $P(TS) = 0.1$ - and perform inference on the network. Submit your network in .bif format and show the inference results for the following probabilities: $P(PV \mid FF \wedge \neg TS)$, $P(TS \mid CF \wedge \neg FF)$, $P(PV \mid \neg FF \wedge TS)$.

```
network Untitled {
  property short = ;
  property detailed = ;
}
variable TS {
  type discrete [2]{T, F};
  property position = (7296.0, 5107.0);
}
variable CF {
  type discrete [2]{T, F};
  property position = (7161.0, 5199.0);
}
variable LT {
  type discrete [2]{T, F};
  property position = (7308.0, 5201.0);
}
variable FF {
  type discrete [2]{T, F};
  property position = (7250.0, 5279.0);
}
variable PV {
  type discrete [2]{T, F};
  property position = (7109.0, 5107.0);
}
probability (TS) {
  table 0.1, 0.9;
}
```

```
probability (CF | TS, PV) {
  (T, T) 0.3, 0.7;
  (T, F) 0.01, 0.99;
  (F, T) 0.7, 0.3;
  (F, F) 0.01, 0.99;
}
probability (LT | TS) {
  (T) 0.25, 0.75;
  (F) 0.02, 0.98;
}
probability (FF | CF, LT) {
  (T, T) 0.3, 0.7;
  (T, F) 0.2, 0.8;
  (F, T) 0.2, 0.8;
  (F, F) 0.01, 0.99;
}
probability (PV) {
  table 0.75, 0.25;
}
```

$P(PV \mid FF \wedge \neg TS) = 0.96533$
$P(TS \mid CF \wedge \neg FF) = 0.04447$
$P(PV \mid \neg FF \wedge TS) = 0.73993$