# Problem solving and search

## Chapter 3

# Outline

◇ Problem-solving agents

◇ Problem types

◇ Problem formulation

◇ Example problems

◇ Basic search algorithms

# Problem-solving agents

Restricted form of general agent:

```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH( problem)
    action ← RECOMMENDATION(seq, state)
    seq ← REMAINDER(seq, state)
    return action
```

Note: this is offline problem solving; solution executed "eyes closed."
Online problem solving involves acting without complete knowledge.

# Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:

be in Bucharest

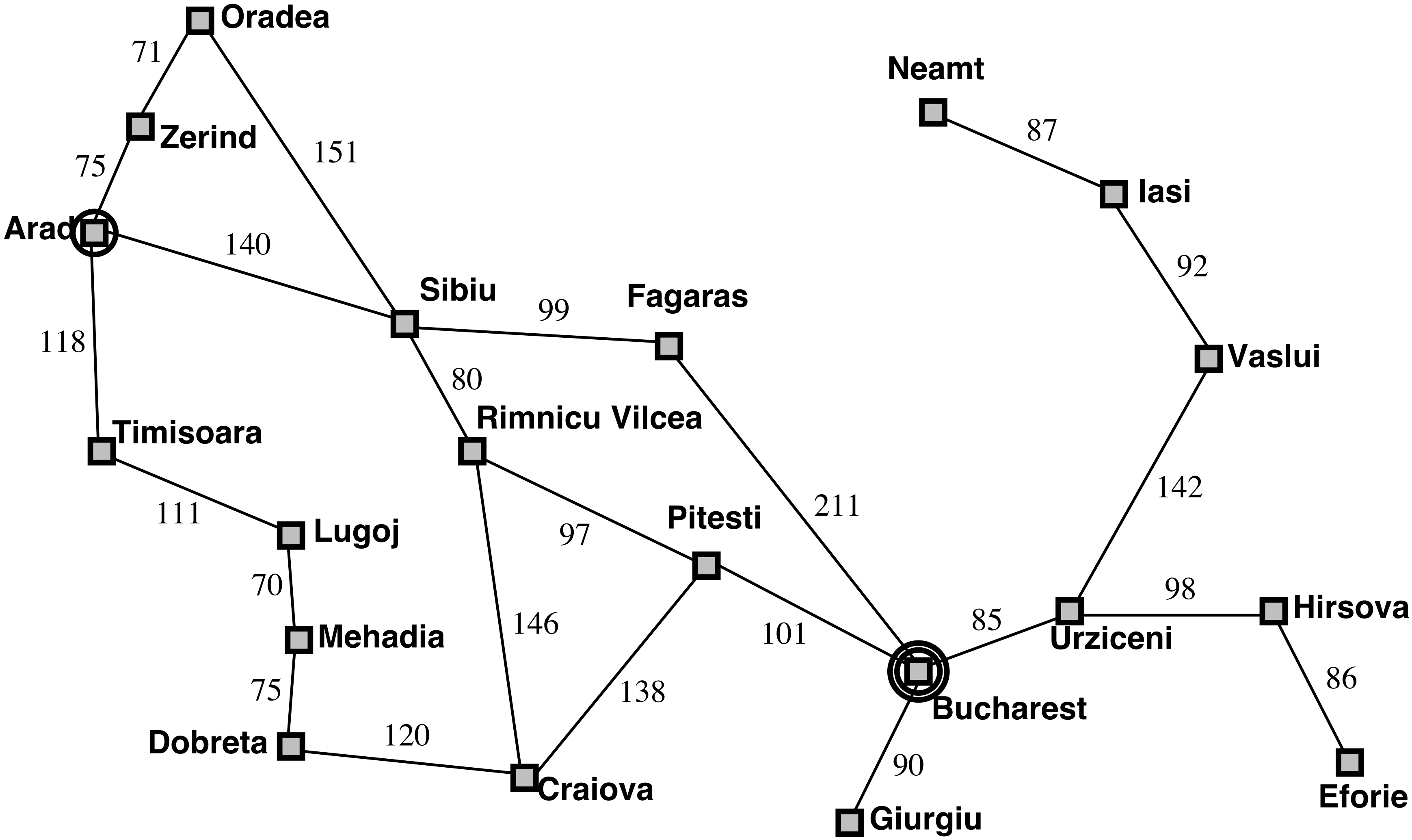
Formulate problem:

states: various cities
actions: drive between cities

Find solution:

sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

# Example: Romania



# Problem types

Deterministic, fully observable $\Longrightarrow$ single-state problem

Agent knows exactly which state it will be in; solution is a sequence

Non-observable $\Longrightarrow$ conformant problem

Agent may have no idea where it is; solution (if any) is a sequence

Nondeterministic and/or partially observable $\Longrightarrow$ contingency problem

percepts provide **new** information about current state
solution is a contingent plan or a policy
often **interleave** search, execution

Unknown state space $\Longrightarrow$ exploration problem ("online")

# Single-state problem formulation

A problem is defined by four items:

initial state    e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs
  e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind\rangle, \ldots\}$

goal test, can be
  explicit, e.g., $x$ = "at Bucharest"
  implicit, e.g., $NoDirt(x)$

path cost (additive)
  e.g., sum of distances, number of actions executed, etc.
  $c(x, a, y)$ is the step cost, assumed to be $\geq 0$

A solution is a sequence of actions
leading from the initial state to a goal state

# Selecting a state space

Real world is absurdly complex
$\Rightarrow$ state space must be **abstracted** for problem solving

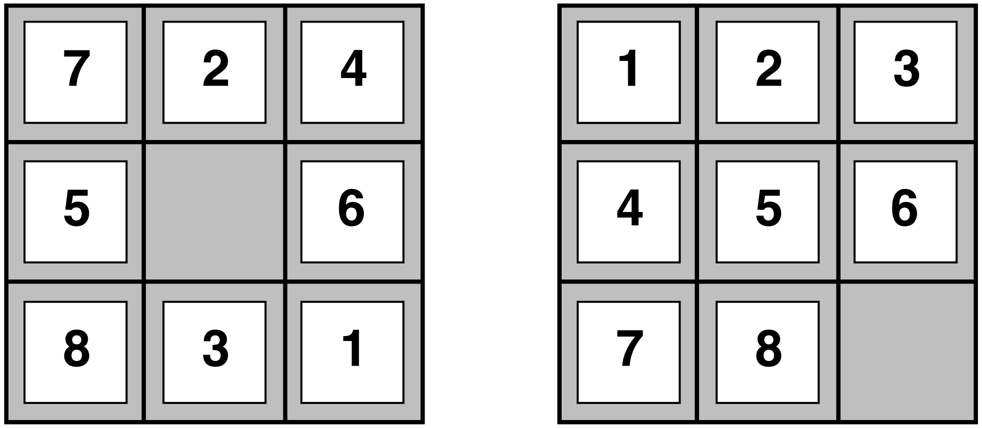
(Abstract) state = set of real states

(Abstract) action = complex combination of real actions
e.g., "Arad $\rightarrow$ Zerind" represents a complex set
of possible routes, detours, rest stops, etc.
For guaranteed realizability, **any** real state "in Arad"
must get to some real state "in Zerind"

(Abstract) solution =
set of real paths that are solutions in the real world

Each abstract action should be "easier" than the original problem!

# Example: The 8-puzzle




**Start State** **Goal State**

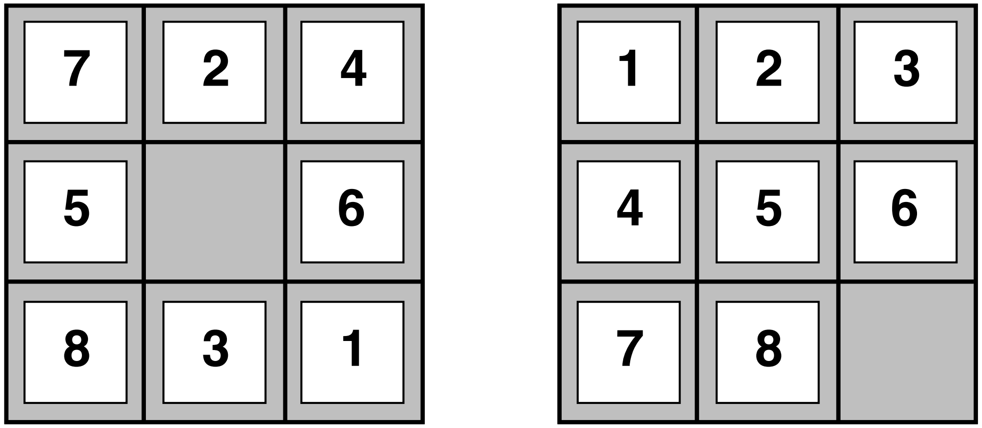
states??

actions??

goal test??

path cost??

# Example: The 8-puzzle




**Start State** **Goal State**

states??: integer locations of tiles (ignore intermediate positions)
actions??
goal test??
path cost??

# Example: The 8-puzzle

| | | |
|---|---|---|
| 7 | 2 | 4 |
| 5 | | 6 |
| 8 | 3 | 1 |

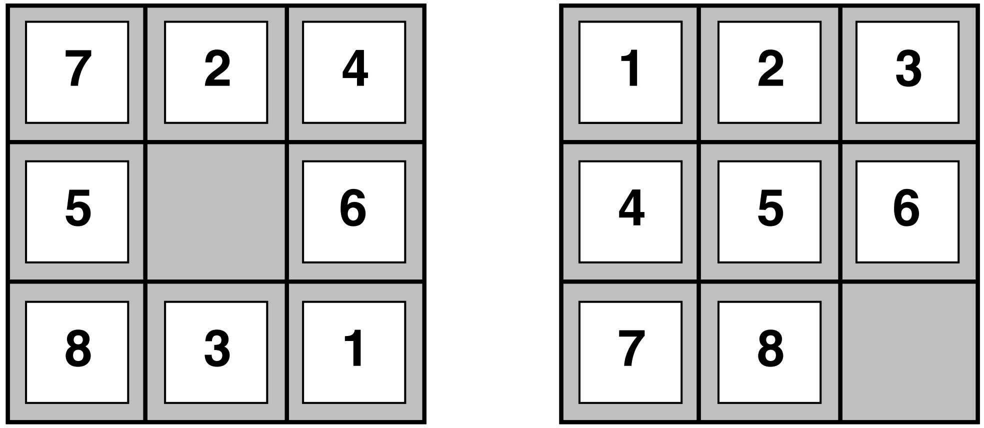
**Start State**

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | |

**Goal State**

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??
path cost??

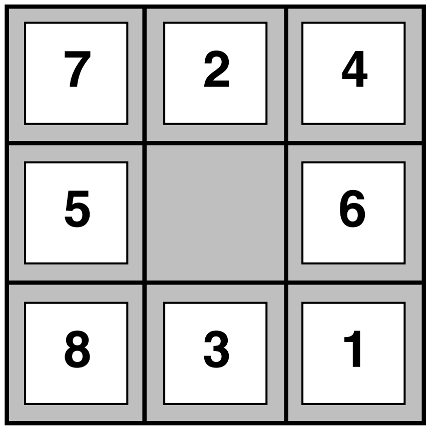
# Example: The 8-puzzle



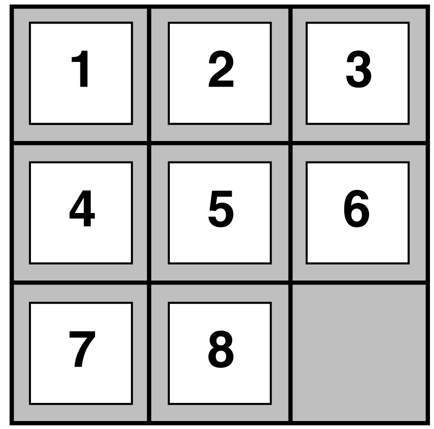

Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??

# Example: The 8-puzzle



Start State

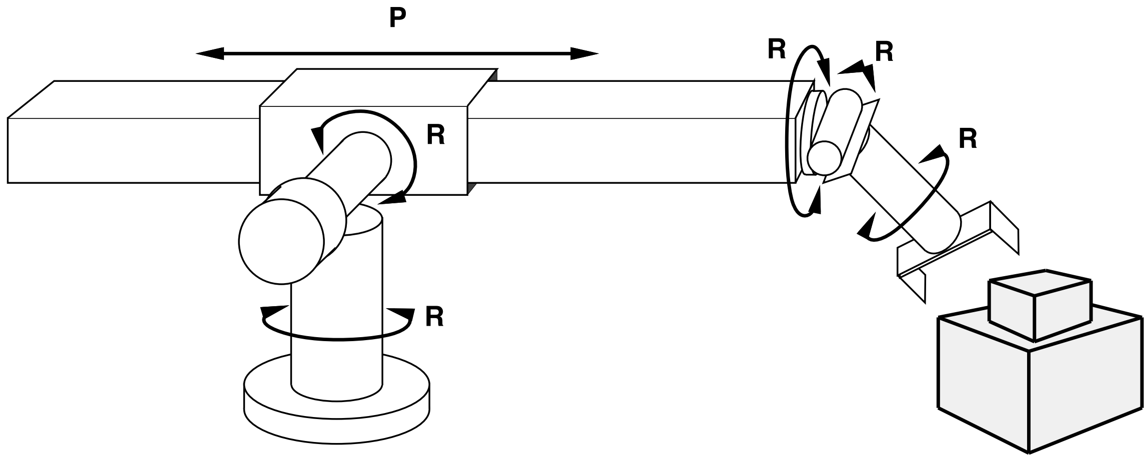


Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: $=$ goal state (given)
path cost??: 1 per move

[Note: optimal solution of $n$-Puzzle family is NP-hard]

# Example: robotic assembly




states??: real-valued coordinates of robot joint angles
parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly **with no robot included!**

path cost??: time to execute

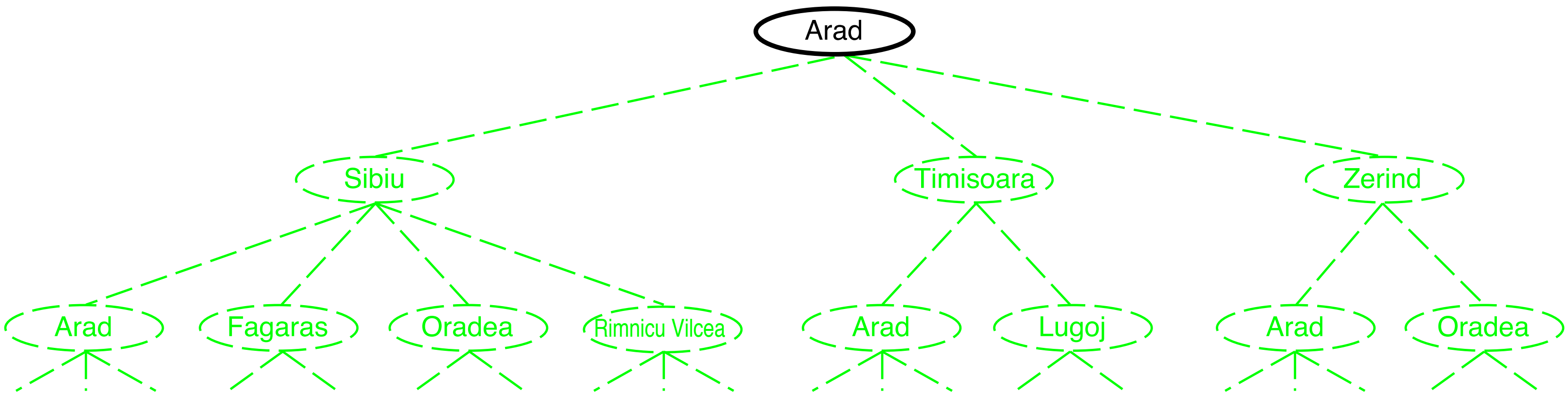
# Tree search algorithms

Basic idea:

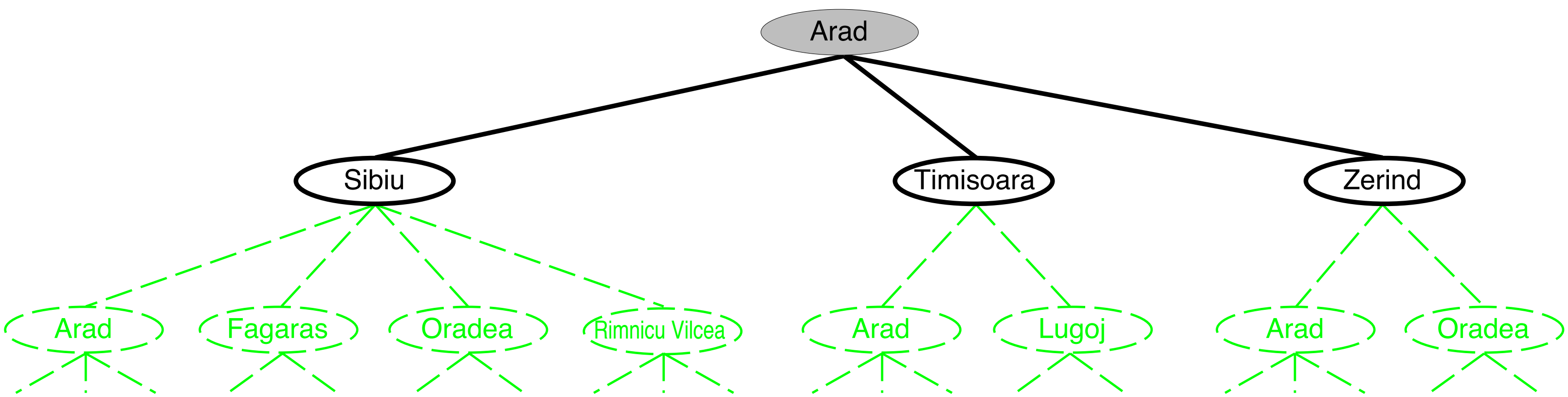
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
  end
```

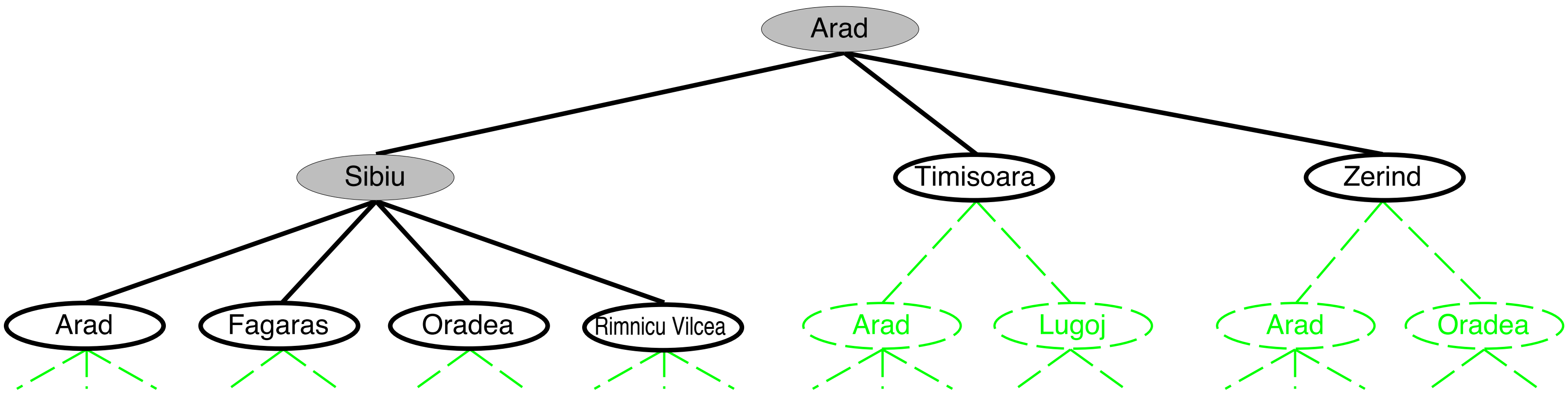
# Tree search example



# Tree search example

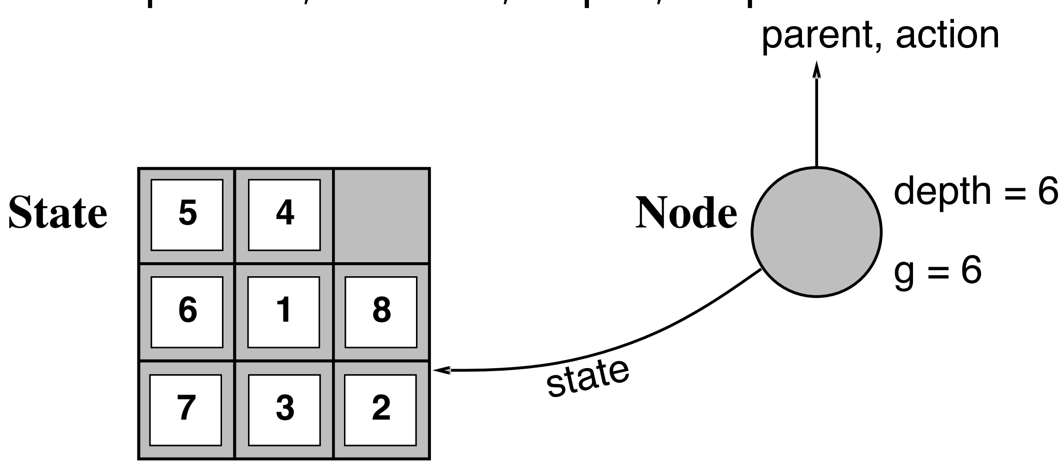


# Tree search example



# Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes parent, children, depth, path cost $g(x)$
States do not have parents, children, depth, or path cost!




The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

# Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST(problem, STATE(node)) then return node
       fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

---

```
function EXPAND( node, problem) returns a set of nodes
   successors ← the empty set
   for each action, result in SUCCESSOR-FN(problem, STATE[node]) do
       s ← a new NODE
       PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
       PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
       DEPTH[s] ← DEPTH[node] + 1
       add s to successors
   return successors
```

# Search strategies

A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- $b$—maximum branching factor of the search tree
- $d$—depth of the least-cost solution
- $m$—maximum depth of the state space (may be $\infty$)

# Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

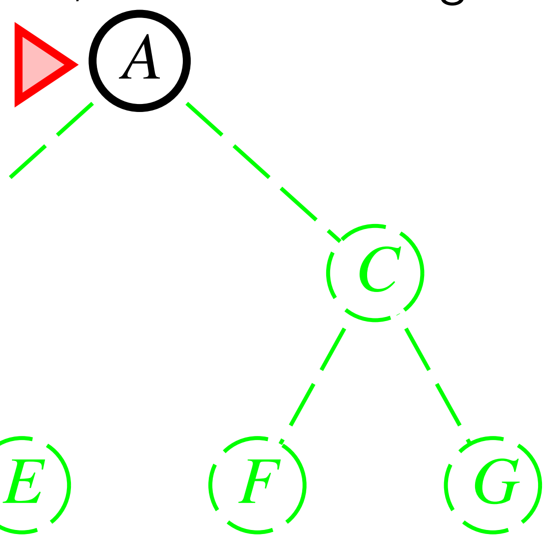
Iterative deepening search

# Breadth-first search

Expand shallowest unexpanded node

**Implementation**:

*fringe* is a FIFO queue, i.e., new successors go at end



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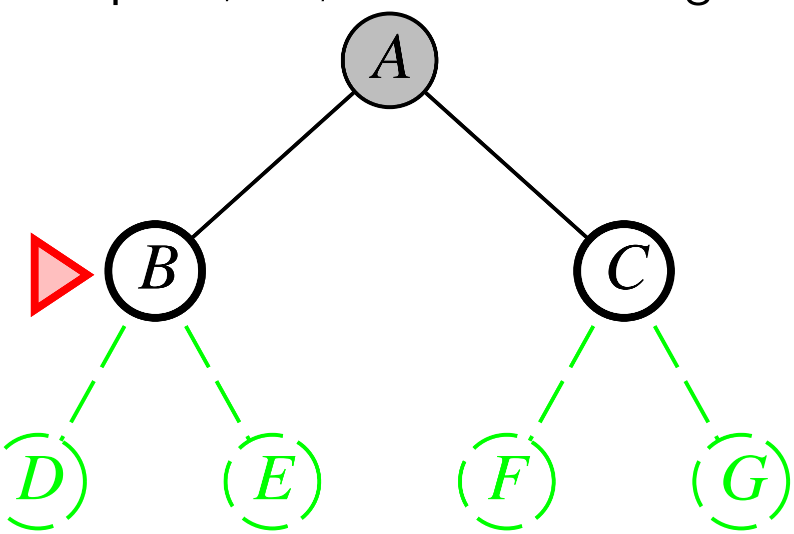

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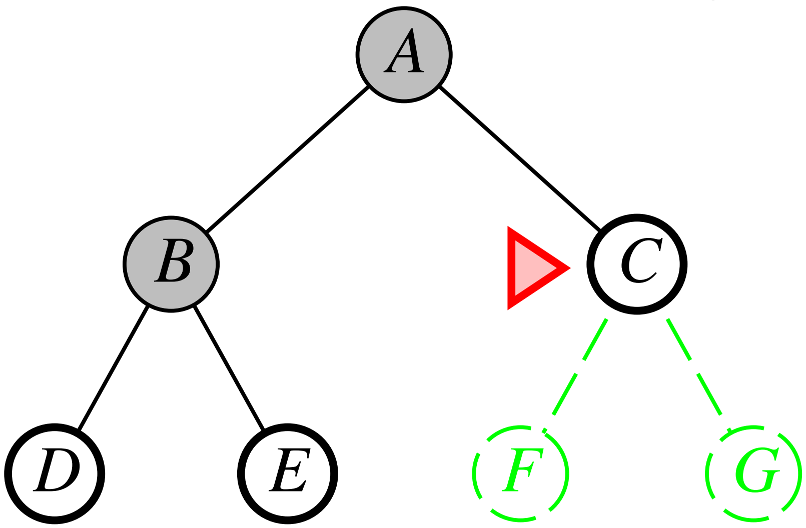

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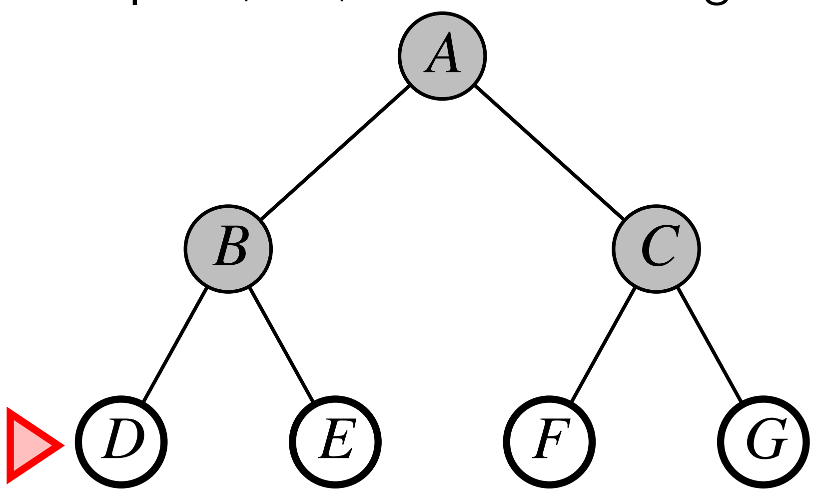

# Properties of breadth-first search

Complete??

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Complete?? Yes (if $b$ is finite)

Time??

# Properties of breadth-first search

Complete?? Yes (if $b$ is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in $d$

Space??

# Properties of breadth-first search

Complete?? Yes (if $b$ is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in $d$

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal??

# Properties of breadth-first search

Complete?? Yes (if $b$ is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in $d$

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

**Space** is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.

# Uniform-cost search

Expand least-cost unexpanded node

**Implementation**:
$fringe =$ queue ordered by path cost, lowest first

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
where $C^*$ is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

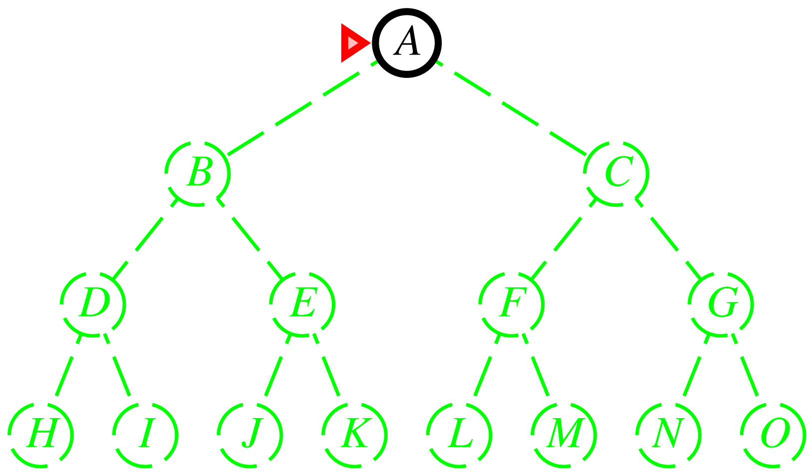
Optimal?? Yes—nodes expanded in increasing order of $g(n)$

# Depth-first search

Expand deepest unexpanded node

**Implementation**:

$fringe$ = LIFO queue, i.e., put successors at front



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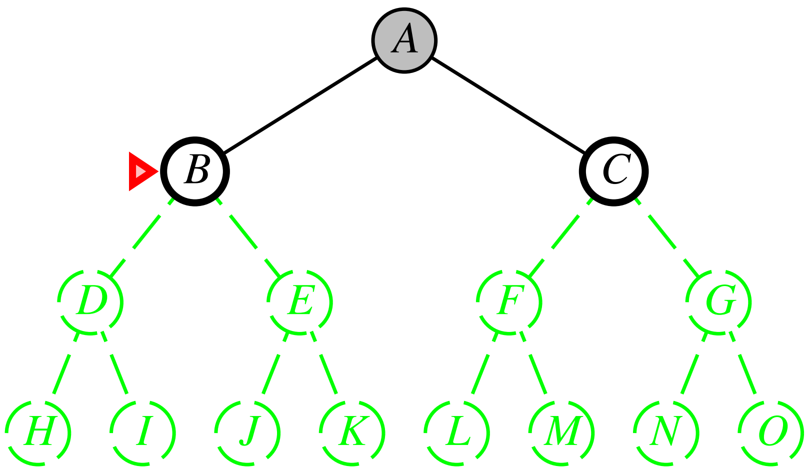

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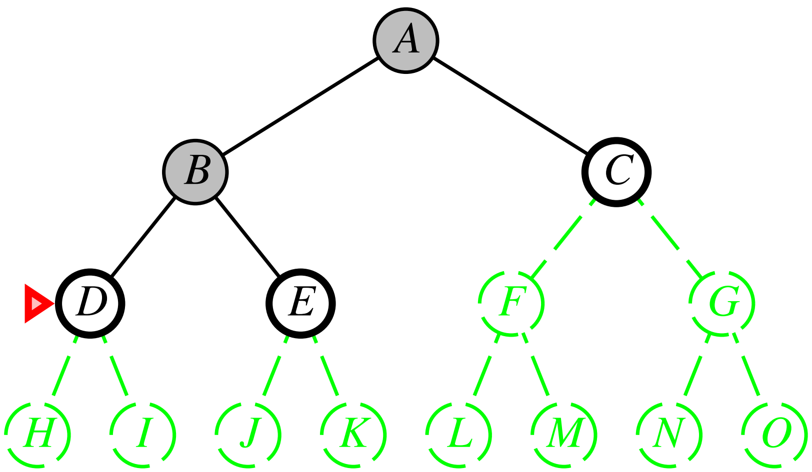

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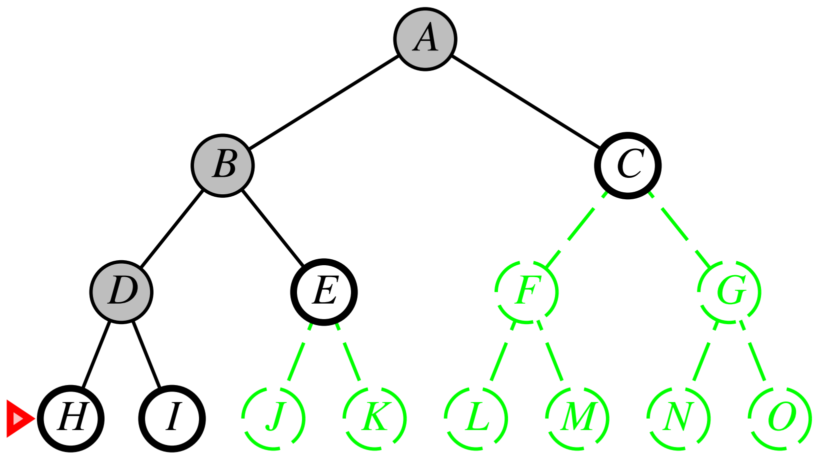

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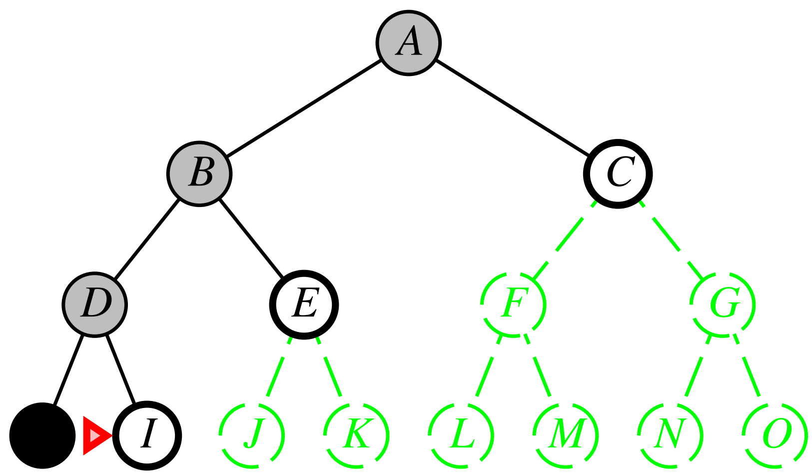

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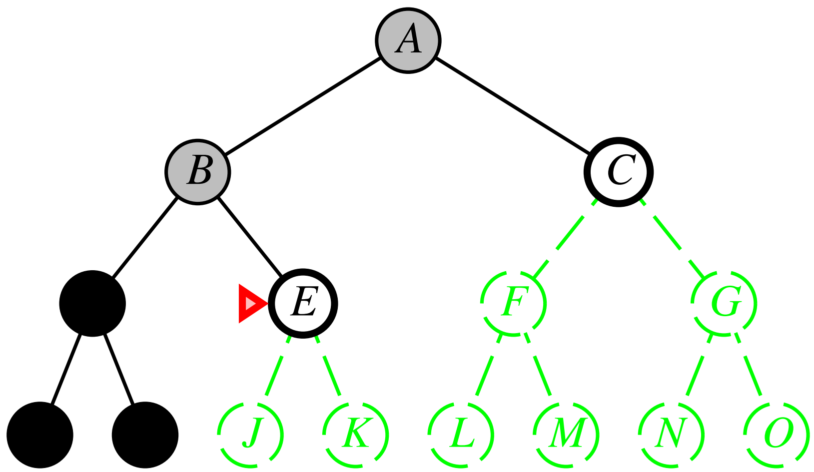

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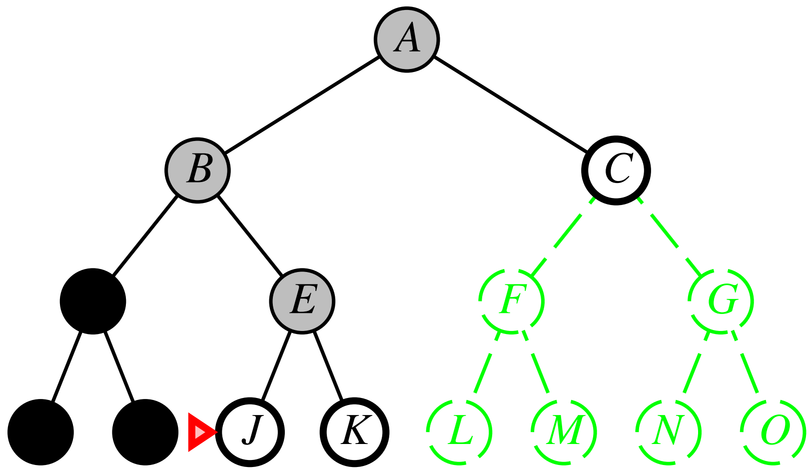

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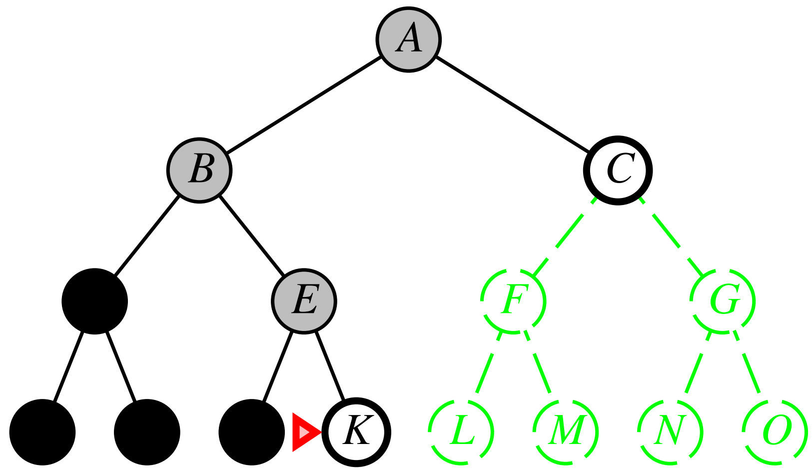

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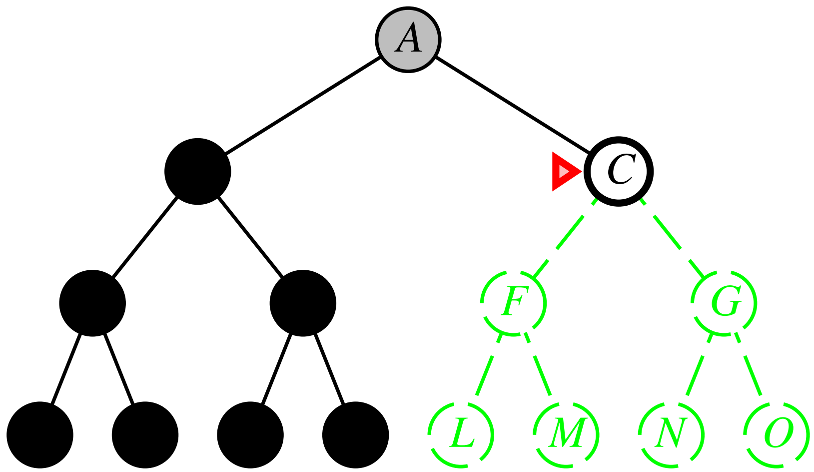

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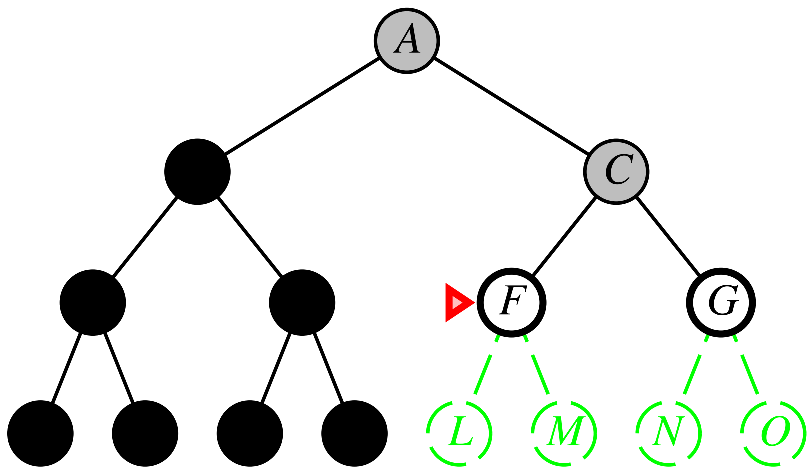

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Expand deepest unexpanded node

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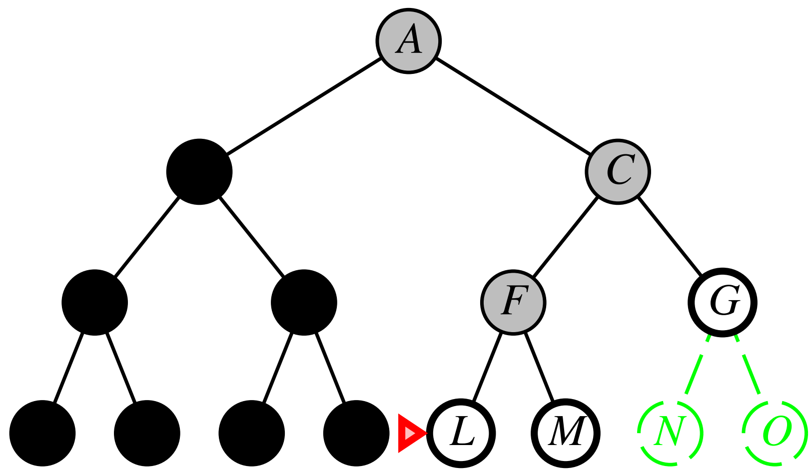

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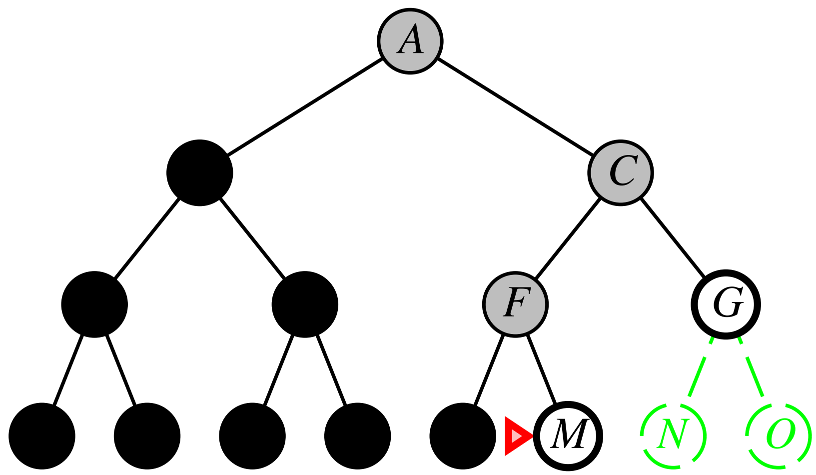

# Properties of depth-first search

Complete??

# Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time??

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Time?? $O(b^m)$: terrible if $m$ is much larger than $d$
but if solutions are dense, may be much faster than breadth-first

Space??

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Time?? $O(b^m)$: terrible if $m$ is much larger than $d$
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal??

# Properties of depth-first search

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Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if $m$ is much larger than $d$
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No

# Depth-limited search

= depth-first search with depth limit $l$,
i.e., nodes at depth $l$ have no successors

**Recursive implementation**:

```
function Depth-Limited-Search( problem, limit) returns soln/fail/cutoff
   Recursive-DLS(Make-Node(Initial-State[problem]), problem, limit)

function Recursive-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if Goal-Test(problem, State[node]) then return node
   else if Depth[node] = limit then return cutoff
   else for each successor in Expand(node, problem) do
        result ← Recursive-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

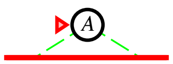
# Iterative deepening search

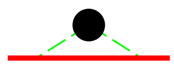
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

# Iterative deepening search $l = 0$

Limit = 0

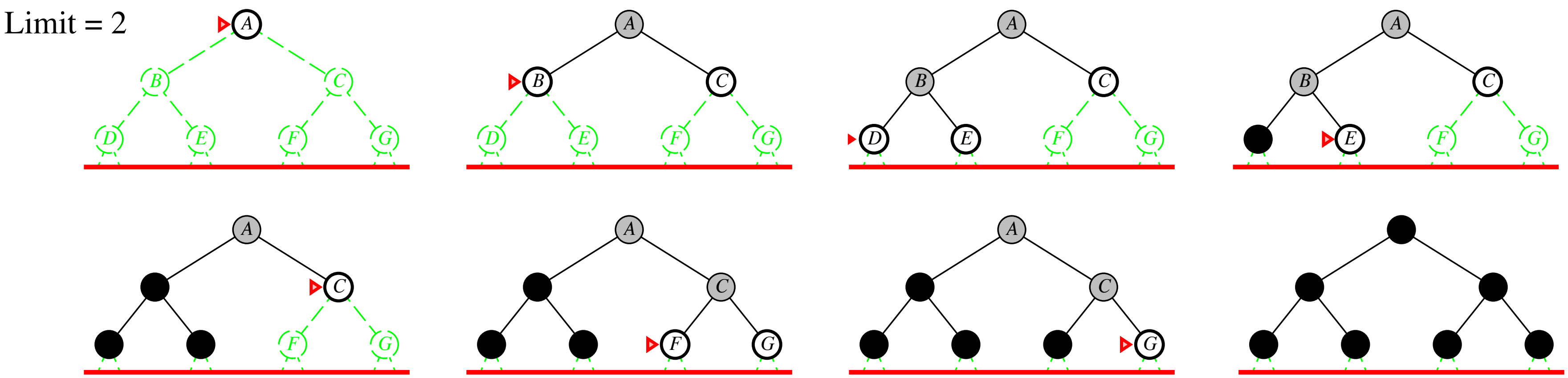


# Iterative deepening search $l = 1$

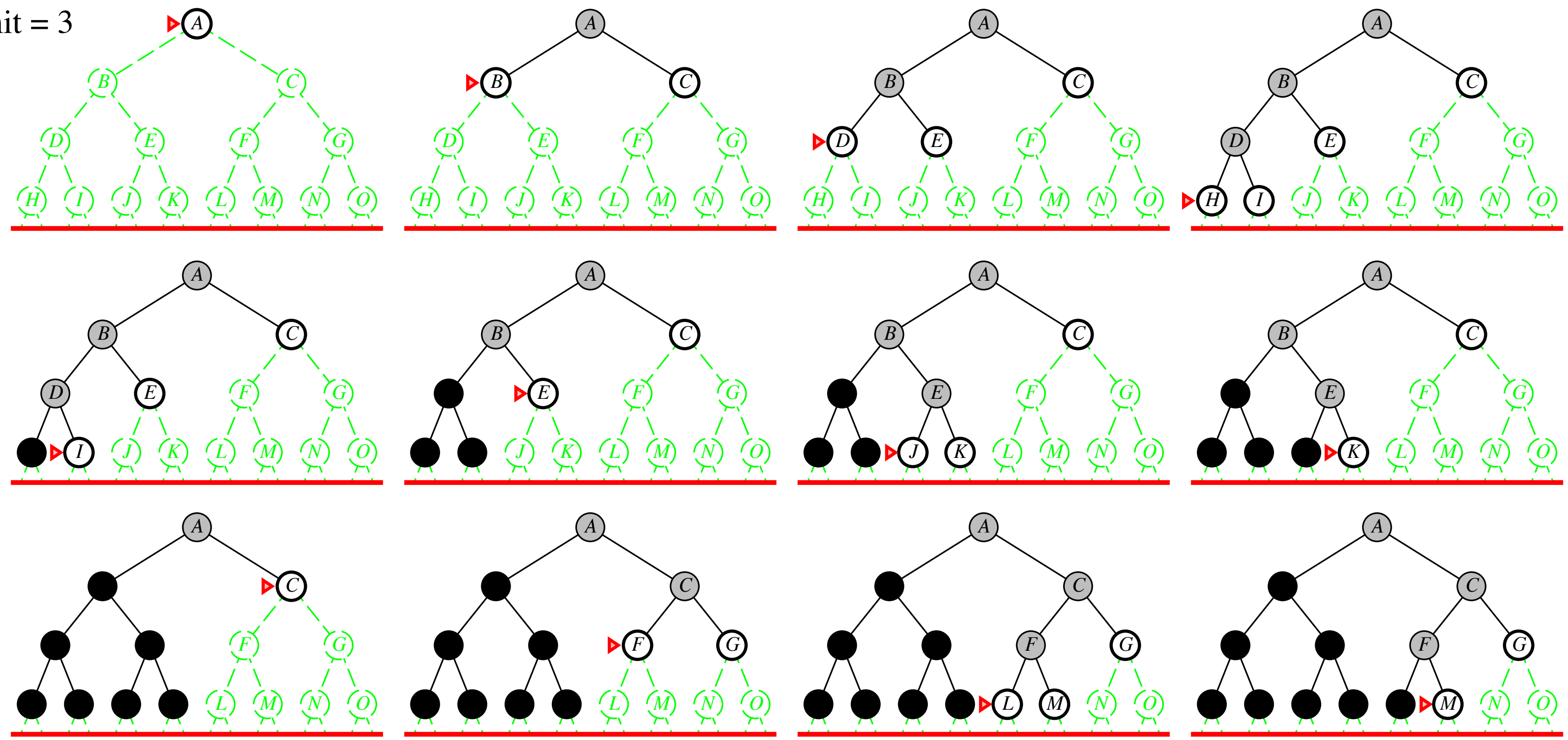
Limit = 1

# Iterative deepening search $l = 2$



# Iterative deepening search $l = 3$

Limit = 3



# Properties of iterative deepening search

Complete??

# Properties of iterative deepening search

Complete?? Yes

Time??

# Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space??

# Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal??

# Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$
Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$\begin{aligned} N(\text{IDS}) &= 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450 \\ N(\text{BFS}) &= 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100 \end{aligned}$$

IDS does better because other nodes at depth $d$ are not expanded

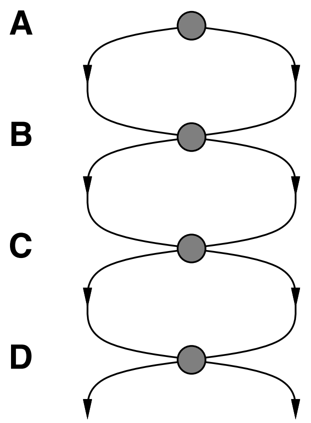
BFS can be modified to apply goal test when a node is **generated**

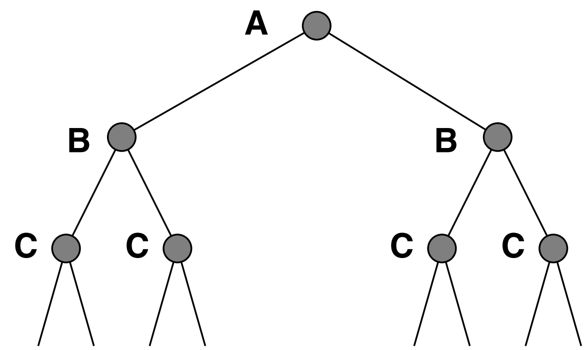
# Summary of algorithms

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening |
|---|---|---|---|---|---|
| Complete? | Yes* | Yes* | No | Yes, if $l \geq d$ | Yes |
| Time | $b^{d+1}$ | $b^{\lceil C^*/\epsilon \rceil}$ | $b^m$ | $b^l$ | $b^d$ |
| Space | $b^{d+1}$ | $b^{\lceil C^*/\epsilon \rceil}$ | $bm$ | $bl$ | $bd$ |
| Optimal? | Yes* | Yes | No | No | Yes* |

# Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!



# Graph search

```
function Graph-Search(problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem, State[node]) then return node
        if State[node] is not in closed then
            add State[node] to closed
            fringe ← InsertAll(Expand(node, problem), fringe)
    end
```

# Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

Variety of uninformed search strategies

Iterative deepening search uses only linear space
and not much more time than other uninformed algorithms

Graph search can be exponentially more efficient than tree search