First-order logic

Chapter 8

Outline

◇ Why FOL?

◇ Syntax and semantics of FOL

◇ Fun with sentences

◇ Wumpus world in FOL

Pros and cons of propositional logic

☺ Propositional logic is **declarative**: pieces of syntax correspond to facts

☺ Propositional logic allows partial/disjunctive/negated information
(unlike most data structures and databases)

☺ Propositional logic is **compositional**:
meaning of $B_{1,1} \land P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

☺ Meaning in propositional logic is **context-independent**
(unlike natural language, where meaning depends on context)

☹ Propositional logic has very limited expressive power
(unlike natural language)
E.g., cannot say "pits cause breezes in adjacent squares"
except by writing one sentence for each square

First-order logic

Whereas propositional logic assumes world contains **facts**,
first-order logic (like natural language) assumes the world contains

- Objects: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .

- Relations: red, round, bogus, prime, multistoried . . . ,
  brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .

- Functions: father of, best friend, third inning of, one more than, end of . . .

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

Syntax of FOL: Basic elements

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\wedge\ \vee\ \neg\ \Rightarrow\quad \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

Atomic sentences

Atomic sentence $= predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term $= function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1, 2) \vee \leq(1, 2)$
$>(1, 2) \wedge \neg>(1, 2)$

Truth in first-order logic

Sentences are true with respect to a model and an interpretation

Model contains ≥ 1 objects (domain elements) and relations among them
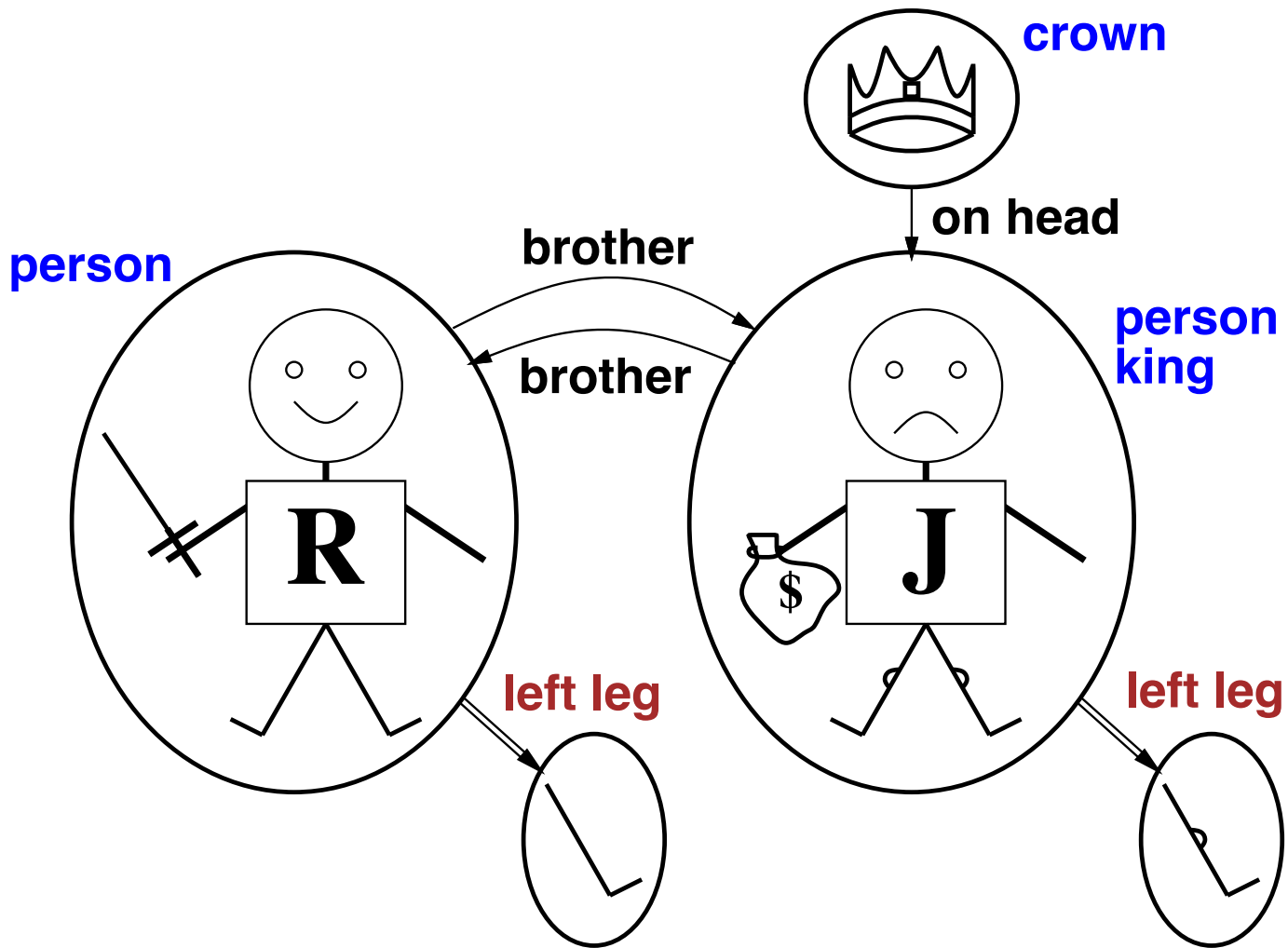
Interpretation specifies referents for
   constant symbols $\rightarrow$ objects
   predicate symbols $\rightarrow$ relations
   function symbols $\rightarrow$ functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

Models for FOL: Example

Truth example

Consider the interpretation in which
$Richard \rightarrow$ Richard the Lionheart
$John \rightarrow$ the evil King John
$Brother \rightarrow$ the brotherhood relation

Under this interpretation, $Brother(Richard, John)$ is true
just in case Richard the Lionheart and the evil King John
are in the brotherhood relation in the model

Models for FOL: Lots!

Entailment in propositional logic can be computed by enumerating models

We **can** enumerate the FOL models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞
    For each k-ary predicate P_k in the vocabulary
        For each possible k-ary relation on n objects
            For each constant symbol C in the vocabulary
                For each choice of referent for C from n objects . . .

Computing entailment by enumerating FOL models is not easy!

Universal quantification

$\forall \langle variables \rangle \ \langle sentence \rangle$

Everyone at Berkeley is smart:
$\forall x \ \ At(x, Berkeley) \ \Rightarrow \ Smart(x)$

$\forall x \ \ P$ is true in a model m iff P is true with x being
each possible object in the model

Roughly speaking, equivalent to the conjunction of instantiations of P

$$(At(KingJohn, Berkeley) \ \Rightarrow \ Smart(KingJohn))$$
$$\wedge \ (At(Richard, Berkeley) \ \Rightarrow \ Smart(Richard))$$
$$\wedge \ (At(Berkeley, Berkeley) \ \Rightarrow \ Smart(Berkeley))$$
$$\wedge \ \ldots$$

A common mistake to avoid

Typically, $\Rightarrow$ is the main connective with $\forall$

Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall\, x \quad At(x, Berkeley) \wedge Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

Someone at Stanford is smart:
$\exists x \ \ At(x, Stanford) \wedge Smart(x)$

$\exists x \ \ P$ is true in a model m iff P is true with x being
some possible object in the model

Roughly speaking, equivalent to the disjunction of instantiations of P

$$
\begin{aligned}
& (At(KingJohn, Stanford) \wedge Smart(KingJohn)) \\
\vee \ & (At(Richard, Stanford) \wedge Smart(Richard)) \\
\vee \ & (At(Stanford, Stanford) \wedge Smart(Stanford)) \\
\vee \ & \ldots
\end{aligned}
$$

Another common mistake to avoid

Typically, $\wedge$ is the main connective with $\exists$

Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x \quad At(x, Stanford) \Rightarrow Smart(x)$$

is true if there is anyone who is not at Stanford!

Properties of quantifiers

$\forall x \ \forall y$ is the same as $\forall y \ \forall x$ (<u>why??</u>)

$\exists x \ \exists y$ is the same as $\exists y \ \exists x$ (<u>why??</u>)

$\exists x \ \forall y$ is **not** the same as $\forall y \ \exists x$

$\exists x \ \forall y \ Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y \ \exists x \ Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x \ Likes(x, IceCream) \qquad \neg \exists x \ \neg Likes(x, IceCream)$

$\exists x \ Likes(x, Broccoli) \qquad \neg \forall x \ \neg Likes(x, Broccoli)$

Fun with sentences

Brothers are siblings

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$$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$$

"Sibling" is symmetric

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"Sibling" is symmetric

$\forall x, y \ \ Sibling(x, y) \ \Leftrightarrow \ Sibling(y, x)$.

One's mother is one's female parent

Fun with sentences

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"Sibling" is symmetric

$\forall x, y \ \ Sibling(x, y) \ \Leftrightarrow \ Sibling(y, x).$

One's mother is one's female parent

$\forall x, y \ \ Mother(x, y) \ \Leftrightarrow \ (Female(x) \land Parent(x, y)).$

A first cousin is a child of a parent's sibling

Fun with sentences

Brothers are siblings

$$\forall\, x, y \ \ Brother(x, y) \ \Rightarrow\ Sibling(x, y).$$

"Sibling" is symmetric

$$\forall\, x, y \ \ Sibling(x, y) \ \Leftrightarrow\ Sibling(y, x).$$

One's mother is one's female parent

$$\forall\, x, y \ \ Mother(x, y) \ \Leftrightarrow\ (Female(x) \wedge Parent(x, y)).$$

A first cousin is a child of a parent's sibling

$$\forall\, x, y \ \ FirstCousin(x, y) \ \Leftrightarrow\ \exists\, p, ps \ \ Parent(p, x) \wedge Sibling(ps, p) \wedge Parent(ps, y)$$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

E.g., $1 = 2$ and $\forall\, x \;\times(Sqrt(x), Sqrt(x)) = x$ are satisfiable
$\quad\quad 2 = 2$ is valid

E.g., definition of (full) $Sibling$ in terms of $Parent$:
$$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; [\neg(x = y) \wedge \exists\, m, f \;\; \neg(m = f) \wedge$$
$$Parent(m, x) \wedge Parent(f, x) \wedge Parent(m, y) \wedge Parent(f, y)]$$

Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB
and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([Smell, Breeze, None], 5))$
$Ask(KB, \exists a \ Action(a, 5))$

I.e., does KB entail any particular actions at $t = 5$?

Answer: $Yes, \{a/Shoot\}$ $\leftarrow$ substitution (binding list)

Given a sentence S and a substitution σ,
$S\sigma$ denotes the result of plugging σ into S; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$

$Ask(KB, S)$ returns some/all σ such that $KB \models S\sigma$