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# CSE 4345 / CSE 5315 - *Computational Methods*

Practice Problems - Exam 1- Fall 2011

Note: This problem set is somewhat larger and more difficult than one exam

Closed book, 2 pages of notes

Date:

Problems marked with $^*$ are required only for students of CSE 5315 but will be graded for extra credit for students of CSE 4345.

1. Briefly explain what the following types of errors measure and how they are affected by the choice of algorithm and its implementation.

   a) Data propagation error.

b) Computation error.

2. Briefly discuss the difference between stability and sensitivity. What do they measure and what are they influenced by ?

3. The equation $(2 + x^2) - (2x/sin(x))$ suffer from loss of significance (cancellation) for $x \rightarrow 0$. Provide a reformulation that avoids this problem.

4. For each of the following fixed point functions determine whether fixed point iteration would converge near the indicated solution.

   a) $g(x) = \frac{2}{x^2}$ , Fixed point: $\sqrt[3]{2}$

   b) $g(x) = 6 - x^2$ , Fixed point: $2$

5. Illustrate the operation of the secant method for root finding for a single equation in one variable by showing two iterations on the function $f(x) = x^2 - 2$ starting with the two initial points $x_0 = 4, x_1 = 3$.

6.* Briefly compare some of the key attributes of the interval bisection method and Newton's method for root finding, detailing their respective advantages and disadvantages. Also briefly discuss how this motivates hybrid methods that combine them.

7. Perform Gaussian Elimination on the following system of equations.

$$
\begin{aligned}
2x + y + 3z &= 0 \\
6x + 4y + 7z &= 2 \\
4x + 4y + 7z &= 14
\end{aligned}
$$

8. The Multivariate Newton method solves a system of nonlinear equations by iteratively solving a sequence of linear systems of equations. List the basic operation steps of the Multivariate Newton method.

9. Solving systems of linear equations, $Ax = b$, only has a solution if the rank of the matrix $A$ is less than or equal to $n$ (the number of variables, $x$), and has a unique solution only if the rank is equal to $n$. Under what conditions does the linear least squares problem, $Ax \cong b$, have a solution and under which conditions is the solution unique ?

10. Consider the linear least squares data fitting problem where a linear function, $f(x) = \alpha x$ is to be fit to the data points, $\{(x,y)_i\} = \{(1,2),(2,3),(3,4)\}$, such that the square difference (residual) between $f(x)$ and $y$ is minimized. (14 / 10 points)

    a) Transform the above problem (and data) into the Normal Equations for the linear least squares problem.

    b) Derive the solution using Normal Equations.

11. Consider the following linear least squares approximation problem where a function, $f(\alpha, x)$ that is linear in the parameters $\alpha$ is to be fitted to a set of data points such that the square residual (difference between $f(\alpha, x)$ and $y$ of the data) is minimized.

$$f(\alpha, x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2 \quad , \quad Data\ (x, y) : \ \{(0,1), (1,3), (2,4), (3,5)\}$$

a) Provide the augmented system formulation for the problem.

b) Discuss some of the differences (in particular in terms of sensitivity and complexity) between the normal equation solution and the solution using the augmented formulation.