A Black-Box Fork-Join Latency Prediction Model for Data-Intensive Applications

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Abstract—The workflows of the predominant datacenter services are underlaid by various Fork-Join structures. Due to the lack of good understanding of the performance of Fork-Join structures in general, today’s datacenters often operate under low resource utilization to meet stringent service level objectives (SLOs), e.g., in terms of tail and/or mean latency, for such services. Hence, to achieve high resource utilization, while meeting stringent SLOs, it is of paramount importance to be able to accurately predict the tail and/or mean latency for a broad range of Fork-Join structures of practical interests. In this article, we propose a black-box Fork-Join model that covers a wide range of Fork-Join structures for the prediction of tail and mean latency, called ForkTail and ForkMean, respectively. We derive highly computational effective, empirical expressions for tail and mean latency as functions of means and variances of task response times. Our extensive testing results based on model-based and trace-driven simulations, as well as a real-world case study in a cloud environment demonstrate that the models can consistently predict the tail and mean latency within 20 and 15 percent prediction errors at 80 and 90 percent load levels, respectively, for heavy-tailed workloads, and at any load levels for light-tailed workloads. Moreover, our sensitivity analysis demonstrates that such errors can be well compensated for with no more than 7 percent resource overprovisioning. Consequently, the proposed prediction model can be used as a powerful tool to aid the design of tail-and-mean-latency guaranteed job scheduling and resource provisioning, especially at high load, for datacenter applications.

Index Terms—Tail latency, mean response time, Fork Join queuing networks, datacenter resource provisioning

1 INTRODUCTION

Fork-Join structures underlay many datacenter services, including web searching, social networking, and big data analytics. A Fork-Join structure is a critical building block in the job processing workflow that constitutes a major part of job processing time and hardware cost, e.g., more than two-third of the total processing time and 90 percent hardware cost for a Web search engine [1]. In a Fork-Join structure (see Fig. 1), each job in an incoming flow spawns multiple tasks, which are forked to, queued and processed at different nodes, called Fork nodes in this paper, in parallel and its task results are then merged at a Join node to yield the final result. Due to barrier synchronization, the job response time is determined by the slowest task, i.e., the tail probability, which is hard to capture, from both modeling and measurement points of view, making it extremely challenging to predict the job performance, e.g., the job tail latency.

Tail latency is considered to be the most important performance measure for user-facing datacenter applications [2], such as web searching and social networking, and normally expressed as a high percentile job response time, e.g.,

1. In the low load region, tail and/or mean latency requirements can be easily satisfied as the available resources are abundant. In contrast, in the heavy load region in which the leftover resource is scarce, resource allocation with high precision must be exercised to meet user requirements.
2. The term “load” can be generally defined as the offered workload per unit time divided by processing capacity per unit time. In the context of Fork-Join structure, it is the maximum of the loads among all the Fork nodes.
Fork-Join model for the prediction of job latency for all the requests, exists, which can pre-process tasks mapped to Fork nodes. Each Fork node is treated as a black box, completely determined by the mean and variance of the task response time, i.e., $E[T]$ and $V[T]$.

Fig. 1. Black-box Fork-Join model. Each job in the incoming flow spawns $k$ tasks mapped to $k$ out of $N$ Fork nodes. Each Fork node is treated as a black box, completely determined by the mean and variance of the task response time, i.e., $E[T]$ and $V[T]$.

and memory resources, respectively, idle almost at all time [6]. Similarly, in a large production cluster at Twitter, aggregate CPU usage is within 20–30 percent even though CPU reservations are up to 80 percent and aggregate memory usage is mostly within 40–50 percent while memory allocation consistently exceeds 75 percent [7]. Hence, how to improve resource utilization or the load from currently less than 50 percent to, say, 80–90 percent, while meeting stringent SLOs has been a challenging issue for datacenter service providers [7]. To this end, a key challenge to be tackled is how to accurately capture the tail and mean latency with respect to various Fork-Join structures at high load.

Fork-Join structures are traditionally modeled by a class of queuing networks, known as Fork-Join queuing network (FJQN) [8], as depicted in Fig. 1. FJQNs are white-box models in the sense that all the Fork nodes are explicitly modeled as queuing systems with given arrival process, queuing discipline, and service time distribution. In this paper, we argue that attempting to use FJQNs to cover a sufficiently wide range of Fork-Join structures of practical interests is not a viable solution. Instead, a black-box solution that can cover a broad range of Fork-Join structures must be sought.

On one hand, FJQNs are notoriously difficult to solve in general. Despite the great effort made for more than half a century, to date, no exact solution is available even for the simplest FJQN where all the nodes are M/M/1 queues [9], i.e., Poisson arrival process and one server with exponential service time distribution. Although empirical solutions for some FJQNs are available, e.g., [10], [11], [12], [13], [14], they can only be applied to a very limited number of Fork-Join structures, e.g., homogeneous case, the case of First-In-First-Out (FIFO) queuing discipline, and a limited number of service time distributions.

On the other hand, the design space of Fork-Join structures of practical interests is vast. It encompasses (a) a wide range of queuing disciplines and service time distributions (e.g., both light-tailed and heavy-tailed) [8]; (b) the case with multiple replicated servers per Fork node for failure recovery, task load balancing, and/or redundant task issues for tail cutting [15], [16] or fast recovery from straggling tasks [17]; (c) the case where the number of spawned tasks per job may vary from one job to another [18]; and (d) the case of consolidated services, where different types of services and applications may share the same datacenter cluster resources [19]. Clearly, the existing FJQNs can hardly cover such a design space in practice.

To tackle the above challenges, in this paper, we propose to study a black-box Fork-Join model for the prediction of job tail and mean latency, called ForkTail and ForkMean, respectively, to cover a broad range of Fork-Join structures of practical interests. By “black-box”, we mean that each Fork node is treated as a black box, regardless of how many replicated servers there are and how tasks are distributed, queued, and processed inside the box. In other words, for a black-box Fork-Join model, one can only use the task statistics measurable from outside of Fork nodes, e.g., the mean and variance of the task response time (see Fig. 1). This is in stark contrast to a white-box Fork-Join model where the exact task queuing discipline and the service model for a Fork node must be known. It also allows the number of spawned tasks per job, $k$, to be a random integer taking values in $[1, N]$, where $N$ is the maximum number of Fork nodes. As we shall see, our black-box model can indeed adequately covers the above design space.

However, general solutions to this model are unlikely to exist, given the limited success in solving the white-box FJQNs. Nevertheless, we found that for the black-box model, empirical solutions under heavy load conditions do exist, known as the central limit theorem for G/G/m queuing systems, where the arrival process is general with independent interarrival times, the queuing discipline is FIFO, and there are $m$ servers with general service time distributions, under heavy load [20], [21]. Inspired by this theorem, we were able to demonstrate [22] that in a load region of 80 percent or higher, where resource provisioning with precision is most desirable and necessary, an empirical expression of the tail-latency for a special case of the black-box model, i.e., $k = N$ for all the requests, exists, which can predict the tail latencies within 15 percent error at any load levels for light-tailed service time distribution and the load level of 90 percent for heavy-tailed one in the cases (a) and (b) in the design space mentioned above. As our sensitivity analysis in Section 4 shows, such prediction errors can be well compensated for with no more than 7 percent resource overprovisioning.

The work in this paper makes the following contributions. First, it generalizes the solution in [22] to also cover cases (c) and (d) in the design space, hence, making it applicable to most Fork-Join structures of practical interests. Second, it gives the first empirical, universal solutions to tail and mean job latencies for both black-and-white-box FJQNs at high load and hence, it makes a contribution to the queuing network theory as well. In fact, for any white-box FJQN with G/G/1 Fork queuing servers, our approach leads to closed-form approximate solutions, which are on par with the most elaborate white-box solutions in terms of accuracy across the entire load range at much lower computational complexity. Third, comprehensive testing and verification of the proposed approximations for tail and mean latency are performed for all (a)–(d) Fork-Join structures, based on model-based and trace-driven simulation, as well as a real-world case study. Fourth, sensitivity analysis indicates that our proposed solutions can lead to accurate resource provisioning for data-intensive services and applications in a consolidated
datacenter environment at high load. Finally, preliminary ideas are provided as to how to use this solution to facilitate SLO-guaranteed job scheduling and resource provisioning.

The rest of the paper is organized as follows. Section 2 introduces our black-box model and ForkTail and Fork-Mean, the empirical approximations for the tail and mean latency, respectively. Section 3 performs extensive testing of the accuracy of these approximations. Section 4 presents the sensitivity analysis for the proposed approximations. Section 5 explores the range of applicability of the proposed solutions. Section 6 discusses how the proposed approximations may be used to facilitate effective job scheduling and resource provisioning with tail-latency-SLO guarantee. Section 7 reviews the related work. Finally, Section 8 concludes the paper and discusses future work.

2 Model and Solutions

2.1 Black-Box Model

The black-box model described in this section greatly extends the scope of the black-box model introduced in [22] to address the entire design space mentioned in Section 1.

Consider a black-box Fork-Join model with each job in the incoming flow spawning $k$ tasks mapped to $k$ out of $N$ Fork nodes, as depicted in Fig. 1. The results from all $k$ tasks are finally merged at a Join node (i.e., the triangle on the right). Jobs arrive following a random arrival process with average arrival rate $\lambda$. Each Fork node may be composed of more than one replicated servers for task-level fault tolerance, load balancing, tail-cutting, and/or straggler recovery. An example Fork node with three server replicas is depicted in Fig. 1.

The above model deals with a general case where $k \leq N$. Note that the traditional FJQNs cover only a small fraction of this design space, i.e., $k = N$, homogeneous Fork nodes with a single server per node, which is modeled as a FIFO queuing system.

General solutions to this model are unlikely to exist. Fortunately, we are most interested in finding solutions in high load regions where precise resource provisioning is highly desirable and necessary. There is a large body of research results in the context of queuing performance in high load regions (e.g., see [23] and the references therein). In particular, a classic result, known as the central limit theorem for heavy traffic queuing systems [20], [21], states that for a G/G/m queue under heavy load, the waiting time distribution can be approximated by an exponential distribution. Clearly, this theorem applies to the response time distribution as well, since the response time distribution converges to the waiting time distribution as the traffic load increases. Inspired by this result, we postulate that for tasks mapped to a black-box Fork node and in a high load region, the task response time distribution $F_T(x)$ for any arrival process and service time distribution can be approximated as a generalized exponential distribution function [24], as follows,

$$F_T(x) = (1 - e^{-x/\beta})^{\alpha}, \quad x > 0, \alpha > 0, \beta > 0,$$

where $\alpha$ and $\beta$ are shape and scale parameters, respectively.

The mean and variance of the task response time are given by [24]

$$\text{E}[T] = \beta(\psi(\alpha + 1) - \psi(1)), \quad \text{V}[T] = \beta^2[\psi'(1) - \psi'(\alpha + 1)],$$

where $\psi(\cdot)$ and its derivative are the digamma and polygamma functions.

From Eqs. (2) and (3), it is clear that the distribution in Eq. (1) is completely determined by the mean and variance of the task response time. In other words, the task response time distribution can be measured by treating each Fork node as a black box as shown in Fig. 1. The rationale behind the use of this distribution, instead of the exponential distribution, is that it can capture both heavy-tailed and light-tailed task behaviors depending on the parameter settings and meanwhile, it degenerates to the exponential distribution at $\alpha = 1$ and $\text{E}[T] = \beta$. In [22], we showed that this distribution significantly outperforms the exponential distribution in terms of tail latency predictive accuracy.

Now, with all the Fork nodes in Fig. 1 being viewed as black boxes, the response time distribution for any job with $k$ tasks can be approximated using the order statistics [9] as follows,

$$F_X^{(k)}(x) = \prod_{i=1}^{k} F_T(x) = \prod_{i=1}^{k} (1 - e^{-x/\beta_i})^{\alpha_i}.$$

Note that the above expression is exact if the response times for tasks mapped to different Fork nodes are independent random variables. This, however, does not hold true for any Fork-Join structures, simply because the sample paths of the task arrivals at different Fork nodes are exactly the same, not independent of one another. This is the root cause that renders the Fork-Join models extremely difficult to solve in general. In what follows, we introduce ForkTail and Fork-Mean, separately, based on this approximation.

2.2 ForkTail

ForkTail was originally presented in [25]. Our postulation is that as load reaches 80 percent or higher where precise resource provisioning is desirable and necessary, the tail-latency prediction errors introduced by the above assumption will become small enough for resource provisioning purpose. Our extensive testing results in this paper provide strong support of the postulation, making our modeling approach the only practically viable one for tail latency prediction.

Tail latency $x_p$, defined as the $p$th percentile job response time, can be written as,

$$x_p = F_X^{(k)-1}(p/100).$$

Eq. (5) simply states that in a high load region, the tail latency can be approximated as a function of the means and variances of task response times for all $k$ tasks at their corresponding Fork nodes, irrespective of what workloads cause the heavy load. The implication of this is significant. It means that this expression is applicable to a consolidated datacenter cluster where more than one service/application share the same cluster resources. Moreover, this expression allows tail latency to be predicted using a limited number of job samples thanks to its dependence on the first two moments of task response times only, i.e., the means and variances.

The results so far are general, applying to the homogeneous case, where task response time distributions may be...
different from one task to another, due to, e.g., the use of heterogeneous Fork nodes and/or uneven background workloads. As a result, the tail latency predicted by Eq. (5) may be different from one job to another or even for two identical jobs, as long as their respective Fork nodes do not completely coincide with one another, or they are issued at different times. In other words, Eq. (5) is a fine-grained tail latency expression. For certain applications, such as offline resource provisioning (see Section 6 for explanations) and coarse-grained, per-service-based tail-latency prediction, one may be more interested in the homogeneous case only. In this case, the response time distribution can be further simplified as,

$$F_X^{(k)}(x) = (1 - e^{-x/jx^k})^{ka}.$$  \hspace{1cm} (6)

This is because the means and variances given in Eqs. (2) and (3) are the same for the homogeneous case. A coarser-grained cumulative distribution function (CDF) of the job response time can then be written as,

$$F_X(x) = \sum_{k_i} F_X|x(k|x_i)P(K = k_i),$$  \hspace{1cm} (7)

where $F_X|x(k|x_i)$ is the conditional CDF of the job response time for jobs with $k_i$ tasks, given by Eq. (6), i.e., $F_X|x(k|x_i) = F_X^{(k_i)}(x)$, and $P(K = k_i) = P_i$ is the probability that a job spawns $k_i$ tasks.

Further assume that there are $m$ job groups with distinct numbers of tasks $k_i’s$, $i = 1, \ldots, m$, and corresponding probabilities $P_i’s$. We then have,

$$F_X(x) = \sum_{i=1}^{m} P_i \cdot F_X^{(k_i)}(x).$$  \hspace{1cm} (8)

Correspondingly, the tail latency for the $m$ groups of jobs as a whole can then be readily obtained, similar to Eq. (5), as follows,

$$x_p = F_X^{-1}(p/100).$$  \hspace{1cm} (9)

For example, the tail latency for a given service can be predicted by collecting statistics for $k_i’s$ and $P_i’s$, as well as mean and variance of task response time and applying them to the tail latency expression in Eq. (9).

### 2.2.1 Application to White-Box FJQNs

Clearly, the above black-box approach leads to closed-form solutions for any white-box models whose analytical expressions for the means and variances of task response times are available, whether it is homogeneous or not. In fact, our solution works for the case where different Fork nodes may have different service time distributions and queuing disciplines. For instance, our approach can be applied to a large class of FJQNs, where each Fork node is an M/G/1 queue or a more general G/G/1 queue, whose mean and variance of the task response time can be computed from Takács recurrence theorem [26] or the queuing network analyzer [27], respectively.

### 2.3 ForkMean

While the approximations in Eqs. (5) and (9) work well for the job tail latency even for the $k < N$ cases, it fails to accurately predict the job mean response time,\(^3\) yielding more than three times larger errors for the same cases studied, especially for the case of light-tailed service time distributions. We find that the reason for this to happen is due to the fact that to accurately predict the job mean response time, the entire job response time distribution including the tail portion must be accurately captured, as the barrier synchronization tends to push the job mean response time towards the tail part of the task response time distribution, as the workload scales out.

On the basis of the above modeling, this section aims at finding solutions to reduce the prediction errors for the job mean latency. To this end, we make the following two key observations.

**Observation 1.** For a wide range of Fork-Join models, the difference between the exact tail-mean ratio and the model-based tail-mean ratio, derived from the CDF in Eq. (4), hereafter called the gap and denoted as $\Delta$, converges to a constant as the number of Fork nodes becomes large enough. Mathematically, we have,

$$\frac{x_{ge}}{x_m} - \frac{x_{ge}}{x_m} = \Delta,$$  \hspace{1cm} (10)

where $x_{ge}$ and $x_m$ are the exact $p$th percentile and mean of job latency, respectively, which can be estimated by experiments, while $x_{ge}$ and $x_m$ are derived from the prediction model, i.e., Eq. (4). Hence, the mean latency can be approximated as follows,

$$x_m = \frac{x_p}{R_{ge} + \Delta} \approx \frac{x_{ge}}{R_{ge} + \Delta},$$  \hspace{1cm} (11)

where $x_p \approx x_{ge}$ at high loads, since ForkTail give accurate predictions for the $p$th percentile at high loads, as indicated in the testing results, and $R_{ge} = x_{ge}/x_{ge}$.

Fig. 2 illustrates the gaps for systems with different task service time distributions, including light-tailed and heavy-tailed ones, where each Fork node is a single server, i.e., without replication. As one can see, the gap converges to a constant as $N$ becomes sufficiently large, say, $N > 100$, for all the cases. Similar trends are also observed for the systems with 3-replica Fork nodes with Round-Robin and redundant-task-issue dispatching policies as well as the systems with variable numbers of forked tasks (not shown here).

**Observation 2.** There is a strong correlation between the tail heaviness of service time distribution and the gap $\Delta$, i.e., the heavier the tail, the smaller the gap. It is evident from Fig. 2 that the light-tailed distributions, including Exponential and Weibull, have larger gaps than the heavy-tailed ones, including the truncated Pareto and empirical (defined in Section 3.1). With this observation, we make the following postulation: The gap is much more of a function of the tail heaviness of a service time distribution than the service time distribution itself.

From the above observations, we propose two empirical solutions, one is white-box and the other black-box, for the approximation of the gap, $\Delta$, and hence, the job mean response time.

\(^3\) We use the terms ‘latency’ and ‘response time’ interchangeably in this paper.
2.3.1 White-Box Approach

This approach is based on the above postulation. Here we consider a homogeneous white-box Fork-Join queuing model where each Fork node can be modeled as a G/G/1 queue. With known interarrival and service time distributions, one can find the job response time distribution and the corresponding tail and mean latencies, and so their ratio $R^\text{E}$ from ForkTail. So to find the job mean latency, $x_m$, all that is left to be done is to find $\Delta$.

To this end, we first define tail heaviness, $w(F_T)$. We use Right Quantile Weight [28] which measures the tail heaviness on the right side of a distribution, the region of interest in all of our experiments. This tail weight measure is defined as,

$$w(F_T) = \frac{F_T^{-1}(\frac{q+1}{2}) + F_T^{-1}(\frac{q}{2}) - 2F_T^{-1}(0.75)}{F_T^{-1}(\frac{q+1}{2}) - F_T^{-1}(\frac{q}{2})},$$

where $0.5 < q < 1$ and $F_T^{-1}(q)$ is quantile $q$ of task service time distribution $F_T$. To capture the tail effect but still retain a reasonable robustness, we set $q = 0.90$.

Based on our postulation, $\Delta = \Delta(\rho, w)$, independent of $F_T(x)$. Here $\rho$ is the load. In other words, as $w(F_T^{(1)}) = w(F_T^{(2)})$, the two homogeneous Fork-Join models with different service time distributions, $F_T^{(1)}$ and $F_T^{(2)}$, respectively, will have the same gap. In other words, if one can find the function, $\Delta(\rho, w)$, using one distribution function with different tail weights, this $\Delta(\rho, w)$ can then be used by any Fork-Join models with other distribution functions to find the gap. In this paper, we use the generalized exponential distribution in Eq. (1) at different coefficients of variance to generate different $\Delta(\rho, w)$. Using one ForkTail service time distribution $F_T$, calculate the tail weight $w$ from Eq. (12), which is then mapped to a $\Delta$ at a given load, e.g., using one of the fitted functions in Table 2.

Approximate the mean latency using Eq. (11).

2.3.2 Black-Box Approach

The white-box approach above leads to closed-form solutions for homogeneous white-box Fork-Join models with known

![Fig. 2. The gaps for Fork-Join systems with different service time distributions at load levels of 80 percent (upper row) and 90 percent (lower row).](image)

![Fig. 3. An example of the gap-vs-tail-weight fitted curve.](image)
service time distributions for Fork nodes. However, in practice, determining those distributions is nontrivial, e.g., for systems with multi-replica Fork nodes. Hence, it is necessary to seek a black-box solution applicable to a wide range of Fork-Join structures of practical interests.

Based on the Observation 1, i.e., $\Delta$ converges to a constant as the number of Fork nodes becomes large enough, i.e., around 100, based on all the testing cases. This suggests that, if for a target application, $\Delta$ can be measured on a small testbed or by simulation, with 100 virtual machines/nodes, or equivalently, a few commodity servers, e.g., 5, then the mean latency can be predicted when the application is deployed on a much larger number of nodes. This approach requires only the means and variances of task response times as inputs, and hence is a hybrid, black-box solution.

The steps taken to find the job mean latency are similar to those for the white-box approach above except for step 2 where $\Delta$ is predicted by running experiments for the target application on a system with a given number of Fork nodes, e.g., 100, and measure the ratio gap between the results from the experiments and the prediction model.

Compared to the white-box solution, the black-box one is simpler and can be applied to a much wider range of Fork-Join structures. However, as a hybrid approach, it requires to run experiments, either via simulation or on a real testbed, with an adequate number of Fork nodes, e.g., 100. Consequently, it should be applied to large-scale systems where a job is forked to at least hundreds of nodes, much larger than the one used for testing. Note that the hybrid approach, which combines analysis and simulation, is not unusual in analyzing performance of the Fork-Join model. Indeed, it has been used in several previous works in the literature [10], [13], [29].

3 Validation

3.1 Tail Latency Prediction Validation

In this section, ForkTail is extensively validated against the results from model-based simulation, trace-driven simulation, and a case study in Amazon EC2 cloud. The validation is performed for the systems with $k = N$, $k \leq N$, and consolidated services, separately. The accuracy of the prediction is measured by the relative error between the value predicted from ForkTail, $t_p$, and the one measured from simulation or real-system testing, $t_m$, i.e.,

$$\text{error} = \frac{100(t_p - t_m)}{t_m}.$$

3.1.1 Case 1: $k = N$

A notable example for this case is Web search engine [30] where a search request looks up keywords in a large inverted index distributed on all the servers in the cluster. We validate ForkTail with three testing approaches, i.e., white-box and black-box model-based testing as well as a real-world case study in Amazon EC2 cloud.

White-Box Model-Based Validation. Here we study the accuracy of ForkTail when applied to homogeneous, single-queuing-server-Fork-node Fork-Join systems with the assumption that the service time distribution is known in advance, the approach taken in all the existing works on performance analysis of FJQNs [9]. The tail latency prediction involves the following steps:

- Find the mean and variance of task response times with the given task service time distribution;
- Substitute the above mean and variance into Eqs. (2) and (3), respectively, and solve that system of equations to find the scale and shape parameters of the generalized exponential distribution in Eq. (1), which is then used to approximate the task response time distribution;
- Calculate the $p$th percentile of request response times from Eq. (9).

First, we compare ForkTail against the state-of-the-art tail latency approximation for homogeneous FJQNs [14], known as EAT, which is derived from analytical results for single-node and two-node systems. Fig. 4 shows the comparative results for three service time distributions studied in [14], i.e., Erlang-2, Exponential, and Hyperexponential-2, at the loads of 10, 50, and 90 percent and numbers of nodes of 100, 500, and 1,000.

EAT provides more accurate (from a few to several percentage points) approximations for the 99th percentiles of response times across all the cases studied. Much to our surprise, our approach yields most of the errors within 10 percent, across the entire load range. Although outperforming our approach, EAT has its limitations. First, it can be applied only to homogeneous FJQNs where each node can be generally modeled as a MAP/PH/1 queuing system, i.e., Markovian arrival processes and phase-type service time distribution with one service center. Second, the method requires the service time distribution to be known in advance and converted into a phase-type distribution, which is nontrivial, especially for heavy-tailed distributions [31]. Third, the method may incur high computational complexity, depending on the selection of a constant $C$, whose value determines the computational runtime and prediction accuracy. It takes

4. For EAT, the case for Hyperexponential-2 at the load of 90 percent is not available, due to a numerical error running the code provided in [14].
Fig. 5. Prediction errors of the 99th percentile response times for white-box systems with single-server Fork nodes.

at least 2 seconds on our testing PC (Core i7-4940MX Quad-core, 32GB RAM) to get the resulting percentiles even at the lesser degree of accuracy with $C = 100$ (more than 300 seconds at $C = 500$). In contrast, our method takes less than 5 millisecond to compute the required percentiles. As a result, similar to other existing white-box solutions, EAT has limited applicability for datacenter job scheduling and resource provisioning in practice.

To cover a sufficiently large workload space, we further consider service time distributions with heavy tails, which are common in practice [32] and cannot be easily dealt with by EAT, including the following,

- Empirical distribution measured from a Google search test leaf node provided in [32], which has a mean service time of 4.22 ms, a coefficient of variance (CV) of 1.12, and the largest tail value of 276.6 ms;
- Truncated Pareto distribution [31] with the same mean service time and a CV of 1.2, whose CDF is given by,

\[
F_S(x) = \frac{1 - (L/x)^\alpha}{1 - (L/H)^\alpha} \quad 0 \leq L \leq x \leq H, \tag{13}
\]

where $\alpha$ is the shape parameter; $L$ is the lower bound; and $H$ is the upper bound, which is set at the maximum value of the empirical distribution above, i.e., $H = 276.6$ ms, resulting in $\alpha = 2.0119$ and $L = 2.14$ ms.
- Weibull distribution [8], also with the same mean service time and a CV of 1.5, whose CDF is defined as,

\[
F_S(x) = 1 - \exp\left[-(x/\beta)^\alpha\right] \quad x \geq 0, \tag{14}
\]

where $\alpha = 0.6848$ and $\beta = 3.2630$ are shape and scale parameters, respectively.

Fig. 5 presents the prediction errors for the 99th percentile response times for the above cases. The Weibull distribution, which is less heavy-tailed, consistently yields smaller errors, well within 5 percent, for the entire load range studied, similar to the light-tailed distribution cases studied earlier. The empirical and truncated Pareto distributions, which are more heavy-tailed, provide good approximations for the 99th percentiles at the load of 80 percent or higher, which is well within 17 and 5 percent at the load of 80 and 90 percent, respectively, agreeing with our postulation.

We also consider the cases with general arrival process and general service time distribution, i.e., $G/G/1$ Fork nodes. Fig. 6 shows the prediction errors for example cases with Erlang-2 (CV = 0.5) and Hyperexponential-2 (CV = 1.2) arrival processes and Truncated Pareto service time distribution (CV = 3.0). Again, ForkTail yields quite accurate approximations for tail latency at high load regions, i.e., above 75 percent. The prediction results also show the same trend for Weibull and Exponential service time distributions, which are not shown here.

Black-Box Model-Based Validation. We now validate ForkTail without making assumption on the service time distribution at each Fork node. We treat each Fork node as a black-box and empirically measure the mean and variance of task response times at each given arrival rate $\lambda$ or load. These measures are then substituted into Eqs. (2) and (3), respectively, to find the shape and scale parameters, which are in turn used to predict the tail latency based on Eq. (9).

For all the three heavy-tailed FJQNs studied above, we consider two types of Fork nodes, i.e., one with single server and the other with three replicated servers. For the one with three servers, we explore two task dispatching policies. The first policy is the Round-Robin (RR) policy, in which the dispatcher will send tasks to different server replicas in an RR fashion. The second policy is still RR, but it also allows redundant task issues, a well-known tail-cutting technique [15], [16]. This policy allows one or more replications of a task to be sent to different server replicas in the Fork node. The replications may be sent in predetermined intervals to avoid overloading the server replicas. In our experiments, at most one task replication can be issued, provided that the original one does not finish within 10 ms, which is around the 95th percentile of the empirical distribution above.

Figs. 7, 8, and 9 present the prediction errors at different load levels and $N$’s for the 99th percentile response times for all three FJQNs with single server and three servers per Fork node, respectively. First, we note that the prediction errors for the cases in Fig. 7 are very close to those in Fig. 5. This is expected as the white-box and black-box results, ideally, should be identical. The differences are introduced due to simulation and measurement errors. Second, the prediction performances of the cases with three replicas and the RR policy in Fig. 8 are also very close to those of the cases in Fig. 7, with errors being well within 20 and 10 percent at the

Fig. 6. Prediction errors of the 99th percentile response times for white-box systems with Erlang-2 (left) and Hyperexponential-2 (right) arrival distributions and Truncated Pareto service time distribution.
loads of 80 and 90 percent, respectively, for all the case studies, further affirming our postulation. The two scenarios have similar performance because they are compared at the same load levels, where the RR policy in the second scenario simply balances the load among three replicas, making each virtually identical to the single-server scenario. In contrast to these two scenarios, Fig. 9 shows that with the application of the tail-cutting technique, the prediction errors are substantially reduced, with less than 10 percent at the load of 80 percent or higher. This is consistent with the earlier observation, i.e., the lighter the tail, the smaller the prediction errors. This suggests that the tail-cutting techniques, often utilized in datacenters to curb the tail effects, can help expand the load ranges in which ForkTail can be applied.

A Case Study in Cloud. We also assess the accuracy of ForkTail for a real case study in Amazon EC2 cloud. We implement a simple Unix grep-like program on the Apache Spark framework (version 2.1.0) [4]. It looks up a keyword in a set of documents and returns the total number of lines containing that keyword, as depicted in Fig. 10. The cluster for the testing includes one master node using an EC2 c4.4xlarge instance and 32 or 64 worker nodes using EC2 c4. large instances. We use a subset of the English version of Wikipedia as the document for lookup. Each worker node holds a shard of the document whose size is 128 MB, corresponding to the default block size on Hadoop Distributed File System (HDFS) [33]. A client, which runs a driver program, sends a flow of keywords, each randomly sampled from a pool of 50K keywords, to the testing cluster for lookup. Each worker searches through its corresponding data block to find the requested keyword and counts the number of lines containing the keyword. The line count is then sent back to the client program to sum up. Clearly, this testing setup matches the black-box model.

We measure the request response time, i.e., the time it takes to finish processing each keyword at the client. We also collect the task response times, composed of the task waiting time and task service time. The task waiting time is the one between the time the request the task belongs to is sent to the cluster and the time the task is sent to a given worker for processing. This is because in the Spark framework, all the tasks spawned by a request are kept in their respective
virtual queues corresponding to their target workers centrally. A task at the head of a virtual queue cannot be sent to its target worker until the worker becomes idle. Hence, to match our black-box model, the task response time must include the task waiting time, i.e., the task queuing time plus the task dispatching time, and the task service time, which is the actual processing time at the worker the task is mapped to. From the collected samples, we compute the means and variances of task response times, which are in turn used to derive the task response time distribution as in Eq. (1).

Ideally, the task response time distributions for all the tasks are the same, given that the workers are identical. In other words, one would expect that this case study is homogeneous. However, our measurement indicates otherwise. A careful analysis reveals that this is mainly due to the task scheduling mechanism in the Spark framework. Each data block has three replicas distributed across different workers. By default, the placement preference is to send a task to an available worker where the data block resides. Unfortunately, as the request arrival rate or load increases, more tasks are mapped to workers that do not hold the required data blocks for the tasks, causing long task response time due to the need to fetch the required data blocks from the distributed file system. This results in higher variability in the task response time distributions among different workers. Therefore, the heterogeneous model given in Eq. (4) is found to be more appropriate in high load regions.

The above observation is confirmed by the experimental results, presented in Fig. 11. As one can see, the heterogeneous model (the blue lines) gives quite accurate prediction for both 95th and 99th percentiles at both \( N = 32 \) and 64 cases, while the prediction from the homogeneous model (the green lines) gets worse as the load becomes higher. Based on the heterogeneous prediction, the prediction errors at both \( N = 32 \) and 64 and the 99th percentile are well within 10 percent in a high load region, i.e., 60 percent or higher. Note that the load here is measured in terms of request arrival rate. Since the system is heterogeneous, we estimated the equivalent loads corresponding to different arrival rates based on the maximum value of means of task service times across all the workers, as given in Table 3.

Finally, we note that to achieve a reasonably good confidence of measurement accuracy for the 99th percentile tail latency, we collected 80K samples in our experiments at the maximum possible sampling rate equal to the average request arrival rate of 5.8 per second, which translates into a measurement time of 13,793 seconds or about 4 hours. It takes even more time to run the experiments at lower arrival rates. The average runtime across all the request arrival rates in the experiments is about 6 hours. Due to the costly cloud services, we have to limit our experiments to 64 worker nodes.

This example clearly demonstrates that it can be expensive and time consuming, if practical at all, to estimate tail latency based on direct measurement. In contrast, ForkTail is able to do so with far fewer number of samples at much lower cost. For example, with 800 samples collectable in less than three minutes, we can estimate the response-time means and variances for all the tasks and hence the tail latency with reasonably good accuracy. This means that our prediction model can reduce the needed samples or prediction time by two orders of magnitude than the direct measurement.

### 3.1.2 Case 2: Variable Number of Tasks \( k \leq N \)

Notable examples for this case are key-value store systems in which a key lookup may touch only a partial number of servers and web rendering which requires to receive web objects or data from a group of servers in a cluster.

In this case study, we assess the accuracy of our prediction model (i.e., Eqs. (8) and (9)) for applications whose jobs may spawn different numbers of tasks with distribution \( P(K = k_i) \). Specifically, we study two scenarios where \( P(K = k_i) \) is non-zero for a specific value of \( K \) and uniformly distributed, respectively. We further consider three different service time distributions: two heavy-tailed ones, the empirical and truncated Pareto as in Section 3.1.1, and a light-tailed exponential distribution, with the same mean service time, i.e., 4.22 ms.

**Scenario 1: Fixed Number of Tasks per Job.** In this scenario, we consider the cases when the number of forked tasks per job is a fixed number \( k \leq N \), i.e., every incoming job is...
split into exactly $k$ tasks which are dispatched to $k$ randomly selected Fork nodes in an $N$-node cluster.

Fig. 12 shows prediction errors for the 99th percentile response times for an 1,000-node cluster with $k = 100$, 500, and 900 tasks. ForkTail provides good prediction in high load regions, with all the errors within 10 percent at the load of 90 and 20 percent at the load of 80 percent for all the cases studied. The case with the light-tailed exponential distribution gives quite accurate prediction for the entire range under study, i.e., all within 6 percent.

**Scenario 2: Uniform Distribution.** Here we deal with cases when an incoming job is forked to $k$ random nodes in the cluster where $k$ is randomly sampled from an integer range $[a, b]$, i.e., $k_i \in \{a, a+1, \ldots, b-1, b\}$ with probability $P_i = P = 1/m$, where $m = b - a + 1$. Therefore, the mean number of tasks is $(a + b)/2$.

Fig. 13 presents prediction errors for an 1,000-node cluster with $k$ in four different ranges, i.e., $[80, 120]$, $[400, 600]$, $[800, 1000]$, and $[10, 90]$. The results again show that ForkTail yields good approximations for the 99th percentile job response times when the system is under heavy load, i.e., 80 percent or higher. Furthermore, again for all the cases with the exponential distribution, ForkTail gives accurate predictions across the entire load range studied.

The above prediction model applies to the case where a single tail-latency SLO is imposed on a service or application as a whole, a practice widely adopted in industry. However, this practice can be too coarse grained. To see why this is true, Table 4 provides the predicted tail latencies for some given jobs with distinct $k$ values in a cluster of size 1,000 and at the load of 90 percent.

Table IV shows that for some given jobs with distinct $k$ values in a cluster of size 1,000 and at the load of 90 percent. As one can see, the 99th percentile tail latencies for jobs at different $k$'s can be drastically different, e.g., the 10-task and 900-task cases. This suggests that even for a single application, finer grained tail latency SLOs may need to be enforced to be effective, e.g., enforcing tail-latency SLOs for job groups with each having $k$’s in a small range. Table 5 shows that ForkTail can indeed provide accurate, finest-grained prediction at given $k$’s, i.e., all well within 10 percent at load of 90 percent.

### 3.1.3 Case 3: Consolidated Services

In this case study, we evaluate the accuracy of ForkTail when applied to the consolidated datacenter where multiple applications, including latency-sensitive user-facing and background batch ones, share cluster resources as illustrated in Fig. 14. We conduct a trace-driven simulation based on a trace file derived from the Facebook 2010 trace, a widely adopted approach in the literature to explore datacenter workloads [19], [34], [35]. We test the accuracy of ForkTail in capturing the tail latency for a given target application.

**Workload.** The trace file is generated based on the description of the Facebook trace in some previously published works [19], [34], [35]. Specifically, we first generate the number of tasks for job arrivals based on the distribution of the job size in terms of the number of tasks per job, as suggested in [35]. It includes nine bins of given ranges of the number of tasks and corresponding probabilities, assuming that the number of tasks is uniformly distributed in the range of each bin. We then generate the mean task service time based on the Forked task processing time information in [34]. Individual task times are drawn from a Normal distribution with the generated mean and a standard deviation that doubles the mean as in [19]. The resulting trace file contains a total of two million requests, each including the following information: request arrival time, number of forked tasks, mean task service time, and the service times of individual forked tasks.

In the experiments, the jobs in the trace file serve as the background workloads, which are highly diverse, involving

---

**TABLE 4**

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Number of forked tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Exponential</td>
<td>291.32</td>
</tr>
<tr>
<td>Truncated Pareto</td>
<td>448.83</td>
</tr>
<tr>
<td>Empirical</td>
<td>391.27</td>
</tr>
</tbody>
</table>

**TABLE 5**

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Number of nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Exponential</td>
<td>-0.861</td>
</tr>
<tr>
<td>Truncated Pareto</td>
<td>-0.571</td>
</tr>
<tr>
<td>Empirical</td>
<td>-2.814</td>
</tr>
</tbody>
</table>

---

Fig. 13. Prediction errors of the 99th percentile response times for an 1,000-node cluster when the number of tasks per job is uniformly distributed.

Fig. 14. Consolidated applications running on a cluster.
a wide range of applications with mean service times ranging from a few milliseconds to thousands of seconds. The target jobs are generated at runtime using the same approach the trace file is generated. The only difference is that the target jobs are statistically similar with the same mean service time, to mimic a given application or simply a group of jobs with similar statistic behaviors. For each simulation run, a predetermined percentage, e.g., 10 percent, of target jobs are created and fed into the cluster at random.

Simulation Settings and Results. In the simulation, the target and background jobs are set at 10 and 90 percent of the total number of jobs, respectively. We evaluate two cases, one with the number of tasks per target job set at one half of the cluster size and the other the same as the cluster size. The tests cover multiple cluster sizes, i.e., 100, 500, 1,000, and 5,000 nodes with each having three replicated servers. All the cases are homogeneous.

The prediction errors for the 99th percentiles of target response times for the two case studies at loads of 50, 75, 80, and 90 percent are shown in Fig. 15. As one can see, the prediction errors are within 15 percent for all the cases studied. Finally, we note that although the validations for tail latency prediction are exclusively focused on the 99th-percentile tail latency, ForkTail offers similar and consistent performance at higher percentiles, which are not shown here due to the lack of space.

3.2 Mean Latency Prediction Validation

In this section, we extensively validate the predicted mean latencies from ForkMean, for both white-box and black-box approaches, against the results from the existing white-box solutions, the event-driven simulation experiments, and a case study on Amazon EC2 as in Section 3.1.

3.2.1 Scenario 1: Single-Server Queues

In this scenario, we compare ForkMean with some well-known closed-form approximations, including NT [10], VMC [36], and VM [37].

Fig. 16 shows the comparison for the systems with 50, 1,000, and 5,000 nodes, each modeled as an M/M/1 queue, at load levels of 50, 75, 80, and 90 percent. Overall, the NT approximation is the most accurate one. The white-box ForkMean yields errors within 5 percent for all the cases studied, which are close to those of the NT approximation. The black-box one that is based on the measured $D$’s at 100 node also gives good approximations to mean latency even for the case of 50 nodes, with errors within 10 percent for all the cases. Note that, due to its high computational complexity, the VM approximation is not included in the cases of 1,000 and 5,000 nodes. With small $n$’s, e.g., 50, it is a little better than the VMC approximation but not as good as the NT one.

The NT and VMC approximations above, which are tailored to M/M/1 queues, could not be applied to general service time distributions as the prediction errors are too large to be useful. Indeed, Fig. 17 shows that while both black-box and white-box ForkMean solutions continue to perform well, with errors within 10 percent, VMC and NT offer extremely poor performance with up to 40 and 50 percent errors for Gamma and Weibull service time distributions, respectively.

The existing methods for the approximation of the mean response time in the case of M/G/1 Fork-Join models are heuristic-based [37] or hybrid-based [13], [29], i.e., combining simulation and analysis. Moreover, these works mainly focus on light-tailed distributions, e.g., Exponential (Exp), Erlang-2 (E2), and Hyperexponential-2 (H2). In contrast, in addition to these distributions, ForkMean solutions are also validated for a wide range of service time distributions.
To test the effectiveness of ForkMean, we first compare our white-box solution with the heuristic approximations in [37] for the cases of Erlang-2 (E2) and Hyperexponential-2 (H2) service time distributions with Poisson arrivals, i.e., M/G/1 queues.

Tables 6 and 7 present the comparative results for Erlang-2 and Hyperexponential-2, respectively. Again due to the computational complexity concerning the VM approximation, we perform comparison only for small $n$’s, i.e., up to 20, the maximum problem size studied by the authors of the VM approximation [37], although our solution offers consistent performance at large $n$’s as well. For the Erlang-2 distribution, the VM approach gives better predictions at load level of 50 percent and lower numbers of nodes, i.e., 5 and 10 nodes, while our solution yields comparable or better predictions for the other settings. The accuracy of our approach outperforms that of the VM for the Hyperexponential distribution. Although yielding good prediction performance for systems with small numbers of Fork nodes, the VM approximation faces the issue of numerical instability and computational complexity due to big binomial coefficients, resulting in higher prediction errors for higher numbers of nodes, as observed from the reported results. In addition, while the VM approximation can in theory be applied to G/G/1 queues, finding light and heavy traffic limits for an arbitrary service time distribution, e.g., Weibull or truncated Pareto, is nontrivial.

Fig. 18 shows the prediction accuracy of ForkMean for the above heavy-tailed service time distributions. Both white-box and black-box solutions yield quite accurate predictions for less heavy-tailed distributions, i.e., Weibull, for all the cases studied, with errors within 12 percent for all the cases. For heavier tailed distributions, i.e., truncated Pareto and empirical, the solutions give good approximations at high load levels, i.e., 80 percent or higher, a region of interest for resource provisioning. Overall, the black-box solution gives comparably close prediction performance to that of the white-box one. The errors are mostly within 20 and 10 percent at the load levels of 80 and 90 percent, respectively.

The predictions for G/G/1 cases as in Section 3.1 also show similar performance, i.e., within 20 percent errors at the load levels of 80 percent or higher, which are not shown here due to the lack of space.

### Scenario 2: Systems With Replicated Servers

We now validate ForkMean for systems with 3-replica Fork nodes. We consider two dispatching policies, i.e., Round-Robin and redundant-task-issue, and heavy-tailed service time distributions as in Section 3.1. The validation is run only for the black-box solution since the exact service time distributions for the Fork nodes are simply unknown for such cases.

Fig. 18 shows the prediction accuracy of ForkMean for the above heavy-tailed service time distributions. Both white-box and black-box solutions yield quite accurate predictions for less heavy-tailed distributions, i.e., Weibull, for all the cases studied, with errors within 12 percent for all the cases. For heavier tailed distributions, i.e., truncated Pareto and empirical, the solutions give good approximations at high load levels, i.e., 80 percent or higher, a region of interest for resource provisioning. Overall, the black-box solution gives comparably close prediction performance to that of the white-box one. The errors are mostly within 20 and 10 percent at the load levels of 80 and 90 percent, respectively.

The predictions for G/G/1 cases as in Section 3.1 also show similar performance, i.e., within 20 percent errors at the load levels of 80 percent or higher, which are not shown here due to the lack of space.

### 3.2.2 Scenario 2: Systems With Replicated Servers

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<table>
<thead>
<tr>
<th>Load</th>
<th>Method</th>
<th>Number of nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VM</td>
<td>5 10 15 20</td>
</tr>
<tr>
<td>50%</td>
<td>VM</td>
<td>5.101 8.110 6.398 5.251</td>
</tr>
<tr>
<td></td>
<td>White-box</td>
<td>7.947 6.312 5.483 4.934</td>
</tr>
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<td>75%</td>
<td>VM</td>
<td>3.440 7.885 9.039 9.538</td>
</tr>
<tr>
<td></td>
<td>White-box</td>
<td>10.101 8.110 6.398 5.251</td>
</tr>
<tr>
<td>80%</td>
<td>VM</td>
<td>10.101 8.110 6.398 5.251</td>
</tr>
<tr>
<td></td>
<td>White-box</td>
<td>11.001 8.110 6.398 5.251</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load</th>
<th>Method</th>
<th>Number of nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VM</td>
<td>1.007 6.446 13.839 17.937</td>
</tr>
<tr>
<td>50%</td>
<td>White-box</td>
<td>0.869 0.945 1.881 2.118</td>
</tr>
<tr>
<td>75%</td>
<td>VM</td>
<td>1.682 6.556 12.601 16.678</td>
</tr>
<tr>
<td></td>
<td>White-box</td>
<td>1.255 0.975 2.091 2.574</td>
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<tr>
<td>80%</td>
<td>VM</td>
<td>0.402 6.361 11.687 14.975</td>
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<tr>
<td></td>
<td>White-box</td>
<td>0.106 1.503 2.563 2.753</td>
</tr>
<tr>
<td>90%</td>
<td>VM</td>
<td>0.111 4.030 6.366 8.976</td>
</tr>
<tr>
<td></td>
<td>White-box</td>
<td>0.081 1.183 1.242 1.825</td>
</tr>
</tbody>
</table>

Fig. 18. Errors for mean response time approximations using the white-box (upper row) and black-box (lower row) solutions.
Fig. 19 presents the results for these cases using the black-box approach, applying the $\Delta$ values measured from the respective systems at $n = 100$ to the ones with 200, 500, and 1,000 nodes. One can see that the results for the Round-Robin cases are close to those in the previous scenario. This is due to the fact that the Round-Robin policy mainly performs load balancing between replica and thus the effective service time distributions on the Fork nodes are almost unchanged. In contrast, the model yields good predictions for the redundant-task-issue policy for the entire load range under study. This is largely because this policy curbs the tail effects and makes the effective service time distributions less heavy-tailed. These results agree with those from the previous scenarios for less heavy-tailed distributions, i.e., Gamma and Weibull.

### 3.2.3 Scenario 3: Systems With Variable Numbers of Tasks

For illustrative purposes, we validate the results on Fork-Join models with homogeneous, single-server Fork nodes with the above service time distributions using the black-box solution, assuming that the tasks for each incoming job is randomly dispatched to 40–60 percent total number of Fork nodes. As a result, the effective load on each Fork node is half of that on the single-server systems in Scenario 3.2.1. Therefore, we double the arrival rate, $\lambda$, to keep the same arrival rate on each node as in the previous cases. The results of this scenario are shown in Fig. 20. Similar to the previous scenarios, the black-box solution gives accurate predictions across the entire load range for light-tailed distributions, e.g., Exponential, Gamma (which is not shown here), while yielding good approximations for the heavy-tailed distributions, i.e., truncated Pareto and empirical, at high load regions, e.g., 80 percent or above.

### 3.2.4 Scenario 4: A Case Study on Amazon EC2

We also evaluate the accuracy of the black-box solution for the case study on AWS EC2 as in Section 3.1.1. To illustrate the effectiveness of the black-box solution for this case study, we compute the gap for the 32-worker cluster and apply it to the approximation of request mean response time for the case of the 64-worker cluster. Table 8 presents the prediction errors for this case study. Again, the black-box method predicts mean response time quite accurately when the system at the effective load of 60 percent or higher, corresponding to arrival rates greater than 3.5 requests/s.

Finally, we note that the tail effect is a recognized issue in datacenter applications and tail-cutting techniques are often exploited in datacenters to reduce the tail effects [1], [15], [16], [38]. As a result, the effective service time distributions tend to be less heavy-tailed. Therefore, ForkTail and Fork-Mean show a great potential to be able to accurately predict the tail and mean latencies in a wide load range in practice, not limited to a high load region.

### 4 Sensitivity Analysis

From all the experiments above, we can see that the proposed approximations can be applied to a wide range of
systems with reasonable prediction errors for the 99th percentile and mean job latency, consistently within 20 and 15 percent at the loads of 80 and 90 percent, respectively. Now, the question yet to be answered is how much impact these errors will have on the accuracy for resource provisioning at high loads. To this end, we conduct a sensitivity analysis of tail and mean latencies as functions of load.

We perform experiments with different load levels in the high load region, i.e., 78 to 95 percent, for FJQNs with different service time distributions, i.e., exponential, Weibull, truncated Pareto, and empirical ones. Figs. 21 and 22 shows results from both simulation and the proposed approximations for 1,000-node systems. First, we note that the proposed models consistently overestimate the tail and mean latencies for the exponential and Weibull cases, while mostly underestimates them for the truncated Pareto and empirical cases. In other words, the former causes resource overprovisioning, whereas the latter leads to resource underprovisioning. Then the question is how much. Take the exponential case as an example, the predicted mean latency at 90 percent load is roughly equal to the simulated one at 91 percent load. This means that the model may lead to 1 percent resource over provisioning for the exponential cases. Following the same logic, it is easy to find that for both exponential and Weibull cases, the prediction models for both tail and mean latency may result in no more than 1 percent resource overprovisioning in the entire 78–95 percent load range. By the same token, we find that for the truncated Pareto and empirical cases, the models may cause up to 4 and 6 percent resource underprovisioning at 80 percent load and 2 and 1 at 90 percent load for tail and mean latency, respectively. This can be well compensated for by leaving a 6 percent resource margin in practice. This implies that in the worst-case when the actual service time distribution is light-tailed, our approximations may cause up to 7 percent resource overprovisioning at the loads of 80 percent or higher, given that we don’t have the knowledge about the tail-heaviness of the workload. With the prediction and the small overprovisioning to compensate the prediction error proposed in this paper, one can expect to run the system at 90 percent instead of 50 percent resource utilization with tail and mean latency guarantee.

Our sensitivity analyses for the other Fork-Join structures, which are not shown here, have led to similar conclusions. This demonstrates the effectiveness of our prediction models as a powerful means to facilitate multi-SLO-guaranteed, e.g., tail and mean latency guaranteed job scheduling and resource provisioning for datacenter applications.

5 Applicability Range

In this section, we want to answer the following question: In what parameter range can our models predict the request latency within 20 percent errors at high load? To this end, we note that we need to focus on identifying the applicability range on the heavy tail end, rather than the light tail end for two reasons. First, from the extensive experiments above, we found that our methods give quite accurate approximations for tail and mean latency for a wide range of loads for light-tailed distributions, e.g., Exponential, Gamma, and Erlang-2. Second, in practice, server workloads in datacenters exhibit heavy-tailed distributions [15], [32]. Also, the heavy-tailed truncated Pareto distribution given in Eq. (13) was found to be a good fit for empirical data from server workloads [31]. Hence, in what follows, we test the applicability range of our approximations based on this distribution.

From extensive experiments with the truncated Pareto distribution, we found that our approximations predict the tail and mean latencies within 20 percent errors at the loads of 80 percent or higher, when the tail index \( \alpha \) in Eq. (13) is less than 2, i.e., \( 0 < \alpha < 2 \). This range of \( \alpha \) was found to be large enough to cover the server workloads in [31].

<table>
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<tr>
<th>Effective load (Arrival rate (requests/s))</th>
<th>#workers</th>
<th>50.0%</th>
<th>58.4%</th>
<th>66.7%</th>
<th>75.1%</th>
<th>83.4%</th>
<th>91.7%</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0%</td>
<td>(3.0)</td>
<td>64</td>
<td>31.678</td>
<td>10.489</td>
<td>7.817</td>
<td>8.874</td>
<td>15.274</td>
</tr>
<tr>
<td>58.4%</td>
<td>(3.5)</td>
<td>64</td>
<td>31.678</td>
<td>10.489</td>
<td>7.817</td>
<td>8.874</td>
<td>15.274</td>
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<tr>
<td>66.7%</td>
<td>(4.0)</td>
<td>64</td>
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<td>7.817</td>
<td>8.874</td>
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Fig. 21. Differences in the 99th percentile response times from simulation and ForkTail for 1000-node systems with different service time distributions and fixed number of Fork tasks.

Fig. 22. Differences in mean response times from the simulation and black-box ForkMean for 1000-node systems with different service time distributions and fixed number of Fork tasks.
To further test if today’s datacenter workloads indeed fall into the above range, we test the fitting of the truncated Pareto distribution to the workload traces from Facebook and Google provided in [19]. These traces include a mixture of different types of workloads placed on datacenter servers. To simulate the workload on one server, we draw 10,000 random samples from each trace and fit them to the truncated Pareto distribution based on the procedure suggested in [39], which uses the \((r+1)\) largest-order statistics and visual check. We found that the fitted values of \(a\) for Google and Facebook samples are mostly within the applicability range of \((0,2)\). Fig. 23 illustrates two examples of the fitted curves.

The above results strongly suggest that our proposed methods can indeed serve as a useful tool for the approximation of tail and mean latency for datacenter workloads.

6 Facilitating Job Scheduling and Resource Provisioning

We now discuss how our proposed approximations may be used to facilitate both SLO-guaranteed job scheduling and resource provisioning. We present here only the procedures for tail latency approximation, i.e., ForkTail. The procedures for mean latency follow similar steps since the approximation of mean latency is based on ForkTail. The proposed ideas are preliminary and somewhat sketchy, but yet, they do help reveal the promising prospects of our proposed model and point directions for future studies on this topic.

Job Scheduling. We describe the ideas of how a tail-latency-SLO-guaranteed hybrid centralized-and-distributed job scheduler can be developed, based on ForkTail. The main idea is to rely on distributed measurement of the means and variances of the task response times and centralized decision making as to how and whether the request tail-latency SLO can be met, as depicted in Fig. 24. In the master server on the left resides the central job scheduler to which users submit their requests with given tail-latency SLOs. All the servers in the cluster measures the means and variances of task response times for tasks of different sizes or in different bins on a continuous basis. All the servers periodically convey their measurements to the central scheduler. Upon the arrival of a request with a given tail-latency SLO and given \(k\) tasks to spawn, based on Eq. (5), the central scheduler will run a Fork-node selection algorithm to determine which \(k\) Fork nodes should be used such that the tail-latency SLO can be met. If such \(k\) Fork nodes are found, the request will be admitted, otherwise, either the tail-latency SLO will be renegotiated or the request will be rejected. At runtime, the central scheduler periodically run the prediction model using the up-to-date means and variances as input to ensure that the tail-latency SLOs for the on-going requests continue to be met.

Resource Provisioning. ForkTail for the homogeneous case (i.e., Eqs. (8) and (9)) naturally enables a resource provisioning solution involving two steps: (a) the evaluation of the task-level performance requirements to achieve a given tail-latency SLO; and (b) the selection of an underlying platform to meet the requirements. Here, step (a) is platform independent and hence is portable to any datacenter platforms.

For example, consider a service deployment scenario with a given tail-latency SLO and a minimum throughput requirement, \(R\). Assuming that \(N\), \(k_i\), and \(P(K = k_i)\) for the given service are known, Eq. (9) can be used to first translate the tail-latency SLO into a pair, i.e., the mean and variance of the task response time. This pair then serves as the task performance requirements, which are platform independent and portable. This completes step (a).

In step (b), a Fork node is set up, e.g., using three virtual machine instances purchased from Amazon EC2 to form a 3-replica Fork node, loaded with a data shard in the memory. Then run tasks at increasing task arrival rate \(\lambda\) until the measured task mean and/or variance are about to exceed the corresponding budget(s). At this arrival rate \(\lambda\), the tail-latency SLO is met without resource over-provisioning. In other words, the \(\lambda\) value at this point would be the maximum sustainable task throughput, or equivalently, the request throughput, in order to meet the tail-latency SLO. If this throughput is greater than \(R\), the minimum throughput requirement is also met. This means that the resource provisioning is successful and a cluster with 3VM instances can be deployed. Otherwise, repeat step (b) by using a more powerful VM instance or with a re-negotiated tail-latency SLO and/or minimum throughput requirement.

7 Related Work

Fork-Join structures are traditionally modeled by FJQNs, which have been studied extensively in the literature. To date, the exact solution exists for a two-Fork-node FJQN only [10], [40]. Most of the previous works primarily focus on the approximation of mean response time [10], [11], [41] and its bounds [42], [43]. For networks with general service time distribution, several works have introduced hybrid approaches that combine analysis and simulation to derive the empirical approximation for mean response time [10], [13].

Some analytic results are available on redundant task issues [44], [45], [46]. They either address only a single replicated
Various works have been done in tail-latency related research, several works dealt with the approximation of response time distribution assuming a simple queuing model for each Fork node, e.g., M/M/1 [47] or M/M/k [12]. Computable stochastic bounds on request waiting and response time distributions for some FJQNs are provided in a recent work [48]. The most interesting and relevant work is given in [14]. The authors of this work proposed a method, called EAT, for the approximation of tail latency for homogeneous FJQNs based on the analytical results from single-node and two-node cases. The approximation applies to FJQNs with any service time distribution that can be transformed into a phase-type distribution. Although outperforming our solutions by a few percentage points in terms of tail prediction, its computational complexity renders it infeasible to facilitate online resource provisioning. Moreover, this work can only cover a small fraction of the aforementioned design space and hence, cannot be used to facilitate resource provisioning in practice.

Mean Latency Approximation. Various works have been proposed for the approximation of mean response time of FJQNs using model-based or hybrid approaches. The work in [10] introduces a hybrid approach for the approximation of mean response time, $R_n$, for a Fork-Join model with $n$ M/M/1 Fork nodes ($2 \leq n \leq 32$) based on the exact solution for the 2-way network [40] and simulation. In [36], the authors proposed an approximation for mean response time based on the optimistic and pessimistic bounds. Another approximation for mean response time of Fork-Join models with general inter-arrival and service time distributions is proposed in [37] based on light traffic interpolation and heavy traffic limit. The light traffic interpolation is computed from the mean response time of the Fork-Join network when there is only a tagged job in the network, which is equivalent to the maximum of task service time random variables. The heavy traffic limit is postulated based on the observation of the relationship between expressions for light and heavy traffic for 1-way and 2-way networks. In [29], the authors proposed a hybrid procedure for the approximation of mean response time for Fork-Join models with M/G/1 queues. Indeed, this work proposed a methodology rather than specific expressions for finding mean response time. In a recent work [49], a simulation study assessed the accuracy of the approximation based on order statistic.

The existing approaches above are white-box solutions targeting at individual Fork-Join models with specific queuing server models. In contrast, in this paper, we propose both white-box and black-box solutions, applicable to Fork-Join networks with arbitrary server models.

SLO-Aware Resource Provisioning. Due to the lack of theoretical underpinning, the existing SLO-aware resource provisioning proposals cannot provide tail and/or mean latencies SLO guarantee by design. Instead, various techniques such as tail-cutting techniques [15], [16], a combination of job priority and rate limiting based on network calculus [50] are employed to indirectly provide high assurance of meeting tail-latency SLOs. As indirect solutions, however, they cannot ensure precise resource allocation to meet tail-latency SLOs, while allowing high resource utilization, and hence may result in resource overprovisioning. Yet, another alternative solution is to track the target tail-latency SLO through online, direct tail-latency measurement and dynamic resource provisioning [51], [52]. This approach, however, may not be effective, especially in enforcing stringent tail latency SLOs. To see why this is true, consider the 99.9th percentile request response time of 200 ms, i.e., probabilistically, only one out of 1,000 requests should experience a response time greater than 200 ms. Assume that the average request arrival rate is 50 per second. To track, through direct tail-latency measurement, whether this tail latency SLO is violated or not with reasonably high confidence, one needs to collect, e.g., 100K samples to see if there are more than 100 requests whose response times exceed 200 ms. This, however, takes about 100K/50 = 2000 seconds or about 33 minutes of measurement time! Given possibly high volatility of datacenter workloads, the tail latency SLO may have been violated multiple times during this measurement period, even though the total number of requests whose response times exceeding 200 ms may be well within 100. In constraint, using our proposed models, with only 20 seconds of measurement time, one can collect 20 × 50 = 1000 task samples at individual Fork nodes to allow a reasonably accurate estimation of the means and variances of task response times. With moving average for a given time window, e.g., 20 seconds, these means and variances and hence, the 99.9th percentile, can be updated every tens of milliseconds, making it possible to enable fast online tail-latency-guaranteed job scheduling and resource provisioning.

In summary, a solution that can predict the tail and/or mean latency using a small number of samples collected in a short period of time as input and that applies to a large design space of Fork-Join structures must be sought, the primary motivation of the current work.

8 CONCLUSION AND FUTURE WORK

A key challenge in enabling tail-latency and/or mean-latency SLOs for data-intensive services and applications in datacenters is how to predict the latencies for a broad range of Fork-Join structures underlying those services and applications. In this paper, we proposed to study a generic black-box Fork-Join model for the approximations of tail and mean latency that covers most Fork-Join structures of practical interests. On the basis of a central limit theorem for queuing servers under heavy load, we were able to arrive at approximate solutions to this model for both tail and mean latencies, called ForkTail and ForkMean, respectively. These approximations were found to be able to predict the tail and mean latencies for most practical scenarios consistently within 20 percent in a load region of 80 percent or higher, resulting in at most 7 percent resource overprovisioning, making it a powerful tool for resource provisioning at high load. Finally, we discussed some preliminary ideas of how to make use of the proposed prediction model to facilitate tail-latency-SLO-guaranteed job scheduling and resource provisioning.

In our future work, based on ForkTail and ForkMean, we shall develop both job scheduling and online/offline resource provisioning solutions with tail-latency and/or mean-latency SLO guarantee.
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ESetStore: An Erasure-Coded Storage System With Fast Data Recovery

Chengjian Liu, Qiang Wang, Xiaowen Chu, Senior Member, IEEE, Yiu-Wing Leung, and Hai Liu, Member, IEEE

Abstract—Erasure codes have been used extensively in large-scale storage systems to reduce the storage overhead of triplication-based storage systems. One key performance issue introduced by erasure codes is the long time needed to recover from a single failure, which occurs constantly in large-scale storage systems. We present ESetStore, a prototype erasure-coded storage system that aims to achieve fast recovery from failures. ESetStore is novel in the following aspects. We proposed a data placement algorithm named ESet for our ESetStore that can aggregate adequate I/O resources from available storage servers to recover from each single failure. We designed and implemented efficient read and write operations on our erasure-coded storage system via effective use of available I/O and computation resources. We evaluated the performance of ESetStore with extensive experiments on a cluster with 50 storage servers. The evaluation results demonstrate that our recovery performance can obtain linear performance growth by harvesting available I/O resources. With our defined parameter recovery I/O parallelism under some mild conditions, we can achieve optimal recovery performance, in which ESet enables minimal recovery time. Rather than being an alternative to improve recovery performance, our work can be an enhancement for existing solutions, such as Partial-parallel-repair (PPR), to further improve recovery performance.

Index Terms—ESetStore, ESet, Erasure coded storage systems, Fast data recovery

1 INTRODUCTION

Recent years have witnessed rapid growth in the amount of data in large-scale distributed storage systems. In 2015, the European Centre for Medium-Range Weather Forecasts revealed that its data had reached 100 PB and had an annual growth rate of 45 percent [1]. A recent study illustrated that genomic big data have reached full storage of a data center with a 100-PB storage capacity [2]. Triplication [3], which is a reliability mechanism used in traditional storage systems, introduces unaffordable storage costs with 3x storage overhead. This makes the reduction of storage overhead an unavoidable task in large-scale storage systems. Many storage systems have begun to use erasure codes as their reliability mechanism [4]. Microsoft’s cloud service Azure [5], Facebook’s warehouse [6], and Web service storage system f4 [7] have already adopted erasure codes to reduce their storage costs. Famous distributed file systems such as HDFS [8] and Ceph [9] also support erasure coding, yielding greater reliability and lower storage overhead.

An erasure-coded storage system is defined by two integer parameters, \( n \) and \( k \). A file stored in the system is divided into \( k \) equally sized data blocks. These data blocks generate \( n - k \) equally sized parity blocks. The blocks are stored in \( n \) storage components to protect data against up to \( n - k \) failures. The storage overhead is \( n/k \). For example, Facebook f4 [7] sets \( k \) as 10 and \( n \) as 14, where its storage overhead is 1.4x. The QFS sets \( k \) as 6 and \( n \) as 9 with 1.5x storage overhead [10]. However, an erasure-coded storage system may suffer from many performance penalties, one of which is the long time needed to recover a failed storage component, which can be a disk device or a storage server. To recover a missing block, \( k \) blocks are retrieved from \( k \) storage components. Because recovering a failed component is a constantly performed task [6], the long recovery time may introduce degraded service quality to the whole system.

Studies have shown that single failures accounted for more than 99 percent of recoveries [5], [11], [12]. Recovery from a single failure is a performance critical operation in erasure-coded storage systems [11], [13]. This motivated us to focus on recovery from a single failure to evaluate recovery performance of erasure-coded storage systems. A single failure can refer to the failure of a single disk and to the failure of a single storage server. Here we regard a single failure as the failure of a single storage server. The recovery of both kinds of failures requires disk I/O and network bandwidth, both of which can be regarded as I/O resources. We aim to gain adequate I/O resources for each recovery.

The key reason behind the long time needed to recover from a single failure is the heavy I/O operations, which is \( k \) times the replication-based storage. To improve recovery performance, researchers have proposed solutions from the following aspects. Some work reduces the I/O operations required to recover a failed component. The Microsoft cloud...
service Azure adopted local repair codes (LRC) that require around \( k/2 \) blocks to recover a missing block, but at the cost of increasing its storage overhead from \( n/k \) to \( (n+2)/k \) [5]. The Hitchhiker reduces I/O operations by 25 to 45 percent for recovery of single failures [14]. Some other works improve I/O utilization of the recovery. Partial-parallel-repair pipelined I/O operations to introduce \( k \) times of improvement of recovery performance [15]. A recent work studied the pipelined recovery for heterogeneous environments [16].

We can conclude that the long recovery time is caused mainly by limited I/O resources for the recovery of a single failure. Data placement algorithms, a promising means to aggregate the desired I/O resources to recover from a single failure, are ignored by studies of erasure-coded storage systems. This motivated us to propose ESetStore, an erasure-coded storage system with fast data recovery via a novel data placement algorithm ESet. To this end, we designed a data placement algorithm named ESet to harvest adequate I/O resources for recovery of a single failure [17]. Our solution can be an enhancement to existing solutions to improve the recovery performance of erasure-coded storage systems. Our major contributions are summarized as follows:

1. We present a placement algorithm named ESet to improve recovery from failures.
2. We conduct a theoretical analysis to illustrate how ESet can achieve desired I/O aggregation for recovery of a single failure with proper configuration.
3. Rather than being an alternative to existing solutions for improving recovery performance, the placement algorithm ESet can be an enhancement to existing solutions to bring better recovery performance.
4. We achieve efficient I/O utilization for the read and write operations in ESetStore by making an efficient utilization of both I/O and computation resources.
5. We conduct extensive experiments to validate the effectiveness of our proposed ESetStore, and compare it with two storage systems: HDFS [8] and Ceph [9].
6. ESetStore is open-source and available to the public.\(^1\)

The remainder of this paper is organized as follows. Section 2 introduces the background and some related work. Section 3 formulates the problem of long recovery time caused by improper data placement. We present the design of ESet in Section 4. The analysis of ESet is presented in Section 5. The design and implementation of ESetStore are presented in Section 6. Section 7 presents the experimental results of ESetStore. We conclude the paper in Section 8.

2 BACKGROUND AND RELATED WORK

In this section, we first briefly introduce the background of erasure-coded storage systems. Then, we present some related work about the optimization of the recovery.

2.1 Erasure-Coded Storage Systems

Fig. 1 presents a concrete example with \( n = 9 \) and \( k = 6 \). In this example, we assume that the system uses the Reed-Solomon code [18] as the erasure code, which satisfies the Maximum Distance Separable (MDS) property [19]. When writing data to the storage system, the raw data are divided into \( k \) equally sized data blocks. A generator matrix serves as another input. The matrix consists of a \( k \times k \) identity matrix and a \( (n-k) \times k \) matrix, where each element in the \( (n-k) \times k \) matrix is an integer. The matrix multiplies with the set of data blocks to generate \( n \) blocks. The set of \( n \) blocks with \( k \) data blocks and \( n-k \) parity blocks together is called a stripe. The set of blocks is then distributed into \( n \) storage components to tolerate up to \( n-k \) failures.

The selected \( n \) devices for storing a stripe are typically from \( n \) distinct failure domains to minimize the probability of data loss. A failure domain is a physical container that includes a set of storage servers. When the container encounters a failure, all servers in the container become unavailable. The failure domain is a frequently used term in discussions of system failures [7], [20], [21], [22]. In this paper, we consider each rack as a failure domain. Stripes are distributed across failure domains to protect data loss against failures from the same failure domain.

When any storage device becomes unavailable, the storage system must recover the data within it. For each missing block in a stripe, \( k \) blocks are gathered together to reproduce it. The process of recovering a missing block is performed repeatedly until all blocks in the failed storage device become available. This recovery process has become a routine job in large-scale storage systems. For example, at Facebook, which has a production cluster of 3,000 storage servers, around 20 storage servers will encounter failures and require recovery each day [23]. It was also revealed that the recovery of a failed server storing about 150 blocks (each block size is 64 MB and \( k = 10 \)) could take more than 50 minutes, which indicates that recovery performance plays a critical role in storage systems.

Recovery occupies heavy I/O resources in erasure-coded storage systems. Thus, recovering a single failure takes a long time if I/O resources are limited. Reducing the required I/O operations is a major approach to improving the recovery performance of erasure-coded storage systems [5], [14]. Some studies have attempted to reduce recovery time by making better use of available I/O resources [15], [16]. However, the performance is still limited by available I/O resources. This motivated us to design and implement a data placement algorithm to improve the available I/O resources for recovery of single failures. Our work can be used with existing solutions such as Partial-parallel-repair (PPR) [15] to further improve the recovery performance of erasure-coded storage systems by better I/O utilization.

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1. Available at https://github.com/stevenlcj/ESetStore

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![Fig. 1. Erasure-Coded storage with n = 9 and k = 6 using RS code.](image-url)
2.2 Related Work

Recovery of missing data comprises two operations: data gathering and decoding. The performance of decoding was improved by some recent studies like those by Gibraltar [24], PErasure [25] and G-CRS [26]. This leaves I/O operations as the major bottleneck of recovery. We classify the existing studies into two categories: one attempts to reduce I/O operations, and the other attempts to improve I/O utilization. We also investigate why some state-of-the-art data placement algorithms are not suited for fast data recovery in erasure-coded storage systems.

Reducing I/O Operations. XOR-based erasure codes access less data when recovering a single failure. Some researchers proposed optimization techniques for specific codes like RDP Code [27] and X-Code [28]. The rotated Reed-Solomon codes hold the reliability and performance properties of standard Reed-Solomon codes and can reduce the data required for recovery by up to 30 percent [11]. Algorithms have also been proposed to find solutions for reading less data for XOR-based erasure codes [13], [29].

Some studies sought solutions to reduce the required I/O operations by half but at the cost of increased storage overhead. The family of LRCs [30] is representative: it encodes each k/2 of data out of a stripe and stores the parity in a new storage device. When performing recovery, only k/2 of data blocks are required to recover a missing block. The solution is used in Microsoft Azure [5]. The XORing Elephants studied the LRCs and noted that the cost of faster recovery is a 14 percent increase in storage overhead [23].

The family of regenerating codes [31], which hold the properties of MDS codes [19], reduced the amount of data needed to recover a failed storage component with special constraints on k and n-k. Recent studies proposed some solutions without special constraints. A new storage code developed from the Piggybacking framework [32] can reduce network and disk use during recovery by 30 percent and is used in Facebook’s warehouse cluster [6]. The Hitchhiker can reduce I/O operations by around 25 to 45 percent for recovery of a single failure [14], where it also holds the property of MDS codes and supports arbitrary n and k.

Improving I/O Utilization. When recovering a block from a stripe, k blocks from the same stripe are retrieved from k storage servers concurrently. Because each server has the same network throughput, the network bandwidth is underutilized in the recovery. The Partial-parallel-repair (PPR) makes use of this and proposed a pipelined mechanism to make full use of the available bandwidth when performing recovery [15].

The repair pipelining technique was proposed in [16] to make better use of bandwidth for small-size units. An ECPipe was developed to allow the pipelined mechanism to work in heterogeneous environments [16].

Table 1 lists the main properties of some state-of-the-art data placement algorithms. A random placement algorithm can work with both replication and erasure-coding based storage systems [35]. It can select an arbitrary number of storage servers to store data. However, this algorithm is not reliable in large-scale storage systems, as revealed in [21].

Copysets addresses the issue of random placement for replication-based storage systems [21]. The probability of data loss is reduced greatly when random placement is replaced with with Copysets. The parameter “scatter width” can help each storage server in a replication-based storage system achieve efficient data reconstruction. However, this algorithm was designed for replication-based storage systems and may not work for erasure-coded storage systems to improve recovery performance when the number of storage servers in each copyset is increased.

CRUSH [22], from the family of RUSH algorithms [36], [37], is a well-known algorithm adopted in the storage system Ceph [9]. It uses a pseudo-random algorithm to select a set of storage servers when storing data. However, it is not designed to satisfy the property of efficient recovery for erasure-coded storage systems.

For erasure-coded storage systems, researchers have considered placement algorithms to reduce cross-rack traffic when recovering a single failure [38]. The study case is the situation in which a rack contains more than one block of a given stripe. However, when we need to simultaneously tolerate disk-level, host-level, and rack-level failures [7], cross-rack traffic may not be reduced by them.

In summary, the state-of-the-art data placement algorithms do not allow efficient reconstruction in erasure-coded storage systems. This motivated us to design and implement the placement algorithm ESet, which allows efficient recovery in erasure-coded storage systems.

3 Problem Formulation

In this section, we formulate the problem of efficient reconstruction for erasure-coded storage systems. Our study focuses on the recovery of the single failure. We first explain some terms, and then illustrate our motivation by presenting...
a simple example. We finish this section by making a formal formulation of the problem.

3.1 Terminologies

**Block.** A block is a sequence of bytes with a fixed-length. Our storage system includes two kinds of blocks: data blocks and parity blocks. Each data block contains raw data stored by users. The parity blocks are generated from data blocks to protect raw data. The erasure-coded storage system record the location of each file by memorizing the location of its blocks in a metadata server.

**Erasure Code.** We use Reed-Solomon (RS) code as an example of erasure code in this paper. In an \((n, k)\) RS code, where \(n\) is greater than \(k\) and \(k\) is greater than 1, \(k\) data blocks are put together to generate \(n - k\) parity blocks (or code blocks). When no more than \(n - k\) blocks fail in these blocks, any \(k\) remaining blocks can be used to restore the missing blocks. Although the encoding and decoding operations involve huge computation, many existing studies have resolved this challenge with multicore CPU and many-core GPU accelerations [24]. Disk and network I/O overheads are currently the major challenge for such storage systems.

**Stripe.** A stripe contains \(n\) blocks in our system, where \(k\) blocks are data blocks and the other \(n - k\) blocks are parity blocks. Any \(k\) blocks in the same stripe can be gathered together to reproduce other \(n - k\) blocks in the system.

**Erasure-Coded Set.** Each stripe can be distributed in \(n\) disks from \(n\) distinct storage servers in the storage system. We define an erasure-coded set as a set of \(n\) storage servers to carry a complete stripe. It can serve as a unit of failure in erasure-coded storage systems. When a storage server fails, all erasure-coded sets that contain the failed storage server will begin to recover the missing blocks in the server.

**Recovery of Single Failures.** A single failure can be a disk failure or a storage server failure. The recovery of a disk or a storage server consumes both disk I/O and network bandwidth from \(k\) storage servers, which can both be regarded as I/O resources. We aim to gain adequate I/O resources for the single failure recovery. We regard the recovery of a single failure as the recovery of a storage server.

**Recovery I/O Parallelism.** Here we define a concept of recovery I/O parallelism. We use the symbol \(I\) to denote the degree of recovery I/O parallelism in the system, which indicates the number of erasure-coded sets to which each storage server belongs. A larger value for \(I\) indicates that more erasure-coded sets will be available to recover a failed storage server. This may leave more I/O resources for recovery of a failed server. Thus, we can achieve efficient data reconstruction from a single failure by setting a proper value of \(I\).

3.2 Motivation Illustration by an Example

A placement algorithm can decide the number of erasure-coded sets for each storage server. The number of erasure-coded sets for a server indicates the amount of I/O resources that can be involved for recovery from a single failure. More I/O resources may result in better recovery performance.

We begin the demonstration of our motivation with an example in Fig. 2. Here we set \(n = 3\) and \(k = 2\). Six storage servers are indexed from 1 to 6 and organized in three columns. We assume that each column belongs to one rack. Blocks in the same stripe are distributed across racks to tolerate disk-level, host-level, and rack-level failures. We have four stripes indexed from 1 to 4. We describe two data placement algorithms to distribute the stripes. Suppose the network bandwidth is \(\phi\) and the block size is \(\omega\). The time needed to recover a block can be calculated as \((k \times \omega)/\phi\), which is the time needed to gather \(k\) blocks into a single server by reading blocks one by one.

Placement algorithm A, which is shown at the bottom-left of Fig. 2, divides storage servers into two erasure-coded sets. Each erasure-coded set carries two stripes. Placement algorithm B (on the right side of Fig. 2) partitions the six storage servers into four erasure-coded sets. In this way, each erasure-coded set carries one stripe.

Both placement algorithms meet the requirement to distribute blocks across racks to tolerate rack-level failures. The data are evenly distributed among six storage servers, which means that each server stores the same amount of data. However, the recovery performance will differ for placement algorithms A and B when any failure occurs. Note that the placement algorithm A and B differ from standard parity declustering [33]. The standard parity declustering will distribute parity across all disks. As a result, it is not able to tolerate rack-level failures. We will clarify the difference between ESet and parity decluster in Section 5.

For algorithm A, each storage server belongs to one erasure-coded set, so the value of \(I\) is 1. For algorithm B, each storage server belongs to two erasure-coded sets, so the value of \(I\) is 2. If storage server 1 fails as illustrated in the figure, the time for recovery is \((2 \times k \times \omega)/\phi\) with placement algorithm A, and the time for recovery is \((k \times \omega)/\phi\) with placement algorithm B. This is because in algorithm B, two erasure-coded sets can proceed in parallel to recover missing blocks.

The recovery performance of placement algorithm B is twice that of placement algorithm A, which means that a higher value for \(I\) can allow better recovery performance. For fast recovery of these systems, a placement algorithm must be able to set a proper \(I\) for each storage server.

Given the above description, we formulate the problem as follows. Given a data center with \(\alpha\) racks and \(\beta\) storage
servers, we build an erasure-coded storage system to store \( s \) stripes. The problem is how to design a data placement algorithm that makes each storage server stay in \( I \) erasure-coded sets, where \( I \) is a parameter configured by the system administrator when initializing a storage system to indicate the desired recovery performance; meanwhile, the algorithm can evenly distribute data in each storage server, i.e., each storage server stores \( ns/\beta \) blocks, and the algorithm must also distribute each stripe across racks to tolerate rack-level failure. In this way, the recovery time of each storage server can approximate \( (ns/\beta) \times ((k \times \omega)/\phi)/I \), where \( ns/\beta \) is the number of blocks in each storage server and \( (k \times \omega)/\phi \) is the time needed to recover a single block. In a word, the solution to the problem should have the recovery time for each storage server approximate \( (ns/\beta) \times ((k \times \omega)/\phi)/I \).

4 DESIGN OF ESet

In this section, we present the design of the data placement algorithm ESet. To minimize the probability of data loss, we partition the storage servers into many ESetGroups, where each ESetGroup contains \( n \times I \) storage servers organized into \( n \) rows and \( I \) columns. We form \( I^2 \) erasure-coded sets in each ESetGroup. Each storage server in an ESetGroup is contained by \( I \) erasure-coded sets.

4.1 Construction of ESetGroups

To minimize the probability of data loss, we must distribute \( n \) blocks from each stripe across failure domains. We first transform the whole data center into a set of ESetGroups. Each ESetGroup is a set with \( n \times I \) storage servers organized into \( n \) rows and \( I \) columns. The storage servers in each ESetGroup are indexed from \( N_{0,0} \) to \( N_{n-1,I-1} \). The first subscript represents storage server’s row, and the second subscript represents its column within its ESetGroup. In each ESetGroup, the storage servers from any two different rows share no common failure domain. We can construct each erasure-coded set by selecting \( n \) storage servers from \( n \) rows of an ESetGroup. In this fashion, we can achieve the minimum data loss probability when storing stripes that distribute blocks across failure domains.

Algorithm 1. Construction of ESetGroups

1: Input: Data Center with \( \beta \) storage servers, \( n, I \)
2: Output: \( g \) ESetGroups indexed from \( G_0 \) to \( G_{g-1} \)
3: Set the value of \( g \) as \( \beta/(n \times I) \)
4: Initialize \( g \) ESetGroups indexed from \( G_0 \) to \( G_{g-1} \)
5: for each \( i \in [0,g-1] \)
6: for each \( j \in [0,n-1] \)
7: Find and select \( I \) servers from the Data Center that share no common failure domain with the storage servers in \( G_i \)
8: Add selected servers to the \( j \)'th row of \( G_i \)
9: Remove selected servers from the Data Center
10: end for
11: end for
12: Return \( g \) ESetGroups

Here, we present Algorithm 1 to divide \( \beta \) storage servers in the data center into \( g \) ESetGroups, where \( g \) is equal to \( \beta/(n \times I) \). The ESetGroups are indexed from \( G_0 \) to \( G_{g-1} \).

Each storage server can only remain in one ESetGroup. The for-loop (from line 6 to line 10 in the Algorithm 1) iterates \( n \) times to generate an ESetGroup. When selecting \( I \) storage servers as one row of an ESetGroup, the selected storage servers and other storage servers in the ESetGroup must share no common failure domain (at line 7).

4.2 Construction of Each Erasure-Coded Set

We generate an erasure-coded set by selecting \( n \) storage servers from \( n \) rows of an ESetGroup. Because each storage server in an ESetGroup remains in \( I \) erasure-coded sets, we must generate \( I^2 \) erasure-coded sets for each ESetGroup.

Here we use the symbol \( V \) to denote a storage server in an erasure-coded set. The storage servers in an erasure-coded set are indexed from \( V_0 \) to \( V_{n-1} \). The subscript refers to the index of the storage server in the erasure-coded set.

The process of generating each erasure-coded set is shown in Algorithm 2 with a given ESetGroup \( G_i \). The first for-loop (from line 4 to line 12) iterates \( I^2 \) to generate \( I^2 \) erasure-coded sets. Each erasure-coded set selects \( n \) storage servers from \( n \) rows of the \( G_i \). The first storage server \( V_0 \) in an erasure-coded set comes from the storage server in the first row of \( G_i \). The \( r \)th storage server \( V_r \) in an erasure-coded set is selected from the \( r \)th row of an ESetGroup.

Algorithm 2. Generate Erasure-Coded Sets

1: Input: \( G_i \), \( n, I \)
2: Output: \( \)Erasure-coded sets indexed from \( E_{i,j} \) to \( E_{(i+1),j-1} \)
3: set startIdx as \( i^2 \)
4: for each \( j \in [0,I^2-1] \)
5: Initialize \( E_{\text{startIdx}+j} \)
6: Set columnIdx as the value of \( j/I \)
7: Select the storage server \( N_{0,\text{columnIdx}} \) in \( G_i \) as the \( V_0 \) in \( E_{\text{startIdx}+j} \)
8: for each \( r \in [1,n-1] \)
9: Set columnIdx as the value of \((jj)(r-1)+j) \mod I \)
10: Select the storage server \( N_{r,\text{columnIdx}} \) in \( G_i \) as the \( V_r \) in \( E_{\text{startIdx}+j} \)
11: end for
12: end for
13: Return \( I^2 \) Erasure-coded sets

The procedure of translating an entire data center with \( a \) racks and \( \beta \) hosts into many erasure-coded sets are presented in Fig. 3. We partition the data center into \( g \)ESetGroups with Algorithm 1. Each ESetGroup contains \( n \times I \) storage servers organized in \( n \) rows and \( I \) columns. The Algorithm 2 then iterates \( g \) times to generate \( gI^2 \) erasure-coded sets. We set \( \epsilon \) as the value of \( gI^2 \). We have \( g \) erasure-coded sets to carry stripes, where each stripe contains \( n \) blocks. So far, we have illustrated how to generate all erasure-coded sets. Now we must assure each storage server appears in \( I \) erasure-coded sets to obtain the required recovery I/O parallelism. In the next section, we analyze the property of ESet algorithm.

5 ANALYSIS OF ESet

To ensure that each storage server has the same recovery I/O parallelism, each storage server is expected to remain in \( I \) erasure-coded sets. In this section, we first validate this property,
and then analyze the condition in which each storage server has the optimal recovery I/O parallelism.

5.1 Answer When $N_{a,b} \in E_j$

We first prove that our proposed ESet algorithm for constructing erasure-coded sets can make each storage server from any ESetGroup belong to I erasure-coded sets.

**Theorem 5.1.** Given a storage server $N_{a,b} \in G_0$, there are I and only I erasure-coded sets that for each erasure-coded set $E_j$, $N_{a,b} \in E_j$, that is $V_a$ from $E_j$ is selected from $N_{a,b}$, where $0 \leq a < n, 0 \leq b < I, 0 \leq i < g$, and $0 \leq j < I^2$.

**Proof of Theorem 5.1.** Based on Algorithm 2, the erasure-coded sets that contain the storage server $N_{a,b} \in G_0$ are indexed from $E_0$ to $E_{I^2-1}$. We can have $0 \leq j < I^2$ in Theorem 5.1 as $N_{a,b} \in E_j$. We divide the rest of the proof into two parts. First, we prove that I and only I erasure-coded sets contain $N_{a,b}$ when $a$ is equal to 0. We then discuss the case when $a \neq 0$.

**Case 1.** $a = 0$. The value of $b$ is equal to $j/I$ if and only if $N_{a,b} \in E_j$ according to line 6 in Algorithm 2. We can have that for any $E_j$ that $b = j/I$, $N_{a,b} \in E_j$. As $0 \leq j < I^2$ and $0 \leq b < I$, and I only I values of $j$ satisfy $b = j/I$, that is, $j$ is any value from $bl$ to $(bl + 1)$, respectively. To this end, we can conclude that I erasure-coded sets and only I erasure-coded sets contain the storage server $N_{a,b}$ when $a = 0$.

**Case 2.** $a \neq 0$. When $N_{a,b} \in E_j$, $b$ must be equal to $((j/I)(a-1)+j)mod I$ according to line 9 in Algorithm 2. If $N_{0,z} \in G_0$ satisfies the conditions that both $N_{0,z} \in E_j$ and $N_{a,b} \in E_j$, then it must satisfy the condition that $b$ is equal to $(x(a-1)+j) mod I$, where $0 \leq x < I$. As $xI \leq j < xI + 1$ according to the above paragraph, there will be one and only one ESet in which both $N_{0,z}$ and $N_{a,b} \in E_j$. Because $0 \leq x < I$, we can deduce that I and only I erasure-coded sets contain $N_{a,b}$ for the case that the when the value of $a$ is not equal to 0.

In a word, when any storage server fails, we have I erasure-coded sets to recover the failed storage server concurrently according to Theorem 5.1.

5.2 Optimal Recovery I/O Parallelism

When a storage server fails, I erasure-coded sets that contain the failed server will begin to recover missing blocks in the server. Each erasure-coded set will select $k$ storage servers to perform recovery. A storage server that participates in recovery may be selected by more than one erasure-coded set. As a result, I/O contention may occur for the selected storage servers during recovery, which will further decrease the recovery performance. To obtain optimal recovery I/O parallelism when recovering a single failure, there should be no I/O contention for the recovery. To this end, we give the theorem of optimal recovery I/O parallelism for a storage server as follows:

**Theorem 5.2.** Given a storage server $N_{x_1,y_1} \in G_0$, where $0 \leq x_1 < n, 0 \leq y_1 < I$ and $0 \leq i < g$, for any other storage server $N_{x_2,y_2} \in G_0$, where $x_1 \neq x_2, 0 \leq x_2 < n, 0 \leq y_2 < I$, if there is only one erasure-coded set that includes both $N_{x_1,y_1}$ and $N_{x_2,y_2}$, then the storage server $N_{x_1,y_1}$ can have optimal recovery I/O parallelism.

**Proof of Theorem 5.2.** Suppose $N_{x_1,y_1} \in E_a$ and $N_{x_1,y_1} \in E_b$, where $0 \leq x_1 < n, 0 \leq y_1 < I$ and $a \neq b$. The I/O contention only exists when a storage server $N_{x_2,y_2} \in E_a$ and $N_{x_2,y_2} \in E_b$ will participate in the recovery, where $x_1 \neq x_2, 0 \leq x_2 < n, 0 \leq y_2 < I$. Because this violates the condition that $N_{x_1,y_1} \in E_a, N_{x_1,y_1} \in E_b$ and $N_{x_2,y_2} \in E_a, N_{x_2,y_2} \in E_b$ only exists when $a = b$. Thus, there is no I/O contention for recovering $N_{x_1,y_1}$, so it can have optimal recovery I/O parallelism. Now we give the condition in which a storage server can have optimal recovery I/O parallelism of our proposed ESet:

**Theorem 5.3.** For a given storage server $N_{x_1,y_1} \in G_0$, where $0 \leq y_1 < I$, it can have optimal recovery I/O parallelism when it satisfies one of the following conditions:

1) $x_1 = 0$. Namely, a storage server in the first row can always have optimal recovery I/O parallelism

2) For $x_1 \neq 0$. When I is a prime number and is equal to or greater than n-1, $N_{x_1,y_1}$ can have optimal recovery I/O parallelism

**Proof of Theorem 5.3.** We first prove that any storage server $N_{0,y_1} \in G_0$, where $0 \leq y_1 < I$, can always have optimal recovery I/O parallelism. Given a storage server $N_{x_2,y_2} \in G_0$, where $0 \leq x_2 < n, 0 \leq y_2 < I$. If $N_{y_1} \in E_a$, $N_{y_1} \in E_b$, and $N_{x_2,y_2} \in E_a, N_{x_2,y_2} \in E_b$, we can have $y_2 = (y_1(x_2 - 1) + a)mod I$ and $y_2 = (y_1(x_2 - 1) + b)mod I$ based on line 9 in Algorithm 2. Then we can have $(a-b)mod I = 0$. Thus, only one erasure-coded set $E_a$ satisfies $N_{0,y_1} \in E_a$ and $N_{x_2,y_2} \in E_a$. Based on Theorem 5.2, $N_{0,y_1}$ can always have optimal recovery I/O parallelism.

For $x_1 \neq 0, x_2 \neq 0$ and $x_1 \neq x_2$, if $N_{x_1,y_1} \in E_a, N_{x_1,y_1} \in E_b$, and $N_{x_2,y_2} \in E_a, N_{x_2,y_2} \in E_b$, we have the following equations:

$$y_1 = \left(\frac{a}{I}(x_1 - 1) + a\right)mod I$$

$$y_2 = \left(\frac{a}{I}(x_2 - 1) + a\right)mod I$$

$$y_1 = \left(\frac{b}{I}(x_1 - 1) + b\right)mod I$$

$$y_2 = \left(\frac{b}{I}(x_2 - 1) + b\right)mod I.$$
Because \((a-b)/I\) is a value between 0 and I-1, we can conclude that when the value of \(I\) is a prime number and is equal to or greater than \(n-1\), each storage server can obtain optimal recovery I/O parallelism. Because the absolute value of \(x_1 - x_2\) is smaller than \(n-1\), the only condition that satisfies Equation (5) is that \(a\) is equal to \(b\).

The storage servers in the first row of a given ESetGroup can always have optimal recovery I/O parallelism, and when \(I\) is a prime number and is equal to or greater than \(n-1\), any storage server from other rows can obtain optimal recovery I/O parallelism. In this way, the recovery time of each storage server can approximate \((\text{ns}/\beta) \times ((k \times \omega)/\phi)/I\), where \(\text{ns}/\beta\) is the number of blocks in each storage server and \((k \times \omega)/\phi\) is the time needed to recover a single block, which was mentioned in the last paragraph of Section 3.

We consider the case for \(G_0\) in this section. The storage servers for any ESetGroup \(G_i\) still satisfy the conditions of both Theorems 5.1 and 5.3.

The algorithm of standard parity declustering [33] also seeks to obtain an optimized recovery performance. However, our ESet differs from standard parity declustering in the following three aspects based on our analysis of ESet:

1) The standard parity declustering is designed for disk-level failures. ESet can tolerate rack-level, host-level, and disk-level failures.

2) The standard parity declustering relies on the Balanced Incomplete Block Design algorithm to distribute blocks, which will perform a complex calculation before distribution. For ESet, each erasure-coded set is one unit for distributing blocks in each stripe.

3) As we have proved, when \(I\) is a prime number and no less than \(n-1\), it can achieve optimal recovery performance. However, standard parity declustering relies on the Balanced Incomplete Block Design algorithm to give optimal recovery performance, which is difficult to find.

In summary, our placement algorithm can obtain the desired recovery I/O parallelism. It can achieve optimal recovery I/O parallelism under some mild conditions. Our algorithm differ from existing algorithms such as standard parity declustering and is appropriate to achieve fast data recovery performance for erasure-coded storage systems.

6 DESIGN AND IMPLEMENTATION OF ESETSTORE

We illustrate the data placement algorithm ESet to obtain efficient reconstruction for erasure-coded storage systems. In this section, we present the design and implementation of a prototype erasure-coded storage system that integrates the data placement algorithm ESet to harvest fast data recovery. We also consider how to achieve good read and write performance for erasure-coded storage systems. We developed ESetStore in C language with around 40,000 lines of source code.

We first present the overall architecture of our ESetStore. Then we illustrate how to make efficient utilization of available I/O and computation resources in ESetStore to obtain good read and write performance. We also detail the efficient reconstruction with the placement algorithm ESet. Finally, we make a discussion about how to handle various kinds of I/O pressure in storage systems.

6.1 ESetStore Architecture

Fig. 4 illustrates the overview of ESetStore’s system architecture. ESetStore consists of three major components: ECClient, a client library that allows users to interact with the storage system; ECMeta, the metadata service that manages the metadata information; and a Storage Cluster that contains many storage servers where each one is deployed with the storage service ECServer to store raw data.

ECClient. The ECClient is a library that provides fundamental operations for users to interact with the storage system. The three functions are create, open, and delete. When a client issues a create request to the system, the command is sent to the ECMeta. ECMeta returns a set of \(n\) storage servers for storing files and some additional information about how to encode the files. To open a file, a set of \(k\) storage servers that contain any data or parity blocks of the file will be returned. When it sends a delete command, the file information is deleted from the metadata server, and the file’s content is deleted by the storage servers.

Two fundamental functions for accessing file content are read and write. When \(n\) storage servers are provided, the write function writes \(n\) data and parity blocks of a file to these storage servers. To read the content of a file from the servers, the ECClient first selects \(k\) storage servers that contain the data blocks to retrieve data from the storage system. In case that any storage server that contains data blocks failed, some parity blocks will be downloaded, and the decoding operation is performed to restore the file.

ECMeta. Each operation issued by users will first interact with the ECMeta. It records the file created by users via file management. It also manages the relationship between stripes and files by Block Management module. Monitoring the state of each storage server is also performed by the metadata service. It interacts with each storage server to periodically check its status. It is also responsible for conducting recovery of any failed storage server.

The metadata service is responsible for managing the metadata information of each file, and it records the location of each block and manages the relationship between blocks and stripes. When any storage server fails, it selects related erasure-coded sets for which each erasure-coded set contains the failed server to perform recovery.

Storage Cluster. The storage cluster contains many storage servers to store raw data. Each storage server contains a
storage service named ECServer. ECServer provides interfaces that help users access data on each storage server. When users write data to a storage server, it receives the data and writes them to the local disk. If the storage server receives the read request from users, the ECServer reads the data from the local disk and sends the data to the user. ECServer also assists ECMeta in recovering missing data.

### 6.2 Efficient Read and Write Operations

Accessing data is a constantly performed task in storage systems. Read and write operations are two fundamental operations for accessing data in a storage system. A write operation in erasure-coded storage systems involves three steps. First, the data are read from disk to memory. The encoding operation is then performed to generate k data chunks and n-k parity chunks. Finally, these chunks are written to n storage servers. When reading a file, the ECClient gathers data from k storage servers. If any of the k chunks retrieved from storage servers belong to the n-k parity chunk, the decoding operation must be performed before constructing the file. When decoding is required to read a file, we call it degrade read. The read operation without decoding is called normal read. To this end, effectively utilizing both I/O and computational resources are the key factors in achieving good read and write performance.

Fig. 5 presents a simple example of read and write operations. In this example, we set n as 3 and k as 2. The file content is denoted with a string ABCDEF. We can see that H is equal to A ⊕ D, I is equal to B ⊕ E, and J is equal to C ⊕ F. The right side of Fig. 5 demonstrates the timeline of read and write operations. One read or write operation is performed at each time slot. Note that we assume the ECServer 2 failed in the read operation in the example. We perform a degrade read to retrieve the file with decoding operation involved. As for a normal read when ECServer 2 is available, we need only retrieve data chunks from ECServer 1 and ECServer 2. The read and write operations in this way may take a rather long time because each step takes an exclusive time slot.

One solution to improve read and write performance is to utilize the I/O resources of both the client side and the server side. Fig. 6 illustrates the method of overlapping operations to accelerate the performance of read and write operations. We divide each chunk into many small sub-chunks for each read and write step in our operation. To write data to servers, we read A and B and then encode them into O. We then write A, B, and O into three servers. We write C, D, and P in a similar way. The timeline at the bottom of Fig. 6 reveals that the writing, coding, and reading steps can proceed in parallel to better utilize the I/O resources.

In this way, we can make better use of I/O resources. Via numeric calculation, our proposed method takes 3/4 of the way in Fig. 5 for writing, and the time for reading is 9/13 of that in Fig. 5 for reading. To effectively overlap I/O and coding operations as illustrated in Fig. 6, we define a concept called streaming size as follows:

The streaming size is a configured fixed-length (e.g., 64 KB) to manipulate data. The block size is a multiple of the streaming size. When writing a file, the file is divided into one or more sets of k data blocks. Each block is further divided into many small chunks whose size is the streaming size. The streaming size can be the minimum size for handling both read and write operations.

For read and write operations, we have two sets of buffers. When we allocate a set of buffers to handle read and write operations in ESetStore, the size of each buffer is a multiple of the streaming size. Each set of buffers has two states. One state is idle when no thread is performing a task on them. Another state is busy when a thread is conducting a read or write operation on it. We have three threads to carry read and write operations with these buffers. One thread is the disk thread, which is responsible for reading data from disks into the buffers or writing data from buffers to a disk. The coding thread is responsible for coding. The network thread is responsible for sending data to other storage servers or receiving data from other storage servers.

The procedure of the write operation in ESetStore is presented in Fig. 7a. The disk thread first selects a set of n buffers that remain idle and mark the set’s state as busy. It then reads data from the disk into the k buffers. After completing reading data into k buffers, it notifies the coding thread to handle the set of buffers. The disk thread continues to acquire a set of idle buffers and reading data into them until the file data are read from disks. The coding thread then generates parity data and fills them into n-k buffers. It then
notifies the coding thread to send data to \( n \) storage servers. The network thread receives the set of buffers from the coding thread and sends data from \( n \) buffers to \( n \) storage servers. It then sets the state of the set of buffers to idle.

Reading works in the opposite way. The procedure of the read operation in ESetStore is presented in Fig. 7b. The network thread continues to request a set of buffers, marks the set’s state as busy, and receives data from \( n \) storage servers until all tasks are complete. The coding thread performs decoding when any data block is missing. The disk thread writes data into a disk when a set of buffers is ready for writing. It then sets the set of buffers’ state to idle when all data in the buffer have been written to a disk.

6.3 Data Distribution and Efficient Reconstruction

We use the data placement algorithm ESet to decide where to store each stripe in our ESetStore. After partitioning the storage servers into many erasure-coded sets, we use an array to store the entire erasure-coded sets with each erasure-coded set containing \( n \) storage servers. When storing a file to ESetStore, the file is encoded into a stripe, and we then select an erasure-coded set to carry the stripe for the file. To evenly distribute the data across storage servers, we select the erasure-coded sets in a round-robin fashion.

When a storage server fails, we must recover the missing blocks. Algorithm 3 presents a high-level workflow of the recovery process. First, it finds \( I \) erasure-coded sets that contain the failed server. Each of the \( I \) erasure-coded sets then selects \( k \) available storage servers to recover missing blocks. Each erasure-coded set recovers one missing block from the set at a time, and it continues to recover missing blocks until all missing blocks from the erasure-coded sets are recovered. The recovery is complete when each erasure-coded set has recovered its missing blocks.

![Algorithm 3. High-Level Workflow of Recovery](image)

### 6.3.1 Erasure-Coded Sets for a Failed Storage Server

To recover a failed server, we need to locate \( I \) erasure-coded sets. We first consider the case in which we have only one group in our storage system. Erasure-coded sets are indexed from \( E_0 \) to \( E_{I-1} \). The storage servers are indexed from \( N_{0,0} \) to \( N_{I-1,I-1} \). The algorithm to find \( I \) erasure-coded sets for a given storage server \( N_{i,j} \) is presented in Algorithm 4.

When the failed server is in the first row of the group, its erasure-coded sets are located from line 5 to line 7 in Algorithm 4. They are indexed from \( I \times j \) to \( (I + 1) \times j - 1 \). For the case in which any failed storage server is in other rows, any erasure-coded set with its index satisfies the condition that at line 10 contains the storage server \( N_{i,j} \). Here, the index of erasure-coded sets is from 0 to \( I^2 - 1 \). When the storage system contains many groups, locating \( I \) erasure-coded sets for a storage system with many \( ESetGroups \) is similar to the case in which we have only one \( ESetGroup \). We must add the value, which is calculated by multiplying the index of the given \( ESetGroup \) by \( I^2 \), with each index of the located erasure-coded set by Algorithm 4.

![Algorithm 4. Find I Erasure-Coded Sets That Contain N_{i,j}](image)

6.3.2 Recovery of Each Erasure-Coded Set in ESetStore

We use a simple approach to recover missing blocks. After selecting \( k \) available storage servers from the given erasure-coded set, we select one storage server responsible for recovering missing blocks. To recover a failed stripe in an erasure-coded set, the selected storage server in each erasure-coded set reads \( k-1 \) blocks of the stripe from other \( k-1 \) storage servers and reads one local block from its own disk. It then reproduces the missing block and caches the recovered block in its own disk. The selected storage servers in each erasure-coded set repeat the process until all missing blocks are recovered. The ESetStore is expected to have \( I \) times of performance growth to recover a failed storage server in an optimal situation compared with the case in which the value of \( I \) is 1.

![Algorithm 4. Find I Erasure-Coded Sets That Contain N_{i,j}](image)

After locating \( I \) erasure-coded sets to recover a failed storage server, we select one storage server from each erasure-coded set responsible for recovering missing blocks. To recover a failed stripe in an erasure-coded set, the selected storage server in each erasure-coded set reads \( k-1 \) blocks of the stripe from other \( k-1 \) storage servers and reads one local block from its own disk. It then reproduces the missing block and caches the recovered block in its own disk. The selected storage servers in each erasure-coded set repeat the process until all missing blocks are recovered. The ESetStore is expected to have \( I \) times of performance growth to recover a failed storage server in an optimal situation compared with the case in which the value of \( I \) is 1.
6.3.3 Recovery With PPR in ESetStore

We can see that the above recovery performance for each erasure-coded set is limited by the head server’s network bandwidth. A study named PPR [15] proposed a solution to make better use of the available network bandwidth from each set of \( k \) selected storage servers.

Our implementation of the PPR is illustrated in Fig. 9. The storage servers are indexed from ECServer 0 to ECServer \( k-1 \). The last storage server sends the required subblock to ECServer \( k-2 \) to generate a temporary block. The temporary block is then sent to the former storage server. The head server continues to receive temporary blocks and restore missing small blocks, and the other servers continue to generate temporary blocks and send the temporary block to its former server until all missing data are restored. The entire process can be well pipelined so that we can make full use of the available I/O resources to perform recovery.

6.4 Limitations and Discussions

Our current design of ESetStore mainly addresses the issue of storage efficiency (i.e., using erasure coding) and recovery performance (i.e., through ESet), and meanwhile optimizes the read and write throughput. We consider recovery performance as the major design factor because in an erasure-coded system, the required I/O for recovery is \( k \)-times of that of replication based system. Currently we have not considered other factors like data access dynamics and data locality.

Storage systems may encounter various kinds of I/O pressure. A storage system may have poor read performance when providing limited I/O resources for hot data, which are frequently accessed by clients in a short period. ESetStore does not implement any strategy to deal with hot data. We discuss some existing solutions that could be used to tackle this issue. As revealed in the study of Facebook’s system f4 [7], newly created data in storage systems typically have a higher access rate. It is possible to distribute data in a round-robin manner to provide more I/O resources for hot data. The f4 system also uses a caching stack to reduce the load on the storage system. As a result, system administrators can use in-memory storage systems such as Memcache [39], [40] to reduce I/O pressure on our ESetStore. A recent erasure-coded storage system EC-Store is designed for various kinds of I/O workload [41]. It uses data access patterns and data movement strategies to improve the distribution of workload and provide efficient retrieval for erasure-coded storage systems. It is possible to adopt its strategy and run it on top of our ESet algorithm to provide better I/O performance for various access patterns.

7 Evaluation of ESetStore

In this section, we conduct a set of experiments to evaluate the performance of ESetStore using a cluster of up to 50 computers. We first introduce the experimental setup of our evaluation, and then measure the read and write throughput of ESetStore and compare it with HDFS using different settings of block size. The version of HDFS is Hadoop 3.0.0-alpha2 [8]. Many previous studies used HDFS as a baseline [14], [23], so we also choose it for comparison. We also evaluate the overall throughput of read and write operations on a small cluster and compare it with Ceph of version 13.2.6 [43]. Afterwards, we measure the recovery performance of ESetStore and compare it with both HDFS and Ceph. We demonstrate the recovery performance of ESetStore with the optimal I/O parallelism. Finally, we evaluate the system recovery performance with a single failure.

7.1 Experimental Setup

Fig. 10 presents the testbed used to the evaluate ESetStore, HDFS and Ceph. Each storage server is equipped with one 1 Gbps Ethernet card connected to a 1 Gbps switch. The testbed has 50 servers in total. Each storage server has a disk with the peak read and write throughput around 100 MB/s. The metadata service ECMeta is deployed in a high-performance server with 12 CPU cores and a 10 Gbps Ethernet card to provide low latency metadata service. It is connected to a
is 2. It is obvious that the read performance increases with 18 storage servers and set the file size of ESetStore is 16 MB. We evaluate a small cluster and contains the number of storage servers is seven. We have an ESetGroup as 2. We use a pop-

We write around 1 GB data to each storage server, and then manually shut down one server to measure the recovery performance. The performance of simultaneous data access and data recovery will be presented in Section 7.5.

### 7.2 Read and Write Throughput

In this subsection, we measure the read and write throughput of ESetStore and compare it with HDFS and Ceph. We evaluate both the normal data access and degraded data access when there is a single failure of storage server. In this subsection, data recovery will not be triggered upon the failure of a storage server. The performance of simultaneous data access and data recovery will be presented in Section 7.5.

#### 7.2.1 Single Client Performance: Comparison With HDFS

We first investigate the impact of block size on the read/write throughput of a single client. Fig. 11 presents the read throughput of ESetStore and HDFS with different settings. In Fig. 11, ESetStore R, HDFS R, and Baseline R represent the cases without any storage server failure. On the contrary, ESetStore Degrade R, HDFS Degrade R, and Baseline Degrade R represent the cases with one storage server failure which contains the required data block. The baseline refers to the special version of ESetStore without the optimization strategies in Section 6.

The read throughput of ESetStore is about 50 MB/s when the block size is 1 MB when no server failure occurs. When the value of k is 3, the performance is slightly better than when k is 2. It is obvious that the read performance increases gradually as the data block size increases. The throughput reaches its maximum (which is limited by the network bandwidth) when the size of each block is 64 MB. The performance penalty of the degrade read is about 10 percent of normal read, attributed to the overlapping of I/O operations and computing operations.

We use the baseline and the HDFS for comparison. The results of the baseline reveal that the system can encounter various kinds of performance penalties at both the client side and the server side, when no optimization strategy is applied to our system. The HDFS does not make good use of available I/O resources to obtain high read throughput. It requires a larger block size to obtain better read performance. No obvious difference is observed between its normal read and degrade read. The under-utilization of I/O resources could be the major result of its performance degradation.

<table>
<thead>
<tr>
<th>OS</th>
<th>Block Size</th>
<th>Placement Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESetStore</td>
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<td>64 MB</td>
</tr>
<tr>
<td>HDFS</td>
<td>Ubuntu 14.04</td>
<td>64 MB</td>
</tr>
<tr>
<td>Ceph</td>
<td>Ubuntu 16.04</td>
<td>4 MB</td>
</tr>
</tbody>
</table>

When evaluating the impact of recovery operation on the data access performance in Section 7.5, we write \(256 \times I\) files to our storage system and then retrieve the files using multiple client machines. The size of each file is \(4 \times k\) MB. So each storage server stores 1 GB of data. Both ESetStore and Ceph use \(k \times I\) number of client machines to retrieve all files. We evaluate the performance of two cases: (1) no storage server is failed; (2) a single storage server is failed and the recovery procedure is performed.

10 Gbps switch which is connected to the 10 Gbps uplink port of the 1 Gbps switch.

To measure the read and write throughput achievable by a single client in Section 7.2.1, we vary the values of block size from 1 to 128 MB. We perform one read and write operation of a file for each measurement. The file size is the value of \(k\) multiplied by the block size. The total size written to the storage servers is the value of \(n\) multiplied by the block size, and the total size read from the storage server is the value of \(k\) multiplied by the block size. The throughput is calculated from dividing the file size by the whole execution time. We set the streaming size as 64 KB in each set of experiments. The buffer of each block is 256 KB. Each client can read and write 256 KB of data for each buffer.

When measuring the overall system throughput in Section 7.2.2, we set the size of each object as 16 MB, which means the file size of ESetStore is 16 MB. We evaluate a small cluster with 18 storage servers and set \(n\) as 3 and \(k\) as 2. We use a popular tool IOR [44] to measure the performance of our storage cluster with intensive I/O operations. We implement the necessary APIs for IOR to perform the read and write operations with ESetStore, and the source code can be found at [44]. We measure the read and write performance with up to 16 clients that are deployed on 16 machines in two cases: (1) without storage server failure; (2) with a single storage server failure.

When evaluating the single recovery performance of ESetStore in Sections 7.3 and 7.4, we set the value of \(I\) from one to seven. We have \(I\) hosts in each rack for each measurement. Each host uses one disk to store data. We have \(n\) racks, and the number of storage servers is \(nI\). Our ESetStore has \(n\) racks to form one EsetGroup and contains \(I^2\) erasure-coded sets. We use the same physical configuration to measure the performance of HDFS and Ceph for comparison.

The configuration for recovery experiments is presented in Table 2. The file size is \(64 \times k\) MB. The default setting is used for ESetStore and HDFS. For Ceph, the block size refers to the data size written to each disk when writing an object. We use the default CRUSH setting and set the placement groups as the maximum number allowed, selected from 128, 256, 512, and 1024.

### TABLE 2

The System Configuration for Recovery Performance Comparison

<table>
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</tr>
</tbody>
</table>
when performing reads. This also indicates that our optimization strategy is effective and achieve show good read and write performance in erasure-coded storage systems.

Fig. 12 presents the write throughput of ESetStore and HDFS. The write throughput is \( k/n \) of the raw throughput because writing parity blocks account a part of the total execution time. This is why the case when \( n = 3 \) and \( k = 2 \) has better write throughput than the case when \( n = 5 \) and \( k = 3 \). The case of \( k = 2 \) has nearly the same throughput as that when \( k = 3 \) for the block size is 1 MB because the buffer is set to 256k bytes in each write operation.

The baseline version can have better performance with a larger block size. However, the write throughput is limited to around 45 MB/s because each step occupies a time slot exclusively. The write throughput of HDFS has similar performance to its read throughput. The performance increases as the block size increases. According to our experiments, ESetStore has much better write throughput than either the baseline version or the HDFS.

### 7.2.2 Overall Performance: Comparison With Ceph

The overall read/write throughput is also important to a distributed storage cluster. We use up to 16 client machines to measure the overall throughput of a cluster of 18 storage servers. We set \( I = 6, n = 3 \) and \( k = 2 \) in our storage cluster. We set the transfer size for each read and write to 2 MB, and the size of each file as 16 MB. Each measurement iterates 10 times and we pick the maximum overall throughput to represent the peak performance that the cluster can reach. The experimental results are shown in Fig. 13.

In Fig. 13, Single Failure denotes the case when the storage cluster has one failed storage server. From the figure, we can see that single failure has no obvious impact on the overall read/write throughput of both ESetStore and Ceph. When the number of clients increases from 1 to 2 and from 2 to 4, we can see the throughput is nearly doubled. But when the number of clients grows from 4 to 8 and 8 to 16, there are at most 50 percent performance growth. This could be caused by I/O contention when we have more clients issuing read/write requests. In all cases, we can see that ESetStore achieves much higher overall read/write throughput than Ceph.

### 7.3 Recovery Performance of Single Failure

The recovery performance of ESetStore is presented in Fig. 14. We set the value of \( k \) from 2 to 5. The value of \( n - k \) can be set to 1 for two reasons. First, the value of \( n - k \) as 1 is adequate to tolerate a single failure. Meanwhile, the smaller
value of $n - k$ means a reduction in coding overhead, which could better reveal the overhead of I/O operations.

We conduct two sets of experiments to measure the recovery performance of ESetStore. The first is ESetStore Row 0. We manually shutdown one storage server in the first row (row 0) in the ESetGroup. For ESetStore Row 1, we manually shutdown one storage server in the second row (row 1) in the ESetGroup. The storage servers in the first row always obtain optimal recovery performance because there will be $k$ distinct storage servers to recover missing blocks in each ESet. However, whether the storage servers in other rows can obtain optimal recovery performance relies on the value of $I$ as analyzed in Section 5.

From Fig. 14, we can see that the recovery performance is nearly $1/(k - 1)$ of the maximum read throughput when the value of $I$ is one because we must read $k - 1$ blocks from other servers to recover a missing block.

A storage server from the first row will have $k$ distinct servers in each erasure-coded set to recover all missing blocks that belong to the erasure-coded set. It can thus obtain $I$ times of performance growth as compared to the situation in which $I$ is equal to 1. When $k = 3$ when recovering a failed storage server from the second, a storage server that participates in recovery belongs to two erasure-coded sets when $I$ is 2 and 4. However, because each erasure-coded set has only around 45 MB/s network bandwidth, its recovery performance is nearly the same as that of the first row. We can thus obtain $I$ times of performance growth to recover a failed storage server in all cases compared with that in which the value of $I$ is 1.

The evaluation result in ESetStore is close to our analysis that the recovery time is approximately $(ns/\beta) \times ((k \times \omega)/\phi)/I$ in Section 3. We didn't use the same setting as the one in Fig. 7b and 7c, in which the required machines $I \times n$ for storage servers will exceed the number of our testbed. However, the recovery performance can still approximate $(ns/\beta) \times ((k \times \omega)/\phi)/I$ for these configuration. Because the main performance penalty for the recovery is the limited I/O resources, and our ESet can increase the I/O resources for recovery with the increases of value $I$.

The base performance, where $I$ is 1, of HDFS and Ceph is slightly worse than that of our ESetStore, and we can see that there are fewer cases in which the recovery performance increases as $I$ increases for HDFS. HDFS uses a random placement algorithm, which indicates that the algorithm cannot make as good use of the available disk I/O and network bandwidth resources as our ESetStore.

We set the number of threads for recovery to 8 for Ceph to achieve good recovery performance. Ceph showed slightly better performance than HDFS. It relies on more placement groups to achieve better performance. However, it can only achieve around 3x performance growth when $I$ increases from 1 to 7. Thus, we can conclude that our ESetStore can harvest available I/O resources to improve the recovery performance with the placement algorithm ESet.

### 7.4 Performance of Optimal Recovery I/O Parallelism

We can observe from Fig. 14 that as $k$ increases, the recovery performance suffers a great decrease because the recovery performance is limited by the network bandwidth of a single storage server. Here we implement the PPR algorithm in our ESetStore to evaluate the performance of optimal recovery I/O parallelism. With PPR, each erasure-coded set can make good use of the available I/O resources.

The evaluation results are presented in Fig. 15. We measured the recovery performance of a failed storage server from row 0 and a failed storage server from row 1. The evaluation results differ greatly from that in Fig. 14. The performance is far better due to the better utilization of available I/O resources in each erasure-coded set.

To recover a single failure from row 0, the performance can still show linear growth with the increase of $I$. However, when recovering a failed storage server from row 1, some cases show no obvious performance increase. For example, when $k$ is 5, there is no performance growth for $I$ from 1 to 4 because $I$ is smaller than $n - 1$. In these cases, some storage servers concurrently participate in the recovery of a failed storage server in more than one erasure-coded set, resulting in I/O contention. As a consequence, there is no increase of I/O resources to recover the failed server.

In addition, in some cases the performance of row 1 is around half that of row 0, such as when $k$ is 3 and $I$ is 4, because there is a storage server staying in two erasure-coded sets that contains the failed storage server. As a result, the I/O resources to recover the failed storage server is reduced to half for row 1.

In summary, the recovery performance is always optimal for the storage servers in row 0. The storage servers in other rows can obtain optimal recovery performance only when the configuration satisfies the optimal recovery I/O parallelism, namely, that $I$ is a prime number and its value is equal to or greater than $n - 1$. Our evaluation also indicates that an optimization solution, such as PPR, may become useless in
erasure-coded storage systems if the stripes are not placed in a proper way with a data placement algorithm.

### 7.5 Impact of Recovery on Data Access Performance

Although ESetStore is designed for efficient data recovery, it is still important to evaluate the data access performance while the system is during the recovery process. We present the experimental results in Fig. 16, where we measure the data access throughput while the system is undergoing the recovery process. The Average means the mean throughput of all clients, which is calculated by dividing the single file size by the average file downloading time. The Overall is the total aggregated system throughput for accessing all files, which is calculated by dividing the size of all retrieved files by the total time of downloading all files.

We can observe that the throughput of a single client in ESetStore decreases with the increase of $I$. This is because more clients result in more I/O contentions. It also explains why the overall throughput does not achieve linear performance growth with the increase of $I$. When $I$ is 1 or 2, we observe that the data access performance with ongoing data recovery is slightly worse than the one without server failure. However, for larger values of $I$, the impact of data recovery on the data access performance is negligible.

For Ceph, the read operation cannot be performed when the value of $I$ is 1 when one storage server failed. That is why the throughput is zero in Fig. 16 for Ceph Degraded. For other cases, the Ceph Degraded R and Ceph R have similar performance. In all cases, ESetStore shows superior data access throughput than Ceph when the system is undergoing data recovery.

### 8 CONCLUSION

In this paper, we present the design and implementation of our data placement algorithm, ESet, on the prototype of an erasure-coded storage system ESetStore. Our results demonstrate that ESetStore can effectively improve the single recovery performance by harvesting available I/O resources with ESet. Meanwhile, our recovery solution can be an enhancement to existing optimizations such as the parallel partial repair algorithm. When the value of parameter recovery I/O parallelism $I$ is a prime number and is equal to or greater than $n-1$, each storage server can make good use of the available I/O resources to obtain optimal recovery performance.

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The Design of Fast Content-Defined Chunking for Data Deduplication Based Storage Systems

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Abstract—Content-Defined Chunking (CDC) has been playing a key role in data deduplication systems recently due to its high redundancy detection ability. However, existing CDC-based approaches introduce heavy CPU overhead because they declare the chunk cut-points by computing and judging the rolling hashes of the data stream byte by byte. In this article, we propose FastCDC, a Fast and efficient Content-Defined Chunking approach, for data deduplication-based storage systems. The key idea behind FastCDC is the combined use of five key techniques, namely, gear based fast rolling hash, simplifying and enhancing the Gear hash judgment, skipping sub-minimum chunk cut-points, normalizing the chunk-size distribution in a small specified region to address the problem of the decreased deduplication ratio stemming from the cut-point skipping, and last but not least, rolling two bytes each time to further speed up CDC. Our evaluation results show that, by using a combination of the five techniques, FastCDC is 3-12X faster than the state-of-the-art CDC approaches, while achieving nearly the same and even higher deduplication ratio as the classic Rabin-based CDC. In addition, our study on the deduplication throughput of FastCDC-based Destor (an open source deduplication project) indicates that FastCDC helps achieve 1.2-3.0X higher throughput than Destor based on state-of-the-art chunkers.

Index Terms—Data deduplication, content-defined chunking, storage system, performance evaluation

1 INTRODUCTION

Data deduplication, an efficient approach to data reduction, has gained increasing attention and popularity in large-scale storage systems due to the explosive growth of digital data. It eliminates redundant data at the file- or chunk-level and identifies duplicate contents by their cryptographically secure hash signatures (e.g., SHA1 fingerprint). According to deduplication studies conducted by Microsoft [1], [2] and EMC [3], [4], about 50 and 85 percent of the data in their production primary and secondary storage systems, respectively, are redundant and could be removed by the deduplication technology.

In general, chunk-level deduplication is more popular than file-level deduplication because it identifies and removes redundancy at a finer granularity. For chunk-level deduplication, the simplest deduplication approach is to cut the file or data stream into equal, fixed-size chunks, referred to as Fixed-Size Chunking (FSC) [5]. Content-Defined Chunking (CDC) based approaches are proposed to address the boundary-shift problem faced by the FSC approach [6]. Specifically, CDC declares chunk boundaries based on the byte contents of the data stream instead of on the byte offset, as in FSC, and thus helps detect more redundancy for deduplication. According to some recent studies [1], [2], [7], [8], CDC-based deduplication approaches are able to detect about 10-20 percent more redundancy than the FSC approach.

Currently, the most popular CDC approaches determine chunk boundaries based on the Rabin fingerprints of the content, which we refer to as Rabin-based CDC [6], [9], [10]. Rabin-based CDC is highly effective in duplicate detection but time-consuming, because it computes and judges (against a condition value) Rabin fingerprints of the data stream byte by byte [11]. In order to speed up the CDC process, other hash algorithms have been proposed to replace the Rabin algorithm for CDC, such as SampleByte [12], Gear [13], etc. Meanwhile, the abundance of computation resources afforded by multi and manycore processors [14], [15] or GPU processors [16], [17], [18] has been leveraged for CDC acceleration.

Generally, CDC consists of two distinctive and sequential stages: (1) hashing in which fingerprints of the data contents are generated and (2) hash judgment in which fingerprints are compared against a given value to identify and declare chunk cut-points. Our previous study of delta compression, Ddelta [13], suggests that the Gear hash (i.e., \( fp = (fp << 1) + G(b) \), see Section 3.2) is more efficient as a rolling hash for...
CDC. To the best of our knowledge, Gear appears to be one of the fastest rolling hash algorithms for CDC at present since it use much less calculation operations than others. However, our empirical and analytical studies on Gear-based CDC obtain three important observations:

- Observation ①: the Gear-based CDC has the potential problem of low deduplication ratio (i.e., the percentage of redundant data reduced), about 10-50 percent lower on some datasets (detailed in Section 4.2). This is because, in the hash judgment stage of Gear-based CDC, the sliding window size is very small, only 13 bytes in its current implementation [13].

- Observation ②: the hash judgment stage becomes the new performance bottleneck in Gear-based CDC. This is because the accelerated hashing stage by Gear, has shifted the bottleneck to the hash judgment stage.

- Observation ③: Enlarging the predefined minimum chunk size (used in CDC to avoid the very small-sized chunks [6]) can further speed up the chunking process (called cut-point skipping in this paper) but at the cost of decreasing the deduplication ratio in Gear-based CDC. This is because many chunks with skipped cut-points are not divided truly according to the data contents (i.e., content-defined). Our large scale study (detailed in Section 4.3) suggests that skipping this predefined min chunk size usually increases the chunking speed by the ratio of Predefined min chunk size but decreases the deduplication ratio (about 15 percent decline in the worst case).

Therefore, motivated by the above three observations, we proposed FastCDC, a Fast and efficient CDC approach that addresses the problems of low deduplication efficiency and expensive hash judgement faced by Gear-based CDC. To address the problems observed in the 1st and 2nd observations, we use an approach of enhancing and simplifying the hash judgment to further reduce the CPU operations during CDC for data deduplication. Specifically, FastCDC pads several zero bits into the mask value in its hash-judging statement to enlarge the sliding window size to the size of 48 Bytes used by Rabin-based CDC, which makes it able to achieve nearly the same deduplication ratio as the Rabin-based CDC; Meanwhile, by further simplifying and optimizing the hash-judging statement, FastCDC decreases the CPU overhead for the hash judgment stage in CDC.

For the 3rd observation and to further speed up chunking, FastCDC employs a novel normalized Content-Defined Chunking scheme, called normalized chunking, that normalizes the chunk-size distribution to a specified region that is guaranteed to be larger than the minimum chunk size to effectively address the problem facing the cut-point skipping approach. Specifically, FastCDC selectively changes the number of mask bits ‘1’ in the hash-judging statement of CDC, and thus it normalizes the chunk-size distribution to a small specified region (e.g., 8KB∼16KB), i.e., the vast majority of the generated chunks fall into this size range, and thus minimizes the number of chunks of either too small or large in size. The benefits are twofold. ①, it increases the deduplication ratio by reducing the number of large-sized chunks. ②, it reduces the number of small-sized chunks, which makes it possible to combine with the cut-point skipping technique above to maximize the CDC speed without sacrificing the deduplication ratio.

In addition, we propose a technique called “rolling two bytes each time” for FastCDC, which further reduces the calculation operations in the hashing stage by rolling two bytes each time to calculate the chunking fingerprints in the hashing stage, and then judging the even and odd bytes respectively in the hash judgement stage. This further accelerates the chunking process while achieving exactly the same chunking results.

Our evaluation results based on seven large-scale datasets, suggest that FastCDC is about 3-12× faster than the state of art, while ensuring a comparably high deduplication ratio. In addition, we have incorporated FastCDC in Destor [19], an open source data deduplication system, and evaluation shows that Destor using FastCDC helps achieve about 1.2-3.0X higher system throughput than using other CDC approaches. Meanwhile, due to its simplicity and effectiveness, FastCDC has been adopted as the default chunker by several known open source Github projects to speed up the detection of duplicate contents, such as Rdedup [20], Content Blockchain [21], etc. The released Rdedup version 2.0.0 states: “rdedupe store performance has been greatly improved by implementing many new techniques” and “our default CDC algorithm is now FastCDC”.

2 BACKGROUND

Recently, chunk-level data deduplication becomes one of the most popular data reduction method in storage systems for improving storage and network efficiency. As shown in Fig. 1, it splits a file into several contiguous chunks and removes duplicates by computing and indexing hash digests (or called fingerprints, such as SHA-1) of chunks [5], [6], [22], [23]. The fingerprints matching means that their corresponding chunks are duplicate, which thus simplifies the global duplicates detection in storage systems. In the past ten years, data deduplication technique has been demonstrated its space efficiency functionality in the large-scale production systems of Microsoft [1], [2] and EMC [3], [4].

Chunking is the first critical step in the operational path of data deduplication, in which a file or data stream is divided into small chunks so that each can be duplicate-identified. Fixed-Size Chunking (FSC) [5] is simple and fast but may face the problem of low deduplication ratio that stems from the boundary-shift problem [6], [24]. For example, if one or several bytes are inserted at the beginning of a file, all current chunk cut-points (i.e., boundaries) declared by FSC will be shifted and no duplicate chunks will be detected.

Content-Defined Chunking (CDC) is proposed to solve the boundary-shift problem. CDC uses a sliding-window technique on the content of files and computes a hash value (e.g., Rabin fingerprint [6], [9]) of the window. A chunk cut-point...
that is modified
is a number representing
is defined as a polynomial
$B_1 C_1 2 AST a (\text{here). A typical data}
other relevant deduplication issues
is the number of
bits of the hash value
r
mod
26x87]byte to declare a fingerprint for chunking, in contrast to
ing duplicate chunks as small as 32-64 bytes. It uses one
grained network redundancy elimination, usually eliminat-
tion systems and they can be broadly classified as either algo-
ratio, it incurs heavy CPU overhead
Specifically, in Rabin-based CDC, Rabin
hash for a sliding window containing the byte sequence $B_1, B_2, \ldots, B_n$ is defined as a polynomial
\[
Rabin(B_1, B_2, \ldots, B_n) = A(p) = \left\{ \sum_{i=1}^{a} B_i p^{i-1} \right\} \mod D, \quad (1)
\]
where $D$ is the average chunk size, $a$ is the number of
an irreducible polynomial [9]. Rabin hash is a rolling hash
algorithm since it is able to compute the hash in an iterative
fashion, i.e., the current hash can be incrementally com-
puted from the previous value as
\[
Rabin(B_2, B_3, \ldots, B_{a+1}) = \left\{ \left[ Rabin(B_1, B_2, \ldots, B_n) - B_1 p^{a-1} \right] p + B_{a+1} \right\} \mod S. \quad (2)
\]
However, Rabin-based CDC is time-consuming because it
computes and judges the hashes of the data stream byte by
byte, which renders the chunking process a performance
bottleneck in deduplication systems. There are many
approaches that accelerate the CDC process for deduplica-
tion systems and they can be broadly classified as either algo-
rithmic oriented or hardware oriented. We summarize below
some of these approaches that represent the state of the art.
Algorithmic-Oriented CDC Optimizations. Since the fre-
frequent computations of Rabin fingerprints for CDC are time-
consuming, many alternatives to Rabin have been proposed
to accelerate the CDC process [12], [13], [24]. Sample-
Byte [12] is designed for providing fast chunking for fine-
gained network redundancy elimination, usually eliminat-
ing duplicate chunks as small as 32-64 bytes. It uses one
byte to declare a fingerprint for chunking, in contrast to
Rabin that uses a sliding window, and skips $\frac{1}{3}$ of the
expected chunk size before chunking to avoid generating
extremely small-sized strings or chunks (they called “avoid
oversampling”). Gear [13] uses fewer operations to generate
rolling hashes by means of a small random integer table to
map the values of the byte contents, so as to achieve higher
chunking throughput. AE [24] is a non-rolling-hash-based
chunking algorithm that employs an asymmetric sliding
window to identify extremums of data stream as cut-points,
which reduces the computational overhead for CDC. Yu
et al. [26] adjust the function for selecting chunk boundaries
such that if weak conditions are not met, the sliding win-
dow can jump forward, avoiding unnecessary calculation
steps. RapidCDC [27] leverages the data locality to record
the chunking positions to reduce the CDC computations for
the duplicate chunks to appear next time.

Hardware-Oriented CDC Optimizations. StoreGPU [16], [17]
and Shredder [18] make full use of GPU’s computational
to accelerate popular compute-intensive primitives
(i.e., chunking and fingerprinting) in data deduplication.

P-Dedupe [14] pipelines deduplication tasks and then further
parallelizes the sub-tasks of chunking and fingerprinting with
multiple threads and thus achieves higher throughput. SS-
CDC [28] proposes a two-stage parallel content-defined chunk-
approach without compromising deduplication ratio.

It is noteworthy that there are other chunking approaches
trying to achieve a higher deduplication ratio but introduce
more computation overhead on top of the conventional CDC
approach. TTTD [29] and Regression chunking [2] introdu-
ces one or more additional thresholds for chunking judg-
ment, which leads to a higher probability of finding chunk
boundaries and decreases the chunk size variance. MAXP
[30], [31], [32] treats the extreme values in a fixed-size region
as cut-points, which also results in smaller chunk size var-
iance. In addition, Bimodal chunking [33], Subchunk [34],
and FBC [35] re-chunk the non-duplicate chunks into smaller
ones to detect more redundancy.

For completeness and self-containment we briefly dis-
cuss other relevant deduplication issues here. A typical data
deduplication system follows the workflow of chunking,
fingerprinting, indexing, and storage management [19],
[22], [36], [37]. The fingerprinting process computes the
cryptographically secure hash signatures (e.g., SHA1) of
data chunks, which is also a compute-intensive task but can
be accelerated by certain pipelining or parallelizing techni-
ques [14], [38], [39], [40]. Indexing refers the process of iden-
tifying the identical fingerprints for checking duplicate
chunks in large-scale storage systems, which has been well
explored in many previous studies [19], [22], [41], [42]. Stor-
age management refers to the storage and possibly post-
deduplication processing of the non-duplicate chunks and
their metadata, including such processes as related to fur-
ter compression [13], defragmentation [43], reliability [44],
security [45], etc. In this paper, we focus on designing a
very fast and efficient chunking approach for data dedupli-
cation since the CPU-intensive CDC task has been widely
recognized as a major performance bottleneck of the CDC-
based deduplication system [17], [18], [27], [28].

3 FASTCDC DESIGN AND IMPLEMENTATION

3.1 FastCDC Overview
FastCDC aims to provide high performance CDC. And
there are three metrics for evaluating CDC performance,
namely, deduplication ratio, chunking speed, and the aver-
age generated chunk size. Note that the average generated
chunk size is also an important CDC performance metric since it reflects the metadata overhead for deduplication indexing, i.e., the larger the generated chunk size is, the fewer the number of chunks and thus the less metadata will be processed by data deduplication.

Generally, it is difficult, if not impossible, to improve these three performance metrics simultaneously because they can be conflicting goals. For example, a smaller average generated chunk size leads to a higher deduplication ratio, but at the cost of lower chunking speed and high metadata overheads. Thus, FastCDC is designed to strike a sensible trade-off among these three metrics so as to strive for high performance CDC, by using a combination of the five techniques with their complementary features as shown in Fig. 3.

- Gear-based rolling hashing: due to its hashing simplicity and rolling effectiveness, Gear hash is shown to be one of the fastest rolling hash algorithms for CDC, and we introduce it and discuss its chunking efficiency in detail in Section 3.2.
- Optimizing hash judgment: using a zero-padding scheme and a simplified hash-judging statement to speed up CDC without compromising the deduplication ratio, as detailed in Section 3.3.
- Sub-minimum chunk cut-point skipping: enlarging the predefined minimum chunk size and skipping cut-points for chunks smaller than that to provide a higher chunking speed and a larger average generated chunk size, as detailed in Section 3.4.
- Normalized chunking: selectively changing the number of mask ‘1’ bits for the hash judgment to approximately normalize the chunk-size distribution to a small specified region that is just larger than the pre-defined minimum chunk size, ensuring both a higher deduplication ratio and higher chunking speed, as detailed in Section 3.5.
- Rolling two bytes each time: further speeding up chunking without affecting the chunking results by reducing the calculation operations in the hashing stage by rolling two bytes each time to calculate the chunking fingerprints in the hashing stage, and then judging the even and odd bytes respectively in the hash judgement stage, as detailed in Section 3.7.

In general, the key idea behind FastCDC is the combined use of the above five key techniques for CDC acceleration, especially employing normalized chunking to address the problem of decreased deduplication ratio facing the cut-point skipping, and thus achieve high performance CDC on the three key metrics.

### 3.2 Gear-Based Rolling Hashing

In this subsection, we elaborate on and analyze the Gear-based rolling hash, and then introduce the new challenges and opportunities after we introduce Gear-based CDC. Gear-based rolling hash is first employed by Ddelta [13] for delta compression, which helps provide a higher delta encoding speed and is suggested to be a good rolling hash candidate for CDC.

A good hash function must have a uniform distribution of hash values regardless of the hashed content. As shown in Fig. 4, Gear-based CDC achieves this in two key ways: (1) It employs an array of 256 random 64-bit integers to map the values of the byte contents in the sliding window (i.e., the calculated bytes, whose size is the bit-width of the fp); and (2) The addition (“+”) operation adds the new byte in the sliding window into Gear hashes while the left-shift (“<<”) operation helps strip away the last byte of the last sliding window (e.g., B_{i-1} in Fig. 4). This is because, after the “<<” and modulo operations, the last byte B_{i-1} will be calculated into the fp as the \((G[B_{i-1}] << n) \mod 2^b\), which will be equal to zero. As a result, Gear generates uniformly distributed hash values by using only three operations (i.e., “+”, “<<”, and an array lookup), enabling it to move quickly through the data content for the purpose of CDC. Table 1 shows a comparison among the two rolling hash algorithms: Rabin and Gear, which suggests Gear uses far fewer calculation operations than Rabin, thus being a good rolling hash candidate for CDC.

<table>
<thead>
<tr>
<th>Name</th>
<th>Pseudocode</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rabin (fp = ((fp' U(a)) &lt; &lt; 8)</td>
<td>b'T[fp &gt; &gt; N]</td>
<td>Slow</td>
</tr>
<tr>
<td>Gear (fp = (fp &lt; &lt; 1) + G(b))</td>
<td>Fast</td>
<td></td>
</tr>
</tbody>
</table>

Here ‘a’ and ‘b’ denote contents of the first and last byte of the sliding window respectively, ‘N’ is the length of the content-defined sliding window, and ‘U’, ‘T’, ‘G’ denote the predefined arrays [6], [11], [13]. ‘fp’ represents the fingerprint of the sliding window.

---

**Fig. 4.** A schematic diagram of the Gear hash.

**Table 1**

<table>
<thead>
<tr>
<th>The Hashing Stage of the Rabin- and Gear-Based CDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Rabin (fp = ((fp' U(a)) &lt; &lt; 8)</td>
</tr>
<tr>
<td>Gear (fp = (fp &lt; &lt; 1) + G(b))</td>
</tr>
</tbody>
</table>

In general, the key idea behind FastCDC is the combined use of the above five key techniques for CDC acceleration, especially employing normalized chunking to address the problem of decreased deduplication ratio facing the cut-point skipping, and thus achieve high performance CDC on the three key metrics.
chunking efficiency. However, according to our experimental analysis, there are still challenges and opportunities for Gear-based CDC, such as low deduplication ratio, expensive hash judgment, further acceleration by skipping, etc. We elaborate on these issues as follows.

**Algorithm 1. GearCDC8KB**

**Input:** data buffer, src buffer length, n  
**Output:** chunking breakpoint i  
\[\text{MinSize} \leftarrow 2KB; \quad \text{MaxSize} \leftarrow 64KB; \quad i \leftarrow \text{MinSize}; \quad fp \leftarrow 0; \]
**if** \(n \leq \text{MinSize} \)** then  
\[\text{return } n \]
**while** \(i < n \)** do  
\[fp = (fp < < 1) + \text{Gear}[\text{src}[i]]; \]
**if** \((fp \& 0x1fff == 0x78) \| i >= \text{MaxSize} \)** then  
\[\text{return } i; \]
\[\text{return } i;\]

**Low Deduplication Ratio Due to the Limited Sliding Window Size.** The Gear-based CDC has the potential problem of low deduplication ratio, about 10-50 percent lower on some datasets (see the evaluation results in Section 4.2). This is because, the traditional hash judgment for the Rabin-based CDC, as shown in Fig. 2 (i.e., “fp mod D==r”), is also used by the Gear-based CDC [13] as shown in Algorithm 3.2. But this results in a smaller sized sliding window used by Gear-based CDC since it uses Gear hash for chunking. For example, as shown in Fig. 6, the sliding window size of the Gear-based CDC will be equal to the number of the ‘1’ bits used by the mask value. Therefore, when using a mask value of 0x1fff (i.e., \(2^{13} - 1\), there are thirteen ‘1’ bits) for the expected chunk size of 8 KB, the sliding window for the Gear-based CDC would be 13 bytes while that of the Rabin-based CDC would be 48 bytes [6]. The smaller sliding window size of the Gear-based CDC can lead to more chunking position collisions (i.e., randomly marking the different positions as the chunk cut-points), resulting in the decrease in deduplication ratio.

**The Expensive Hash Judgment.** In Gear-based CDC, the accelerated hashing stage by the fast Gear, has shifted the bottleneck to the hash judgment stage that requires more operations as shown in Algorithm 3.2. Our implementation and in-depth analysis of Gear-based CDC on several datasets (detailed in Section 4) suggest that its hash-judging stage accounts for more than 60 percent of its CPU overhead during CDC after the fast Gear hash is introduced. Thus, there is a lot of room for the optimization of the hash judging stage to further accelerate the CDC process as discussed later in Section 3.3.

**Further Speed up Chunking by Skipping.** Another observation is that the minimum chunk size used for avoiding extremely small-sized chunks, can be also employed to speed up CDC by the cut-point skipping, i.e., eliminating the chunking computation in the skipped region. But this minimum chunk size for cut-point skipping approach decreases the deduplication ratio (as demonstrated in the evaluation results in Fig. 12c in Section 4.3) since many chunks are not divided truly according to the data contents, i.e., not really content-defined.

The last observation from the minimum chunk size skipping motivates us to consider a new CDC approach that (1) keeps all the chunk cut-points that generate chunks larger than a predefined minimum chunk size and (2) enables the chunk-size distribution to be normalized to a relatively small specified region, an approach we refer to as normalized chunking in this paper, as described in Section 3.5.

**3.3 Optimizing Hash Judgment**

In this subsection, we optimize the hash judgment stage on top of the Gear-based CDC, which helps further accelerate the chunking process and increase the deduplication ratio to reach that of the Rabin-based CDC. More specifically, FastCDC incorporates two main optimizations as elaborated below.

**Enlarging the Sliding Window Size by Zero Padding.** As discussed in Section 3.2, the Gear-based CDC employs the same conventional hash judgment used in the Rabin-based CDC, where a certain number of the lowest bits of the fingerprint are used to declare the chunk cut-point, leading to a shortened sliding window for the Gear-based CDC (see Fig. 6) because of the unique feature of the Gear hash. To address this problem, FastCDC enlarges the sliding window size by padding a number of zero bits into the mask value. As illustrated by the example of Fig. 7, FastCDC pads five zero bits into the mask value and changes the hash judgment statement to “fp & mask == r”. If the masked bits of fp match a threshold value r, the current position will be declared as a chunk cut-point. Since Gear hash uses one left-shift and one addition operation to compute the rolling hash, this zero-padding scheme enables 10 bytes (i.e., \(B_1, \ldots, B_{10}\)), instead of the original five bytes, to be involved in the final hash judgment by the five masked one bits (as the red box shown in Fig. 7) and thus makes the
sliding window size equal or similar to that of the Rabin-based CDC [6], minimizing the probability of the chunking position collision. As a result, FastCDC is able to achieve a deduplication ratio as high as that by the Rabin-based CDC.

Simplifying the Hash Judgment to Accelerate CDC. The conventional hash judgment process, as used in the Rabin-based CDC, is expressed in the programming statement of “fp mod D == r” [6], [13]. For example, the Rabin-based CDC usually defines D and r as 0x02000 and 0x78, according to the known open source project LBFS [6], to obtain the expected average chunk size of 8 KB. In FastCDC, when combined with the zero-padding scheme introduced above and shown in Fig. 7, the hash judgment statement can be optimized to “fp & Mask == 0”, which is equivalent to “fp & Mask”. Therefore, FastCDC’s hash judgment statement reduces the register space for storing the threshold value r and avoids the unnecessary comparison operation that compares “fp & Mask” and r, thus further speeds up the CDC process as verified in Section 4.2.

3.4 Cut-Point Skipping
Most of CDC-based deduplication systems impose a limit of the maximum and minimum chunk sizes, to avoid the pathological cases of generating many extremely large- or small-sized chunks by CDC [1], [6], [33], [34], [37], [46]. A common configuration of the average, minimum, and maximum parameters follows that used by LBFS [6], i.e., 8, 2, and 64 KB. Our previous study [13] and experimental observations (see Fig. 11 in Section 4.2, using curve fitting) suggest that the cumulative distribution of chunk size X in Rabin-based CDC approaches with an expected chunk size of 8 KB (without the maximum and minimum chunk size requirements) generally follows an exponential distribution as follows:

$$P(X \leq x) = F(x) = (1 - e^{-\frac{x}{8192}}), \ x \geq 0. \quad (3)$$

Note that this theoretical exponential distribution in Equation (3) is based on the assumption that the data content and Rabin hashes of contents (recall Equation (1) and Fig. 2 for CDC) follow a uniform distribution. Equation (3) suggests that the value of the expected chunk size will be 8 KB according to exponential distribution.

According to Equation (3), the chunks smaller than 2 KB and larger than 64 KB would account for about 22.12 and 0.03 percent of the total number of chunks respectively. This means that imposing the maximum chunk size requirement only slightly hurts the deduplication ratio but skipping cut-points before chunking to avoid generating chunks smaller than the prescribed minimum chunk size, or called sub-minimum chunk cut-point skipping, will impact the deduplication ratio significantly as evidenced in Fig. 12c. This is because a significant portion of the chunks are not divided truly according to the data contents, but forced by this cut-point skipping.

Given FastCDC’s goal of maximizing the chunking speed, enabling the minimum chunk size and skipping sub-minimum chunk cut-point will help FastCDC achieve a higher CDC speed by avoiding the operations for the hash calculation and judgment in the skipped region. This gain in speed, however, comes at the cost of reduced deduplication ratio. To address this problem, we will develop a normalized chunking approach, to be introduced in the next subsection.

It is worth noting that this cut-point skipping approach, by avoiding generating chunks smaller than the minimum chunk size, also helps increase the average generated chunk size. In fact, the average generated chunk size exceeds the expected chunk size by an amount equal to the minimum chunk size. This is because the F(x) in Equation (3) is changed to $$(1 - e^{-\frac{x - minimum\ chunk\ size}{8192}})$$ after cut-point skipping, thus the value of the expected chunk size becomes 8 KB + minimum chunk size, which will be verified in Section 4.3. The speedup achieved by skipping the sub-minimum chunk cut-point can be estimated by $$1 + \frac{the\ minimum\ chunk\ cut-point}{the\ expected\ chunk\ size}$$. The increased chunking speed comes from the eliminated computation on the skipped region, which will also be evaluated and verified in Section 4.3.

3.5 Normalized Chunking
In this subsection, we propose a novel chunking approach, called normalized chunking, to solve the problem of decreased deduplication ratio facing the cut-point skipping approach. As shown in Fig. 8, normalized chunking generates chunks whose sizes are normalized to a specified region centered at the expected chunk size. After normalized chunking, there are almost no chunks of size smaller than the minimum chunk size, which means that normalized chunking enables skipping cut-points for subminimum chunks to reduce the unnecessary chunking computation and thus speed up CDC.

In our implementation of normalized chunking, we selectively change the number of effective mask bits (i.e., the number of ‘1’ bits) for the hash-judging statement. For the traditional CDC approach with expected chunk size of 8 KB (i.e., 2^13), 13 effective mask bits are used for hash judgment...
NC reduces the number of small-sized chunks, which makes it possible to combine it with the cut-point skipping approach to achieve high chunking speed without sacrificing the deduplication ratio as suggested in Fig. 8.

NC further improves the deduplication ratio by reducing the number of large-sized chunks, which compensates for the reduced deduplication ratio caused by reducing the number of small-sized chunks in FastCDC.

The implementation of FastCDC does not add additional computing and comparing operations. It simply separates the hash judgment into two parts, before and after the expected chunk size.

Fig. 9 shows the chunk-size distribution after normalized chunking in comparison with FastCDC without NC on the TAR dataset (whose workload characteristics are detailed in Table 2 in Section 4.1). The normalization levels 1, 2, and 3 indicate that the normalized chunking uses the mask bits of (14, 12), (15, 11), (16, 10), respectively, where the first and the second integers in the parentheses indicate the numbers of effective mask bits used in the hash judgment before and after the expected chunk size (or normalized chunk size) of 8 KB. Fig. 9 also suggests that the chunk-size distribution is approximately normalized to a specified region always larger than the minimum chunk size, instead of following the exponential distribution (see Fig. 5).

Generally, there are three benefits or features of normalized chunking (NC):

- NC reduces the number of small-sized chunks, which makes it possible to combine it with the cut-point skipping approach to achieve high chunking speed without sacrificing the deduplication ratio as suggested in Fig. 8.
- NC further improves the deduplication ratio by reducing the number of large-sized chunks, which compensates for the reduced deduplication ratio caused by reducing the number of small-sized chunks in FastCDC.
- The implementation of FastCDC does not add additional computing and comparing operations. It simply separates the hash judgment into two parts, before and after the expected chunk size.

Table 2 Workload Characteristics of the Seven Datasets Used in the Performance Evaluation

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Workload descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAR</td>
<td>56 GB</td>
<td>215 tarred files from several open source projects such as GCC, GDB, Emacs, etc. [47]</td>
</tr>
<tr>
<td>LNX</td>
<td>178 GB</td>
<td>390 versions of Linux source code files (untarred). There are totally 16, 381, 277 files [48].</td>
</tr>
<tr>
<td>WEB</td>
<td>237 GB</td>
<td>102 days’ snapshots of the website: news.sina.com, which are collected by crawling software wget with a maximum retrieval depth of 3.</td>
</tr>
<tr>
<td>VMA</td>
<td>138 GB</td>
<td>90 virtual machine images of different OS release versions, including CentOS, Fedora, etc. [49]</td>
</tr>
<tr>
<td>VMB</td>
<td>1.9 TB</td>
<td>125 backups of an Ubuntu 12.04 virtual machine image in use by a research group.</td>
</tr>
<tr>
<td>RDB</td>
<td>1.1 TB</td>
<td>198 backups of the Redis key-value store database snapshots, i.e., dump.rdb files.</td>
</tr>
<tr>
<td>SYN</td>
<td>2.1 TB</td>
<td>300 synthetic backups. The backup is simulated by the file create/delete/modify operations [50].</td>
</tr>
</tbody>
</table>

As shown in Fig. 9, there are only a very small number of chunks smaller than 2 or 4 KB after normalized chunking while FastCDC without NC has a large number of chunks smaller than 2 or 4 KB (consistent with the discussion in Section 3.4). Thus, when combining NC with the cut-point skipping to speed up the CDC process, only a very small portion of chunk cut-points will be skipped in FastCDC, leading to nearly the same deduplication ratio as the conventional CDC approaches without the minimum chunk size requirement. In addition, normalized chunking allows us to enlarge the minimum chunk size to maximize the chunking speed without sacrificing the deduplication ratio.

It is worth noting that the chunk-size distribution shown in Fig. 9 is not truly normal distribution but an approximation of it. Actually, it follows an improved exponential distribution calculating from Equation (3) as follows (taking the NC 2 normalization level, deduplication ratio, and chunking speed, it means that there will be a “sweet spot” among the normalization level, deduplication ratio, and chunking speed, which will be studied and evaluated in Section 4.

3.6 Putting It All Together

To put things together and in perspective. Algorithm 3.6 describes FastCDC combining the three key techniques: optimizing hash judgment, cut-point skipping, and normalized chunking (with the expected chunk size of 8 KB). The
data structure “Gear” is a predefined array of 256 random 64-bit integers with one-to-one mapping to the values of byte contents for chunking [13].

**Algorithm 2. FastCDC8KB (with NC)**

**Input:** data buffer, src; buffer length, n
**Output:** chunking breakpoint i

```plaintext
MaskS ← 0x0000d9f903530000LL; // 15 '1' bits;
MaskA ← 0x0000d903530000LL; // 13 '1' bits;
MaskL ← 0x0000d90000350000LL; // 11 '1' bits;
MinSize ← 2 KB; MaxSize ← 64 KB;
fp ← 0; i ← MinSize; NormalSize ← 8 KB;
if n ≤ MinSize then
  return n;
if n ≥ MaxSize then
  n ← MaxSize;
else if n ≤ NormalSize then
  NormalSize ← n;
for : i < NormalSize; i++; do
  fp = (fp << 1) + Gear[src[i]];
  if (! (fp & MaskS) ) then
    return i; // if the masked bits are all '0';
  if i < n then
    fp = (fp << 1) + Gear[src[i]];
  if ! (fp & MaskL) then
    return i; // if the masked bits are all '0';
return i;
```

As shown in Algorithm 3.6, FastCDC uses normalized chunking to divide the chunking judgment into two loops with the optimized hash judgment. Note that FastCDC without normalized chunking is not shown here but can be easily implemented by using the new hash-judging statement “…fp & MaskA” where the MaskA is padded with 35 zero bits to enlarge the sliding window size to 48 bytes as that used in the Rabin-based CDC [6]. Note that MaskA, MaskS, and MaskL are three empirically derived values where the padded zero bits are almost evenly distributed for slightly higher deduplication ratio according to our large scale tests.

FastCDC implements normalized chunking by using mask value MaskS and MaskL to make the chunking judgment harder or easier (to generate chunks smaller or larger than the expected chunk size) when the current position is smaller or larger than the expected chunk size, respectively. And the number of ’1’ bits in MaskS and MaskL can be changed for different normalization levels. The minimum chunk size used in Algorithm 3.6 is 2 KB, which can be enlarged to 4 KB or 8 KB to further speed up the CDC process while combining with normalized chunking. Tuning the parameters of minimum chunk size and normalization level will be studied and evaluated in the next section.

In addition, we implement the normalized chunking scheme in Rabin-based CDC as shown in Algorithm 3.6. This improved Rabin-based CDC is also evaluated in the next section. Note that the hash judgment optimization for Gear is not applied for Rabin. This is because there are many zero hash values generated by Rabin [9], which results too many positions satisfying the chunking judgment “(fp & MaskValue) == false” and thus the generated average chunk size after Rabin-based CDC will be far blow the expected average chunk size.

### 3.7 Rolling Two Bytes Each Time

Besides the techniques mentioned in the above subsection, we also propose another independent technique called “rolling two bytes each time” on top of FastCDC. As shown in Algorithm 3.7, the core idea of this technique is that the fingerprint fp is rolling two bytes each time (i.e., “fp << 2”) in contrast to the traditional way of rolling one byte each time (see Algorithm 3.2), and then we judge the even and odd bytes respectively to determine the chunks’ boundaries. Specifically, ① for the even bytes, we use the Gear_Is table (Gear_Is contains elements from the Gear table, which are all left shift one bit) and the mask value MaskA (i.e., MaskA < < 1) for the hash judgment in FastCDC, this is because when we judge the fp for the even bytes, fp has been already left shift two bits (as described in Algorithm 3.7); ② for the odd bytes, we process the fingerprints using Gear table and the mask value MaskA in the traditional way.

**Algorithm 3. RabinCDC8KB (With NC)**

**Input:** data buffer, src; buffer length, n
**Output:** chunking breakpoint i

```plaintext
MinSize ← 2 KB; MaxSize ← 64 KB;
fp ← 0; i ← MinSize; NormalSize ← 8 KB;
if n ≤ MinSize then
  return n;
if n ≥ MaxSize then
  n ← MaxSize;
else if n ≤ NormalSize then
  NormalSize ← n;
for : i < NormalSize; i++; do
  fp = ((fp’U(a) << 8)|b’T[fp >> N];
  if fp & (8192 * 2 – 1) == 0x78 then
    return i;
  if fp & (8192/2 – 1) == 0x78 then
    return i;
```

**Algorithm 4. Rolling Two Bytes Each Time on FastCDC8KB (Without NC for Simplicity)**

**Input:** data buffer, src; buffer length, n
**Output:** chunking breakpoint i

```plaintext
MaskA ← 0x0000d9035300000LL; // 13 '1' bits;
MaskA_I's ← (MaskA < < 1); fp ← 0; i ← MinSize;
MinSize ← 2 KB; MaxSize ← 64 KB;
fp = (fp < < 2) + Gear_I's[ src[2 * i] ];
if ! (fp & MaskA_I's) then
  return 2 * i;
fp+ = Gear[src[2 * i + 1] ];
if ! (fp & MaskA ) then
  return 2 * i + 1;
return n;
```
This technique is simple but very useful for FastCDC since it reduces one shift operation when chunking each two bytes compared with the traditional approach (rolling one byte each time as shown in Algorithm 3.2) while ensuring exactly the same chunking results. Note that it requires to lookup one more table and increases additional computation operations of \( \frac{2}{C_3 i} \) and \( \frac{2}{C_3 i+1} \), but these overheads are minor and FastCDC using this technique is about 30-40 percent faster than rolling one byte each time according to our evaluation results discussed in the next section.

### 4 Performance Evaluation

#### 4.1 Experimental Setup

**Experimental Platform.** To evaluate FastCDC, we implement a prototype of the data deduplication system on the Ubuntu 18.04.1 operating system running on an Intel Xeon(R) Gold 6130 processor at 2.1 GHz, with a 128 GB RAM. To better evaluate the chunking speed, another Intel i7-8700 processor at 3.2 GHz is also used for comparison.

**Configurations for CDC and Deduplication.** Three CDC approaches, Rabin-, Gear-, and AE-based CDC, are used as the baselines for evaluating FastCDC. Rabin-based CDC is implemented based on the open-source project LBFS [6] (also used in many published studies [7], [19] or project [51]), where the sliding window size is configured to be 48 bytes. The Gear- and AE-based CDC schemes are implemented according to the algorithms described in their papers [13], [24], and we obtain performance results similar to and consistent with those reported in these papers. Here all the CDC approaches are configured with the maximum and minimum chunk sizes of \( 8 \times \) and \( \frac{1}{4} \times \) of the expected chunk size, the same as configured in LBFS [6]. The deduplication prototype consists of approximately 3,000 lines of C code, which is compiled by GCC 7.4.0 with the “-O3” compiler option to maximize the speed of the resulting executable.

**Performance Metrics of Interest.** *Chunking speed* is measured by the in-memory processing speed of the evaluated CDC approaches and obtained by the average speed of five runs. *Deduplication ratio* is measured in terms of the percentage of duplicates detected after CDC, i.e., the size of duplicate data detected in terms of the percentage of total data size before deduplication.

**Average chunk size after CDC** is \( \frac{\text{Total data size}}{\text{Number of chunks}} \), which reflects the metadata overhead for deduplication indexing.

**Evaluated Datasets.** Seven datasets with a total size of about 6 TB are used for evaluation as shown in Table 2. These datasets consist of the various typical workloads of deduplication, including the source code files, virtual machine images, database snapshots, etc., whose deduplication ratios vary from 40 to 98 percent, which will be detailed in Table 3 in the next subsection.

#### 4.2 A Study of Optimizing Hash Judgment

This subsection discusses an empirical study of FastCDC using techniques of the optimized hash judgment and ‘rolling two bytes each time’. Fig. 10 shows the chunking speed of the four CDC approaches running on the RDB dataset, as a function of the expected chunk size and all using the minimum chunk size of \( \frac{1}{4} \times \) of that for cut-point skipping. In general, the Rabin-based CDC has the lowest speed, and Gear-based CDC are about \( \frac{3}{2} \) faster than Rabin. FastCDC using optimized hash judgement (i.e., FC’ in Fig. 10) is
about 5× faster than Rabin and 1.5× faster than Gear regardless of the speed of the CPU processor and the expected chunk size. The high chunking speed of FastCDC stems from its simplification of the hash judgment after the fast Gear hash is used for chunking as described in Section 3.3. Meanwhile, FastCDC using ‘rolling two bytes each time’ (i.e., FC’ in Fig. 10) further increases the chunking speed by 40-50 percent since it further reduces calculation operation during CDC. Note that FC’ achieves exactly the same chunking results as FC, thus we do not discuss metrics of deduplication ratio and generated chunk size for FC’ in the remainder of this paper.

Table 3 shows the deduplication ratio and the average size of generated chunks (post-chunking) achieved by the three CDC approaches. We compare the Gear-based CDC (GC), and FastCDC (FC) approaches against the classic Rabin-based CDC (i.e., the baseline: RC) and record the percentage differences (in parentheses).

In general, FastCDC achieves nearly the same deduplication ratio as Rabin regardless of the expected chunk size and workload, and the difference between them is tiny as shown in the 3rd, 5th, 7th columns in Table 3 except on the WEB dataset. On the other hand, the Gear-based CDC has a much lower deduplication ratio on the datasets TAR and WEB due to its limited sliding window size as discussed in Section 3.2.

For the metric of the average size of generated chunks, the difference between the Rabin-based CDC and FastCDC is smaller than ±1.0 percent on most of the datasets. For the datasets WEB, FastCDC has 7.89 percent larger average chunk size than Rabin-based CDC, which is acceptable since the larger average chunk size means fewer chunks and fingerprints for indexing in a deduplication system (without sacrificing deduplication ratio) [3]. But for the Gear-based CDC, the average chunk size differs significantly in some datasets while its deduplication ratio is still a bit lower than other CDC approaches due to its smaller sliding window size.

We also compare the chunk-size distributions of the three tested chunking approaches in Fig. 11: FastCDC has nearly the same chunk-size distribution as Rabin on datasets TAR, VMA, and RDB, which generally follows the exponential distribution as discussed in Section 3.4. Note that the results in Fig. 11b are very different from others. This is because there are many zero bytes in this dataset according to our observation, which makes the chunking fingerprints not so random (thus not follow the exponential distribution). However, comparing with Rabin and Gear, FastCDC’s chunk-size distribution on WEB is the most similar to other datasets, which explains why FastCDC achieves the highest deduplication ratio on WEB among the three tested chunking approaches (see Table 3).

In summary, FastCDC with the optimized hash judgment achieves a chunking speed that is 5× higher than Rabin-based CDC while satisfactorily solving the problems of low deduplication ratio and smaller sliding window size faced by Gear-based CDC.

### 4.3 Evaluation of Cut-Point Skipping

This subsection discusses the evaluation results of cut-point skipping technique. Figs. 12a and 12b show the impact of applying different minimum chunk sizes on the chunking speed of FastCDC. Since the chunking speed is not so sensitive to the workloads, we only show the three typical workloads in Fig. 12. In general, cut-point skipping greatly accelerates the CDC process since the skipped region will not be hash-processed by CDC. The speedup of the FastCDC applying the minimum chunk sizes of 4 and 2 KB over the FastCDC without the constraint of the minimum chunk size (i.e., Min-0 KB) is about 1.25× and 1.50× respectively, which is almost consistent with the equation: 1 + \( \frac{\text{the minimum chunk size}}{\text{the expected chunk size}} \) as discussed in Section 3.4.

Figs. 12c and 12d show the impact of applying different minimum chunk sizes on the deduplication ratio and average
generated chunk size of FastCDC. In general, deduplication ratio declines with the increase of the minimum chunk size applied in FastCDC, but not proportionally. For the metric of the average generated chunk size in FastCDC, it is approximately equal to the summation of the expected chunk size and the applied minimum chunk size. This means that the MIN-4 KB solution has the average chunk size of \(8 + 4 = 12\) KB, leading to fewer chunks for fingerprints indexing in deduplication systems. Note that the increased portion of the average generated chunk size is not always equal to the size of the applied minimum chunk size, because the Rabin hashes of contents may not strictly follow the uniform distribution (as described in Equation (3) in Section 3.4) on some datasets. In addition, the average chunk sizes of dataset LNX are smaller than the minimum chunk size, which results from the many very small files whose sizes are much smaller than the minimum chunk size in LNX.

In summary, the results shown in Fig. 12 suggest that cut-point skipping helps obtain higher chunking speed and increase the average chunk size but at the cost of decreased deduplication ratio. The decreased deduplication ratio will be addressed by normalized chunking as evaluated in the next two subsections.

### 4.4 Evaluation of Normalized Chunking

In this subsection, we conduct a sensitivity study of normalized chunking (NC) on the TAR dataset, as shown in Fig. 13. Here the expected chunk size of FastCDC without NC is 8 KB and the normalized chunk size of FastCDC with NC is configured as the 4 KB + minimum chunk size. The normalization levels 1, 2, 3 refer to the three pairs of numbers of effective mask bits \((14,12),(15,11),(16,10)\) respectively that normalized chunking applies when the chunking position is smaller or larger than the normalized (or expected) chunk size, as discussed in Section 3.5.

Figs. 13a and 13b suggest that normalized chunking (NC) detects more duplicates when the minimum chunk size is about 4, 6, and 8 KB but slightly reduces the average generated chunk size, in comparison with FastCDC without NC. This is because NC reduces the number of large-sized chunks as shown in Fig. 9 and discussed in Section 3.5. The results also suggest that NC touches the “sweet spot” of deduplication ratio at the normalization level of 2 when the minimum chunk size is 4, 6, or 8 KB. This is because the very high normalization levels tend to have a similar chunk-size distribution to the Fixed-Size Chunking as shown in Fig. 9 in Section 3.5, which fails to address the boundary-shift problem and thus detects fewer duplicates. Figs. 13c and 13d suggest that NC, when combined with the approach of enlarging the minimum chunk size for cut-point skipping, greatly increases the chunking speed on the two tested processors.

In general, considering the three metrics of chunking speed, average generated chunk size, and deduplication ratio as a whole, as shown in Fig. 13, NC-2 with MinSize of 8 KB maximizes the chunking speed without sacrificing the deduplication ratio. Note that NC-2 with MinSize of 6 KB achieves the highest deduplication ratio among those NC approaches whose average chunk size \(\geq\) that of Rabin and FastCDC tested in Table 3 (with the minimum chunk size of 2 KB).

### 4.5 Comprehensive Evaluation of FastCDC

In this subsection, we comprehensively evaluate the performance of FastCDC with the combined capability of the five key techniques: Gear-based rolling hash, optimizing hash judgment, cut-point skipping, rolling two bytes each time, and normalized chunking using “NC-2” and minimum chunk size of 6 KB/8 KB as suggested by the last subsection. Finally, twelve CDC approaches are tested for evaluation:

- **RC-v1** (or **RC-MIN-2 KB**) is Rabin-based CDC used in LBFS [6]; RC-v2 and RC-v3 refer to Rabin-based CDC using normalized chunking with a minimum chunk size of 4 and 6 KB respectively.
- **FC-v1** is FastCDC uses the techniques of optimizing hash judgment and cut-point skipping with a minimum chunk size of 2 KB; FC-v2 and FC-v3 refer to FastCDC using all the four techniques with a minimum chunk size of 6 and 8 KB, respectively.
- **FC’-v1**, **FC’-v2**, and **FC’-v3** are FastCDC using the technique of rolling two bytes each time on top of FC-v1, FC-v2, and FC-v3 respectively.
- **AE-v1** and **AE-v2** refer to AE-based CDC [24] and its optimized version [52];
- **Fixed-Size Chunking (FIXC)** is also tested for comparison using the average chunk size of 10 KB (to better understand content-defined chunking).

Evaluation results in Table 4 suggest that FC-v1, FC-v2, AE-v2, and RC-v2 achieves nearly the same deduplication ratio as RC-v1 in most cases, which suggests that the normalized chunking scheme works well on both Rabin and FastCDC. Note that FIXC works well on the datasets LNX and VMB, because LNX has many files smaller than the fixed-size chunk of 10 KB (and thus the average generated chunk size is also smaller than 10 KB) and VMB has many structured backup data (and thus VMB is suitable for FIXC).

Table 5 shows that RC-v1, RC-v2, AE-v1, AE-v2, FC-v1, and FC-v2 generate similar average chunk size. But the approaches of RC-v3 and FC-v3 has a much larger average chunk size, which means that it generates fewer chunks and thus less metadata for deduplication processing. Meanwhile,
RC-v3 and FC-v3 still achieves a comparable deduplication ratio, slightly lower than RC-v1 as shown in Table 4, while providing a much higher chunking speed as discussed later.

Table 4 suggests that FC’-v3 has the highest chunking speed, about 12\times faster than the Rabin-based approach, about 2.5\times faster than FC-v1. This is because FC’-v3 is the final FastCDC using all the five techniques to speed up the CDC process. In addition, FC’-v2 is also a good CDC candidate since it has a comparable deduplication ratio while also working well on the other two metrics of chunking speed and average generated chunk size. But this acceleration is limited since the main bottleneck for Rabin-based CDC is still the rolling hashing computation.

Table 6 further studies the CPU overhead among the eight CDC approaches. The CPU overhead is averaged on 1,000 test runs by the Linux tool “Perf”. The results suggest that FC’-v3 has the fewest instructions for CDC computation, the higher instructions per cycle (IPC), and thus the least CPU time overhead, i.e., CPU cycles. Generally, FastCDC greatly reduces the number of instructions for CDC computation by using the techniques of Gear-based hashing, optimizing hash judgment, and rolling two bytes each time (i.e., FC’-v1), and then minimizes the number of computation instructions by enlarging the minimum chunk size for cut-point skipping and combining normalized chunking (i.e., FC’-v2 and FC’-v3). In addition, FastCDC increases the IPC for the CDC computation by well pipelining the instructions of hashing and hash-judging tasks in up-to-date processors. Therefore, these results clearly reveal the reason why FastCDC is much faster than Rabin- and AE-based CDC is that the former not only reduces the number of instructions and branches, but also increases the IPC for the CDC process.
In summary, as shown in Tables 4, 5, 6 and Fig. 14, FastCDC (i.e., FC'-v2 recommended) significantly speeds up the Content-Defined Chunking process and achieves a comparable and even higher deduplication ratio with the similar average chunk size by using a combination of the five key techniques proposed in Section 3.

4.6 Impact of CDC on Overall System Throughput
To understand the impact of the different CDC algorithms on the overall throughput of data deduplication system, we implemented them in the open-source Destor deduplication system [19]. In this evaluation, we use a Ramdisk-driven emulation to avoid the performance bottleneck caused by disk I/O. And for each dataset, we only use 5 GB, a small part of its total size in the evaluation. In addition, to examine the maximum impact of different CDC algorithms on the system throughput, we configure Destor with: (1) using the fast intel ISA-L library for SHA1 computation [53] (SHA1 speed would be about 3-4 GB/s on our tested CPUs); (2) indexing all fingerprints in RAM; (3) pipelining the deduplication subtasks (i.e., chunking, fingerprinting, indexing, etc.).

Fig. 15 shows that FastCDC (i.e., FC'-v2) helps achieve about 1.2-3.0X higher overall system throughput than RC-v1, RC-v2, AE-v1, and AE-v2, while achieving a comparable or even higher deduplication ratio as shown in Table 4. This is because when Destor pipelines the deduplication subtasks and the CDC becomes the bottleneck of the system, acceleration of the CDC can directly benefit the overall system throughput before the system meets another performance bottleneck.

5 Conclusion
In this paper, we propose FastCDC, a much faster CDC approach for data deduplication than the state-of-the-art CDC approaches while achieving a comparable deduplication ratio. The main idea behind FastCDC is the combined use of five key techniques, namely, Gear-based fast rolling hashing, optimizing the hash judgment for chunking, subminimum chunk cut-point skipping, normalized chunking, and rolling two bytes each time. Our experimental evaluation demonstrates that FastCDC obtains a chunking speed that is about 3-12× higher than that of the state-of-the-art CDC approaches while achieving nearly the same deduplication ratio as the classic Rabin-based CDC. In addition, our study of overall system throughput shows that Destor [19] using FastCDC helps achieve about 1.2-3.0X higher overall system throughput than using other CDC approaches.

FastCDC has been adopted as the default chunker in several Github projects (for quickly detecting duplicate contents), such as Rdedup [20], Content Blockchain [21], etc. We have also released the FastCDC source code at https://github.com/Borelset/destor/tree/master/src/chunking to be shared with the deduplication and storage systems research community.

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An Event-Driven Approach to Serverless Seismic Imaging in the Cloud

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Abstract—Adapting the cloud for high-performance computing (HPC) is a challenging task, as software for HPC applications hinges on fast network connections and is sensitive to hardware failures. Using cloud infrastructure to recreate conventional HPC clusters is therefore in many cases an infeasible solution for migrating HPC applications to the cloud. As an alternative to the generic lift and shift approach, we consider the specific application of seismic imaging and demonstrate a serverless and event-driven approach for running large-scale instances of this problem in the cloud. Instead of permanently running compute instances, our workflow is based on a serverless architecture with high throughput batch computing and event-driven computations, in which computational resources are only running as long as they are utilized. We demonstrate that this approach is very flexible and allows for resilient and nested levels of parallelization, including domain decomposition for solving the underlying partial differential equations. While the event-driven approach introduces some overhead as computational resources are repeatedly restarted, it inherently provides resilience to instance shut-downs and allows a significant reduction of cost by avoiding idle instances, thus making the cloud a viable alternative to on-premise clusters for large-scale seismic imaging.

1 INTRODUCTION

Seismic imaging of the earth’s subsurface is one of the most computationally expensive applications in scientific computing, as state-of-the-art imaging methods such as least-squares reverse time migration (LS-RTM), require repeatedly solving a large number of forward and adjoint wave equations during numerical optimization (e.g., [1], [2], [3]). Similar to training neural networks, the gradient computations in seismic imaging are based on backpropagation and require storage or re-computations of the state variables (i.e., of the forward modeled wavefields). Due to the large computational cost of repeatedly modeling wave propagation over many time steps using finite difference modeling, seismic imaging requires access to high-performance computing (HPC) clusters, but the high cost of acquiring and maintaining HPC cluster makes this option only viable for a small number of major energy companies [4]. For this reason, cloud computing has lately emerged as a possible alternative to on-premise HPC clusters, offering many advantages such as no upfront costs, a pay-as-you-go pricing model and theoretically unlimited scalability. Outside of the HPC community, cloud computing is today widely used by many companies for general purpose computing, data storage and analysis or machine learning. Customers of cloud providers include major companies such as General Electric (GE), Comcast, Shell or Netflix, with the latter hosting their video streaming content on Amazon Web Services (AWS) [5].

However, adapting the cloud for high-performance computing applications such as seismic imaging, is not straightforward, as numerous investigations and case studies have shown that performance, latency, bandwidth and mean time between failures (MTBF) in the cloud can vary significantly between platform providers, services and hardware, and are often inferior compared to on-premise HPC resources. Especially in the early days of cloud computing, performance and network connections were considerably slower than on comparable on-premise HPC systems, as discussed in a number of publications. An early performance analysis by Jackson [6] of a range of typical NERSC HPC applications on Amazon’s Elastic Compute Cloud (EC2) found that, at the time of the comparison in 2010, applications on EC2 ran several orders of magnitude slower than on comparable HPC systems, due to low bandwidth and high latency. Other performance benchmarks from the late 2000s and early 2010s, similarly conclude that poor network performance severely limited the HPC capabilities of the cloud at that time [7], [8], [9], [10], [11], [12]. Cloud providers have since then responded by making significant improvements regarding network connections, now offering technologies such as InfiniBand, specialized network adapters such as Amazon’s elastic fabric adapter (EFA, [13]) and improved virtualization techniques to improve performance (e.g., AWS Nitro [14]). Accordingly, more recent benchmarks on various cloud platforms, including AWS and Microsoft Azure, find that the performance on newly introduced HPC
instances is oftentimes on par with modern on-premise HPC systems [15], [16]. Nevertheless, enhanced network technologies are typically limited to a small subset of specialized HPC instances, which are not as widely available as general purpose instances and which are accordingly more expensive [17].

Aside from network communication, several investigations [16], [18], [19] point out that embarrassingly parallel applications show very good performance that is comparable to (non-virtualized) HPC environments, even in the early days of the cloud and using standard (non-HPC-optimized) nodes. Similarly, performance tests on single cloud nodes and bare-metal instances using HPCC and high-performance LINPACK benchmarks demonstrate good performance and scalability as well [20], [21]. These findings underline that the lift and shift approach for porting HPC applications to the cloud is unfavorable, as most HPC codes are based on highly synchronized message passing (i.e., MPI [22]) and rely on stable and fast network connections, which are only available on certain (limited) instance types and which are thus more expensive. On the other hand, individual compute nodes and architectures offered by cloud computing are indeed comparable to current supercomputing systems [21] and the cloud offers a range of novel technologies such as cloud object storage or event-driven computations [23]. These technologies are not available on traditional HPC systems and make it possible to address computational bottlenecks of HPC in fundamentally new ways. Porting HPC applications to the cloud in a way that is financially viable therefore requires a careful re-architecture of the corresponding codes and software stacks to take advantage of these technologies, while minimizing communication and idle times. This process is heavily application dependent and requires the identification of how specific applications can take advantage of specialized cloud services such as serverless compute or high-throughput batch processing to mitigate resilience issues and minimize cost, while avoiding idle instances and fast network fabrics where possible.

Based on these premises, we present a workflow for large-scale seismic imaging on AWS, which does not rely on a conventional cluster of virtual machines, but is instead based on a serverless workload that takes advantage of the mathematical properties of the seismic imaging optimization problem [24]. Similar to deep learning, objective functions in seismic imaging consist of a sum of (convex) misfit functions and iterations of associated optimization algorithms exhibit the structure of a MapReduce program [25]. The map part corresponds to computing the gradient of each element in the sum and is embarrassingly parallel to compute, but individual gradient computations are expensive as they involve solving partial differential equations (PDEs). The reduce part corresponds to the summation of the gradients and update of the model parameters and is comparatively cheap to compute, but I/O intensive. Instead of performing these steps on a cluster of permanently running compute instances, our workflow is based on specialized AWS services such as AWS Batch and Lambda, which are responsible for automatically launching and terminating the required computational resources [23], [26]. EC2 instances are only running as long as they are utilized and are shut down automatically as soon as computations are finished, thus preventing instances from sitting idle. This stands in contrast to alternative MapReduce cloud services, such as Amazon’s Elastic Map Reduce (EMR), which is based on Apache Hadoop and relies on a cluster of permanently running EC2 instances [27]. In our approach, expensive gradient computations are carried out by AWS Batch, a service for processing embarrassingly parallel workloads, but with the possibility of using (MPI-based) domain decomposition for individual solutions of partial differential equations (PDEs). The cheaper gradient summations are performed by Lambda functions, a service for serverless computations, in which code is run in response to events, without the need to manually provision computational resources [23].

The following section provides an overview of the mathematical problem that underlies seismic imaging and we identify possible characteristics that can be taken advantage of to avoid the aforementioned shortcomings of the cloud. In the subsequent section, we describe our seismic imaging workflow, which has been developed specifically for AWS, but the underlying services are available on other cloud platforms (Google Compute Cloud, Azure) as well. We then present a performance analysis of our workflow on a real-world seismic imaging application, using a popular subsurface benchmark model [28]. Apart from conventional scaling tests, we also consider specific cloud metrics such as resilience and cost, which, aside from the pure performance aspects like scaling and time-to-solution, are important practical considerations for HPC in the cloud. An early application of our workflow is presented in an expanded conference abstract [29].

2 Problem Overview

Seismic imaging and parameter estimation are a set of computationally challenging inverse problems with high practical importance, as they are today widely used for geophysical exploration and monitoring geohazards. Exploration seismology is based on the manual excitation of seismic sources, which trigger sound and/or elastic waves that travel through the subsurface. At geological interfaces, waves are scattered and reflected, causing parts of the wavefield to travel back to the surface, where it is recorded by an array of receivers (Fig. 1). In a seismic survey, the source is fired repeatedly as it moves across the survey area and the observed data that is collected for each source location is denoted by $d_i$. The objective of seismic imaging is to recover a physical parametrization of the subsurface from the recorded seismic data. In the setting of inverse problems, this is achieved by minimizing the misfit between recorded data and data that is predicted using numerical modeling. The forward problem is defined as the computation of a predicted seismic shot record through a forward modeling operator $F(m, q)$, where $m$ denotes the discretized unknown parameters, such as the seismic image or the acoustic wave speed and the vector $q$, represents the location and the time signature of the seismic source. The evaluation of the forward modeling operator corresponds to numerically solving a discretized version of the wave equation for the given set of model parameters and current
source location using for example finite differences (details are given in Appendix A, which can be found on the Computer Society Digital Library at http://doi.ieeecomputer society.org/10.1109/TPDS.2020.2982626.).

In the inverse problem, we are interested in recovering the parameters $\mathbf{m}$ from the observed seismic data $\mathbf{d}_i$. Mathematically, this is achieved by formulating an unconstrained optimization problem in which we minimize the $\ell_2$-misfit between the observed and numerically modeled data [30], [31]:

$$
\text{minimize } \Phi(\mathbf{m}) = \sum_{i=1}^{n_s} \frac{1}{2} || F(\mathbf{m}, \mathbf{q}_i) - \mathbf{d}_i ||^2_2. \tag{1}
$$

In essence, the goal of seismic inversion is to find a set of model parameters $\mathbf{m}$, such that the numerically modeled data matches the observed data from the seismic survey. The total number of individual source experiments $n_s$ for realistic surveys, i.e., the number of PDEs that have to be solved for each evaluation of $\Phi(\mathbf{m})$, is quite large and lies in the range of $10^4$ for 2D surveys and $10^5$ for 3D surveys.

Seismic inverse problems of this form are typically solved with gradient-based optimization algorithms such as (stochastic) gradient descent, (Gauss-)Newton methods, sparsity-promoting minimization or constrained optimization (e.g., [32], [33]) and therefore involve computing the gradient of Equation (1) for all or a subset of indices $i$. The gradient of the objective function is given by:

$$
\mathbf{g} = \sum_{i=1}^{n_s} \nabla F(\mathbf{m}, \mathbf{q}_i), \tag{2}
$$

where the linear operator $\nabla F(\mathbf{m}, \mathbf{q}_i)$ is the partial derivative of the forward modeling operator with respect to the model parameters $\mathbf{m}$ and $\nabla$ denotes the matrix transpose. Both the objective function, as well as the gradient exhibit a sum structure over the source indices and are embarrassingly parallel to compute. Evaluating the objective function and computing the gradient are therefore instances of a MapReduce program [25], as they involve the parallel computation and subsequent summation of elements of the sum. However, computing the gradient for a single index $i$ involves solving two PDEs, namely a forward wave equation and an adjoint (linearized) wave equation (denoted as a multiplication with $\nabla^T$). For realistically sized 3D problems, the discretized model in which wave propagation is modeled has up to $10^9$ variables and modeling has to be performed for several thousand time steps. The observed seismic data $\mathbf{d}_i, (i = 1, \ldots, n_s)$ is typically in the range of several terabytes and a single element of the data (a seismic shot record) ranges from several mega- to gigabytes.

The problem structure of Equation (1) is very similar to deep learning and the parallels between convolutional neural networks and PDEs have lately attracted strong attention [34]. As in deep learning, computing the gradient of the objective function (Equation (2)) is based on backpropagation and in principle requires storing the state variables of the forward problem. However, in any realistic setting the wavefields are too big to be stored in memory and therefore need to be written to secondary storage devices or recomputed from a subset of checkpoints [35]. Alternatively, domain decomposition can be used to reduce the domain size per compute node such that the forward wavefields fit in memory, or time-to frequency conversion methods can be employed to compute gradients in the frequency domain [3], [36]. In either case, computing the gradient for a given index $i$ is expensive both in terms of necessary floating point operations, memory and I/O and requires highly optimized finite-difference modeling codes for solving the underlying wave equations. Typical computation times of a single (3D-domain) gradient $\mathbf{g}$, i.e., one element of the sum) are in the range of minutes to hours, depending on the domain size and the complexity of the wave simulator, and the computations have to be carried out for a large number of source locations and iterations.

The high computational cost of seismic modeling, in combination with the complexity of implementing optimization algorithms to solve Equation (1), leads to enormously complex inversion codes, which have to run efficiently on large-scale HPC clusters. A large amount of effort goes into implementing fast and scalable wave equation solvers [37], [38], as well as into frameworks for solving the associated inverse problem [39], [40], [41], [42]. Codes for seismic inversion are typically based on message passing and use MPI to parallelize the loop of the source indices (Equation (1)). Furthermore, a nested parallelization is oftentimes used to apply domain-decomposition or multi-threading to individual PDE solves. The reliance of seismic inversion codes on MPI to implement an embarrassingly parallel loop is disadvantageous in the cloud, where the mean-time-between-failures (MTBF) is much shorter than on HPC systems [6] and instances using spot pricing can be arbitrarily shut down at any given time [43]. Another important aspect is that the computation time of individual gradients can vary significantly and cause load imbalances, which is problematic in the cloud, where users are billed for running instances by the second, regardless of whether the instances are in use or idle. For these reasons, we present an alternative approach for seismic imaging in the cloud based on batch processing and event-driven computations.

3 EVENT-DRIVEN SEISMIC IMAGING ON AWS

3.1 Workflow

Optimization algorithms for minimizing Equation (1) essentially consists of three steps. First, the elements of the
gradient \( g_i \) are computed in parallel for all or a subset of indices \( i \in n_h \), which corresponds to the map part of a Map-Reduce program. The number of indices for which the objective is evaluated defines the batch size of the gradient. The subsequent reduce part consists of summing these elements into a single array and using them to update the unknown model/image according to the rule of the respective optimization algorithm (Algorithm 1). Optimization algorithms that fit into this general framework include variations of stochastic/full gradient descent (GD), such as Nesterov’s accelerated GD [44] or Adam [45], as well as the nonlinear conjugate gradient method [46], projected GD or iterative soft thresholding [47]. Conventionally, these algorithms are implemented as a single program and the gradient computations for seismic imaging are parallelized using message passing. Running MPI-based programs of this structure in the cloud require that users request a set of EC2 instances and establish a network connection between all workers [48]. Tools like StarCluster [49] or AWS HPC [50] facilitate the process of setting up a cluster and even allow adding or removing instances to a running cluster. However, adding or dropping instances/nodes during the execution of an MPI program is not easily possible, so the number of instances has to stay constant during the entire length of the program execution, which, in the case of seismic inversion, can range from several days to weeks. This makes this approach not only prone to resilience issues, but it can result in significant cost overhead, if workloads are unevenly distributed and instances are temporarily idle.

**Algorithm 1.** Generic Algorithm Structure for Gradient-Based Minimization of Equation (1), Using a Fixed Number of Iterations \( n \).

1: Input: batch size \( n_b \), max. number of iterations \( n \), step size \( \alpha \), initial guess \( m_1 \)
2: for \( i = 1 \) to \( n \) do
3: Compute gradients \( g_i, i = 1, ..., n_b \) in parallel
4: Sum gradients: \( g = \sum_{i=1}^{n_b} g_i \)
5: Update optimization variable, e.g., using SGD:
   \[ m_{k+1} = m_k - \alpha g \]
6: end for

Instead of implementing and running optimization algorithms for seismic inverse problems as a single program that runs on a cluster of EC2 instances, we express the steps of a generic optimization algorithm through AWS Step Functions (Fig. 2) and deploy its individual components through a range of specialized AWS services [51]. Step functions allow the description of an algorithm as a collection of states, which are used to implement an iterative optimization loop. Each iteration involves computing the gradient of Equation 1 using AWS Batch (2) and the subsequent event-driven summation of all gradient components using Lambda functions (3). The final Lambda function uses the summed gradient to update the optimization variable (i.e., the seismic image). Once the updated variable is detected by the \texttt{WaitForUpdate} state, the workflow automatically progresses to the next iteration.

is that the execution of the workflow itself is managed by AWS and does not require running any EC2 instances, which is why we refer to this approach as serverless. During execution time, AWS automatically progresses the workflow from one state to the next and users are only billed for transitions between states, but the cost is negligible compared to the cost of running EC2 instances (0.025$ per 1,000 state transitions).

States can be simple if-statements such as the \texttt{IsCount Reached} state, which keeps track of the iteration number and terminates the workflow after a specified number of iterations, but states can also be used invoke other AWS services. Specifically, states can be used to invoke AWS Lambda functions to carry out serverless computations. Lambda functions allow users to run code in response to events, such as invocations through AWS Step Functions, and automatically assign the required amount of computational resources to run the code. Billing is based on the execution time of the code and the amount of used memory. Compared to EC2 instances, Lambda functions have a much shorter startup time in the range of milliseconds rather than minutes, but they are limited to 3 GB of memory and an execution time of 15 minutes. As such, Lambda functions themselves are not suitable for carrying out the gradient computations, but they can be used to manage other AWS services. In our workflow, we use Lambda functions invoked by the \texttt{ComputeGradient} state (Fig. 2) to launch AWS Batch jobs for computing the gradients. During the gradient computation, which can take up to several hours, the \texttt{Step Functions} check in a user-defined interval if the full gradient has been computed, before advancing the workflow to the next state. The \texttt{WaitForUpdate} state pauses...
the workflow for a specified amount of time, during which no additional computational resources are running other than the AWS Batch job itself.

### 3.2 Computing the Gradient

The gradient computations (Equation (2)) are the major workload of seismic inversion, as they involve solving forward and adjoint wave equations, but the embarrassingly parallel structure of the problem lends itself to high-throughput batch computing. On AWS, embarrassingly parallel workloads can be processed with AWS Batch, a service for scheduling and running parallel containerized workloads on EC2 instances [26]. Parallel workloads, such as computing a gradient of a given batch size, are submitted to a batch queue and AWS Batch automatically launches the required EC2 instances to process the workload from the queue. Each job from the queue runs on an individual instance or set of instances, with no communication being possible between individual jobs.

In our workflow, we use the Lambda function invoked by the ComputeGradient state (Fig. 2) to submit the gradient computations to an AWS Batch queue. Each element of the gradient $g_i$ corresponds to an individual job in the queue and is run by AWS Batch as a separate Docker container [53]. Every container computes the gradient for its respective source index $i$ and writes its resulting gradient to an S3 bucket (Fig. 3). Amazon’s cloud object storage system [54]. The gradients computed by our workflow are one-dimensional numpy arrays of the size of the vectorized seismic image and are stored in S3 as so-called objects [55]. Once an individual gradient $g_i$ has been computed, the underlying EC2 instance is shut down automatically by AWS Batch, thus preventing EC2 instances from idling.

Since no communication between jobs is possible, the summation of the individual gradients is implemented separately using AWS Lambda functions. For this purpose, each job also sends its S3 object identifier to a message queue (SQS) [56], which automatically invokes the reduction stage (Fig. 4). For the gradient computations, each worker has to download the observed seismic data of its respective source index from S3 and the resulting gradient has to be uploaded to S3 as well. The bandwidth with which objects are uploaded and downloaded is only limited by the network bandwidth of the EC2 instances and ranges from 10 to 100 Gbps [17]. Notably, cloud object storage such as S3 has no limit regarding the number of workers that can simultaneously read and write objects, as data is redundantly distributed among physically separated data centers, thus providing essentially unlimited IO scalability [54].

AWS Batch runs jobs from its queue as separate containers on a set of EC2 instances, so the source code of the application has to be prepared as a Docker container. Containerization facilitates portability and has the advantage that users have full control over managing dependencies and packages. Our Docker image contains the code for solving acoustic wave equations to compute gradients of a respective seismic source location. Since this is the most computationally intensive part of our workflow, it is important that the wave equation solver is optimized for performance, but is also implemented in a programming language that allows interfacing other AWS services such as S3 or SQS. In our workflow, we use a domain-specific language compiler called Devito for implementing and solving the underlying wave equations using time-domain finite-difference modeling [38], [57]. Devito is implemented in Python and provides an application programming interface (API) for implementing forward and adjoint wave equations as high-level symbolic expressions based on the SymPy package [58].
During runtime, the Devito compiler applies a series of performance optimizations to the symbolic operators, such as reductions of the operation count, loop transformations, and introduction of parallelism [57]. Devito then generates optimized finite-difference stencil code in C from the symbolic Python expressions and dynamically compiles and runs it. Devito supports both multi-threading using OpenMP, as well as generating code for MPI-based domain decomposition. Its high-level API allows expressing wave equations of arbitrary stencil orders or various physical representations without having to implement and optimize low-level stencil codes by hand. The complexity of implementing highly optimized and parallel wave equation solvers is therefore abstracted and vertically integrated into the AWS workflow.

By default, AWS Batch runs the container of each job on a single EC2 instance, but recently AWS introduced the possibility to run multi-node batch computing jobs [59]. Thus, individual jobs from the queue can be computed on a cluster of EC2 instances and the corresponding Docker containers can communicate via the AWS network. In the context of seismic imaging and inversion, multi-node batch jobs enable nested levels of parallelization, as we can use AWS Batch to parallelize the sum of the source indices, while using MPI-based domain decomposition and/or multi-threading for solving the underlying wave equations. This provides a large amount of flexibility in regard of the computational strategy for performing backpropagation and how to address the storage of the state variables. AWS Batch allows to scale horizontally, by increasing the number of EC2 instances of multi-node jobs, but also enables vertical scaling by adding additional cores and/or memory to single instances. In our performance analysis, we compare and evaluate different strategies for computing gradients with Devito regarding scaling, costs and turnaround time.

### 3.3 Gradient Reduction

Every computed gradient is written by its respective container to an S3 bucket, as no communication between individual jobs is possible. Even if all gradients in the job queue are computed by AWS Batch in parallel at the same time, we found that the computation time of individual gradients typically varies considerably (up to 10 percent), due to varying network performance or instance capacity. Furthermore, we found that the startup time of the underlying EC2 instances itself is highly variable as well, so jobs in the queue are usually not all started at the same time. Gradients therefore arrive in the bucket over a large time interval during the batch job. For the gradient reduction step, i.e., the summation of all gradients into a single array, we take advantage of the varying time-to-solutions by implementing an event-driven gradient summation using Lambda functions. In this approach, the gradient summation is not performed by as single worker or the master process who has to wait until all gradients have been computed, but instead summations are carried out by Lambda functions in response to gradients being written to S3.

The event-driven gradient summation is automatically invoked through SQS messages, which are sent by the AWS Batch workers that have completed their computations and have saved their respective gradient to S3. Before being shut down, every batch worker sends a message with the corresponding S3 object name to an AWS SQS queue, in which all object names are collected (Fig. 4). Sending messages to SQS invokes AWS Lambda functions that read up to 10 messages at a time from the queue. Every invoked Lambda function that contains at least two messages, i.e., two object names, reads the corresponding arrays from S3, sums them into a single array, and writes the array as a new object back to S3. The new object name is sent to the SQS queue, while the previous objects and objects names are removed from the queue and S3. The process is repeated recursively until all $n_b$ gradients have been summed into a single array, with $n_b$ being the batch size for which the gradient is computed.

Since Lambda functions are limited to 3 GB of memory, it is not always possible to read the full gradient objects from S3. Gradients that exceed Lambda’s available memory are therefore streamed from S3 using appropriate buffer sizes and are re-uploaded to S3 using the `multipart_upload` functions of the S3 Python interface [60]. As the execution time of Lambda functions is furthermore limited to 15 minutes, the bandwidth of S3 is not sufficient to stream and re-upload objects that exceed a certain size within a single Lambda invocation. For this case, we include the possibility that the workers of the AWS Batch job split the computed gradients into smaller chunks that are saved separately in S3, with the respective objects names being sent to multiple SQS queues. The gradient summation is then performed in chunks by separate queues and Lambda functions. The `CreateQueues` task of our Step Functions workflow (Fig. 2) automatically creates the required number of queues before starting the optimization loop and the `CleanUp` state removes them after the final iteration.

The advantage of the event-based gradient reduction is that that the summation is executed asynchronously, as soon as at least two S3 objects are available, while other batch jobs are still running. Therefore, by the time the last batch worker finishes the computation of its respective gradient, all remaining gradients have already been summed into a single object, or at least a small number of objects. Furthermore, summing files of a single queue happens in parallel (if enough messages are in the queue), as multiple Lambda functions can be invoked at the same time. Furthermore, splitting the gradients itself into chunks that are processed by separate queues leads to an additional layer of parallelism. In comparison to a fixed cluster of EC2 instances, the event-driven gradient summation using Lambda function also takes advantage of the fact that the summation of arrays is computationally considerably cheaper than solving wave equations and therefore does not require to be carried out on the expensive EC2 instances used for the PDE solves.

### 3.4 Variable Update

Once the gradients have been computed and summed into a single array that is stored as an S3 object, the gradient is used to update the optimization variables of equation 1, i.e., the seismic image or subsurface parameters such as velocity. Depending on the specific objective function and optimization algorithm, this can range from simple operations like multiplications with a scalars (gradient descent)
to more computational expensive operations such as sparsity promotion or applying constraints [61]. Updates that use entry-wise operations only and are cheap to compute such as multiplications with scalars or soft-thresholding, can be applied directly by Lambda functions in the final step of the gradient summation. I.e., the Lambda function that sums the final two gradients, also streams the optimization variable of the current iteration from S3, uses the gradient to update it and directly writes the updated variable back to S3.

Many algorithms require access to the full optimization variable and gradient, such as Quasi-Newton methods and other algorithms that need to compute gradient norms. In this case, the variable update is too expensive and memory intensive to be carried out by Lambda functions and has to be submitted to AWS Batch as a single job, which is then executed on a larger EC2 instance. This can be accomplished by adding an extra state such as UpdateVariable to our Step Functions workflow. However, to keep matters simple, we only consider a simple stochastic gradient descent example with a fixed step size in our performance analysis, which is computed by the Lambda functions after summing the final two gradients [62]. The CheckUpdateStatus state of our AWS Step Functions advances the workflow to the next iteration, once the updated image (or summed gradient) has been written to S3. The workflow shown in Fig. 2 terminates the optimization loop after a predefined number of iterations (i.e., epochs), but other termination criteria based on gradient norms or function values are possible too. The update of the optimization variable concludes a single iteration of our workflow, whose performance we will now analyze in the subsequent sections.

4 Performance Analysis

In our performance analysis, we are interested in the performance of our workflow on a real-world seismic imaging application regarding scalability, cost and turn-around time, as well as the computational benefits and overhead introduced by our event-driven approach. We conduct our analysis on a popular 2D subsurface velocity model (Fig. 5a), called the 2004 BP velocity estimation benchmark model [28]. The seismic data set of this model contains 1,348 seismic source locations and corresponding observations \(d_i\), \((i = 1, \ldots, 1,348)\). The (unknown) seismic image (Fig. 5b) has dimensions of \(1,911 \times 10,789\) grid points, i.e., a total of almost 21 million parameters.

4.1 Weak Scaling

In our first performance test, we analyze the weak scaling behavior of our workflow by varying the batch size (i.e., the number of source locations) for which the gradient of the LS-RTM objective function (Equation (1)) is computed. For this test, we perform a single iteration of stochastic gradient descent (SGD) using our workflow and measure the time-to-solution as a function of the batch size. The workload per instance, i.e., per parallel worker, is fixed to one gradient. The total workload for a specified batch size is submitted to AWS Batch as a so-called array job, where each array entry corresponds to a single gradient \(g_i\). AWS Batch launches one EC2 instance per array entry (i.e., per gradient), runs the respective container on the instance and then terminates the instance.

In the experiment, we measure the time-to-solution for performing a single iteration of our workflow, i.e., one stochastic gradient descent update. Therefore, each run involves the following steps:

1) A Lambda function submits the AWS Batch job for specified batch size \(n_b\) (Fig. 3)
2) Compute gradients \(g_i\) \((i = 1, \ldots, n_b)\) in parallel (Fig. 3)
3) Lambda functions sum the gradients (Fig. 4):
   \[ g = \sum_{i=1}^{n_b} g_i \]
4) A Lambda function performs the SGD update of the image: \(x = x - \alpha g\)

We define the time-to-solution as the the time interval between the submission of the AWS Batch job by a Lambda function (step 1) and the time stamp of the S3 object containing the updated image (step 4). This time interval represents a complete iteration of our workflow.

The computations of the gradients are performed on m4.4xlarge instances (Appendix B, available in the online supplemental material) and the number of threads per instance is fixed to 8, which is the number of physical cores that is available on the instance. The m4 instance is a general purpose EC2 instance and we chose the instance size (4xlarge) such that we are able to store the wavefields for backpropagation in memory. The workload for each batch worker consists of solving a forward wave equation to model the predicted seismic data and an adjoint wave equation to backpropagate the data residual and to compute the gradient. For this and all remaining experiments, we use the acoustic isotropic wave equation with a second order finite difference (FD) discretization in time and 8th order in space. We model wave propagation for 12 seconds, which is the recording length of the seismic data. The time stepping interval is given by the Courant-Friedrichs-Lewy condition with 0.55 ms, resulting in 21,889 time steps. Since it is not possible for the waves to propagate through the whole

![Image](image-url)
domain within this time interval, we restrict the modeling grid to a size of $1,911 \times 4,001$ grid points around the current source location. After modeling, each gradient is extended back to the full model size ($1,911 \times 10,789$ grid points). The dimensions of this example represent a large-scale 2D example, but all components of our workflow are agnostic to the number of physical dimensions and are implemented for three-dimensional domains as well. The decision to limit the examples to a 2D model was purely made from a financial viewpoint and to make the results reproducible in a reasonable amount of time.

The timings ranging from a batch size of 1 to 128 are displayed in Fig. 6a. The batch size corresponds to the number of parallel EC2 instances on which the jobs are executed. The time-to-solution consists of three components that make up the full runtime of each job:

1) The average time for AWS Batch to request and launch the EC2 instances and to start the Docker containers on those instances.

2) The runtime of the containers

3) The additional gradient reduction and image update time, given by the time interval between the termination of the AWS Batch job and the time stamp of the updated image.

The sum of these components makes up the time-to-solution as shown in Fig. 6a and each component is furthermore plotted separately in Figs. 6b, 6c, and 6d. All timings are the arithmetic mean over 10 individual runs and error bars represent the 90 percent confidence interval. The container runtimes of Fig. 6c are the arithmetic mean of the individual container runtimes on each instance (varying from 1 to 128). The average container runtime is proportional to the cost of computing one individual gradient and is given by the container runtime times the price of the m4.4xlarge instance, which was $0.2748 per hour at the time of testing. No extra charges occur for AWS Batch itself, i.e., for scheduling and launching the batch job.

The timings indicate that the time-to-solution generally grows as the batch size, and therefore the number of containers per job, increases (Fig. 6a). A close up inspection of the individual components that make up the total time-to-solution shows that this is mostly due to the increase of the startup time, i.e., the average time it takes AWS Batch to schedule and launch the EC2 instances for each job (Fig. 6b). We found that AWS Batch does generally not start all instances of the array job at the same time, but instead in several stages, over the course of 1 to 3 minutes. The exact startup time depends on the batch size and therefore on the number of instances that need to be launched, but also on the availability of the instance within the AWS region. The combination of these factors leads to an increase of the average startup time for an increasing batch size, but also to a large variance of the startup time between individual runs. Users have no control over the startup time, but it is important to consider that no cost is incurred during this time period, as no EC2 instances are running while the individual containers remain in the queue.

The average container runtime, i.e., the average computation time of a single gradient, is fairly stable as the batch size increases (Fig. 6c). This observation is consistent with the fact that each container of an AWS Batch array job runs as an individual Docker container and is therefore independent of the batch size. The container runtime increases only slightly for larger batch sizes and we observe a larger variance in some of the container runtimes. This variance stems from the fact that users do not have exclusive access to the EC2 instances on which the containers are deployed. Specifically, our containers run on m4.4xlarge instances, which have 8 cores (16 virtual CPUs) and 64 GB of memory. In practice, AWS deploys these instances on larger physical nodes and multiple EC2 instances (of various users) can run on the same node. We hypothesize that a larger batch size increases the chance of containers being deployed to a compute node that runs at full capacity, thus slightly increasing the average container runtime, as user do not have exclusive access to the full network capacity or memory bandwidth. The average container runtime also represents a lower bound on the time-to-solution (per iteration) that can be achieved by running the example as a classic (non-event driven) program on a fixed cluster. In this case there is no overhead from requesting EC2 instances, but other overhead may still occur, depending on how the source parallelization and gradient summation are implemented.

Finally, we also observe an increase in the additional gradient reduction time, i.e., the interval between the S3 timestamps of the final computed gradient $g$ and the updated image $x$. The batch size corresponds to the number of gradients that have to be summed before the gradient can be used to update the image. The event-driven gradient...
reduction invokes the summation process as soon as the first gradients are written to S3, so most gradients are already summed by the time the final worker finishes its gradient computation. For the event-driven gradient summation, the variance of the startup and container runtime is therefore advantageous, as it allows the summation to happen asynchronously. However, in our example, the time interval between the first two gradients being written to S3 (thus invoking the gradient reduction) and the final gradient being computed does not appear to be large enough to complete the summation of all gradients. Specifically, we see a general increase in the reduction time, as well as widening of the confidence interval. This variance is due to a non-deterministic component of our event-based gradient summation, resulting from a limitation of AWS Lambda. While users can specify a maximum number of messages that Lambda functions read from an SQS queue, it is not possible to force Lambda to read a minimum amount of two messages, resulting in most Lambda functions reading only a single message (i.e., one object name) from the queue. Since we need at least two messages to sum the corresponding gradients, we return the message to the queue and wait for a Lambda invocation with more than one message. The user has no control over this process and sometimes it takes several attempts until a Lambda function with multiple messages is invoked. The likelihood of this happening increases with a growing batch size, since a larger number of gradients need to be summed, which explains the increase of the reduction time and variance in Fig. 6d.

Overall, the gradient summation and variable update finish within a few seconds after the last gradient is computed and the additional reduction time is small compared to the full time-to-solution. In our example, the startup time (Fig. 6b) takes up the majority of the time-to-solution (Fig. 6a), as it lies in the range of a few minutes and is in fact longer than the average container runtime of each worker (Fig. 6c). However, the startup time is independent of the runtime of the containers, so the ratio of the startup time to the container runtime improves as the workload per container increases. For 3D imaging workloads, whose solution times for 3D wave equations are orders of magnitude higher than for two dimensions, it is therefore to be expected that the ratio between startup and computation time will shift considerably towards the latter. Indeed, in a follow-up application of our workflow to a 3D seismic data set on Microsoft Azure, the average container runtime to compute a gradient was 120 minutes, thus shifting the startup to computation time ratio to 1 : 25 [63].

The cost of the batch job only depends on the container runtime and the batch size, but not on the startup time or reduction time. The cost for summing the gradients is given by the cumulative runtime of the Lambda functions, but is negligible compared to the EC2 cost for computing the gradients. This is illustrated in Fig. 7a, which shows a cost breakdown of running our workflow for 30 iterations of stochastic gradient descent with a batch size of 80, which corresponds to approximately two passes through the data set (a). The data misfit as a function of the iteration number. Due to the fact that the underlying linear system is inconsistent and we stop iterating after two epochs only, the misfit only decays by about 50 percent.

AWS Batch contribute by far the largest share. Using spot instances as opposed to on-demand instances for AWS Batch reduces the cost of our example by a factor of 2.5, but the prices of the remaining services are fixed. The seismic image after the final iteration number is shown in Fig. 5b. In this example, every gradient was computed by AWS Batch on a single instance and a fixed number of threads, but in the subsequent section we analyze the scaling of runtime and cost as a function of the number of cores and EC2 instances. Furthermore, we will analyze in a subsequent example how the cost of running the gradient computations with AWS Batch compares to performing those computations on a fixed cluster of EC2 instances.

4.2 Strong Scaling

In the following set of experiments, we analyze the strong scaling behavior of our workflow for an individual gradient calculation, i.e., a gradient for a batch size of 1. For this, we consider a single gradient computation using AWS Batch and measure the runtime as a function of either the number of threads or the number of instances in the context of MPI-based domain decomposition. In the first experiment, we evaluate the vertical scaling behavior, i.e., we run the gradient computation on a single instance and vary the number of OpenMP threads. In contrast to the weak scaling experiment, we model wave propagation in the full domain (1,911 × 10,789 grid points), to ensure that the sub-domain of each worker is not too small when we use maximum number of threads.

Since AWS Batch runs all jobs as Docker containers, we compare the runtimes with AWS Batch to running our application on a bare metal instance, in which case we have direct access to the compute node and run our code without any virtualization. All timings on AWS are performed on a r5.24xlarge EC2 instance, which is a memory optimized instance type that uses the Intel Xeon Platinum 8175M architecture (Appendix B, available in the online supplemental material). The 24xlarge instance has 96 virtual CPU cores (48 physical cores on 2 sockets) and 768 GB of memory. Using the largest possible instance of the r5 class, ensures that our AWS Batch job has exclusive access to the physical compute node, while bare metal instances automatically give users exclusive access. We also include the Optimum HPC
cluster in our comparison, a small research cluster at the University of British Columbia based on the Intel’s Ivy Bridge 2.8 GHz E5-2680v2 processor. Optimum has 2 CPUs per node and 10 cores per CPU. However, all OpenMP timings were conducted on single CPUs only, i.e., the maximum number of threads on each architecture corresponds to the maximum number of available cores per CPU (10 on Optimum and 24 on the r5.24xlarge instances).

Fig. 8a shows the comparison of the kernel runtimes on AWS and Optimum and Fig. 8b displays the corresponding parallel efficiency. As expected, the r5 bare metal instance shows the best scaling, as it uses a newer architecture than Optimum and does not suffer from the virtualization overhead of Docker. We noticed that AWS Batch in its default mode uses hyperthreading (HT), even if we perform thread pinning and instruct AWS Batch to use separate physical cores. As of now, the only way to prevent AWS Batch from performing HT, is to modify the Amazon Machine Image (AMI) of the corresponding AWS compute environment. With HT disabled, the runtimes and speedups of AWS Batch are very close to the timings on the bare-metal instances, indicating that the overhead of Docker affects the runtimes and scaling of our memory-intensive application only marginally, which matches the findings of [64].

Next, we analyze the horizontal strong scaling behavior of running our application with AWS Batch. Once again, we consider the computation of one single gradient, but this time we vary the number of EC2 instances. We would like to emphasize that AWS Batch is used differently than in the premise cluster. Figure (a) shows the kernel runtimes and the corresponding parallel efficiency as a function of the number of cores on a single socket. Figure (b) shows the corresponding parallel efficiency ranging from 1 to 16 instances for AWS Batch and Optimum and 2 on the r5.24xlarge instance.

Fig. 8 shows the comparison of the kernel runtimes on AWS and Optimum and Fig. 8b displays the corresponding parallel efficiency. As expected, the r5 bare metal instance shows the best scaling, as it uses a newer architecture than Optimum and does not suffer from the virtualization overhead of Docker. We noticed that AWS Batch in its default mode uses hyperthreading (HT), even if we perform thread pinning and instruct AWS Batch to use separate physical cores. As of now, the only way to prevent AWS Batch from performing HT, is to modify the Amazon Machine Image (AMI) of the corresponding AWS compute environment. With HT disabled, the runtimes and speedups of AWS Batch are very close to the timings on the bare-metal instances, indicating that the overhead of Docker affects the runtimes and scaling of our memory-intensive application only marginally, which matches the findings of [64].

Finally, we consider the computation of one single gradient, but this time we vary the number of EC2 instances. We would like to emphasize that AWS Batch is used differently than in the weak scaling experiment, where AWS Batch was used to parallelize the sum over source locations and communication between workers of a separate jobs was not possible. Here, we submit a single workload (i.e., one gradient) as a multi-node AWS Batch job, in which case IP-based communication between instances is enabled. Since this involves distributed memory parallelism, we use domain decomposition based on message passing (MPI) to solve the wave equations on multiple EC2 instances [65, 66]. The code with the corresponding MPI communication statements is automatically generated by the Devito compiler. Furthermore, we use multi-threading on each individual instance and utilize the maximum number of available cores per socket, which is 24 for the r5 instance and 18 for the c5n instance. To investigate the possible virtualization overhead of AWS Batch and Docker, we also conduct the same scaling experiments on clusters of EC2 bare metal instances, in which case our applications runs directly on the compute nodes without any form of virtualization.

We compare the r5.24xlarge and r5.24xlarge instances from the last section with Amazon’s recently introduced c5n HPC instance. Communication between AWS instances is generally based on ethernet and the r5 instances have up to 25 GBps networking performance. The c5n instance type uses Intel Xeon Platinum 8142M processors with up to 3.4 GHz architecture and according to AWS provides up to 100 GBps of network bandwidth. The network is based on proprietary AWS technology called elastic fabric adapter (EFA), but AWS has not disclosed whether this technology is based on InfiniBand or Ethernet [13]. Figs. 9a and 9b show the kernel runtimes and the corresponding parallel efficiency ranging from 1 to 16 instances for AWS Batch and bare metal instances. The r5 instance has overall shorter runtimes than the c5n instance, since the former has 24 physical cores per CPU socket, while the c5n instance has 18. However, as expected, the c5n instance exhibits a better parallel efficiency than the r5 instance (in both batch mode and on bare metal), due to the better network performance. Interestingly, speedups and parallel efficiency of multi-node AWS Batch jobs are better than of the corresponding bare metal jobs, which is counter intuitive. To investigate this further, we measured the latency between two manually requested bare metal instances and two compute nodes assigned by AWS Batch and found that the latter set of instances have less than half the amount of latency (Appendix C, available in the online supplemental material). All nodes
were requested in the same availability zone and EC2 placement group. This indicates that AWS possibly places instances for AWS Batch jobs closer to each other than manually requested EC2 instances, or that AWS Batch instances use a more efficient network gateway, but both of these explanations are purely speculative. Overall, the observed scaling and parallel efficiency of the AWS Batch jobs on both instance types are in the expected range of performance, as our application represents a strongly memory bound workload with an operational intensity of three FLOPs/Byte only, as shown in a roofline analysis of Devito’s generated code for acoustic modeling [38].

The timings given in Fig. 9a are once again the pure kernel times for solving the PDEs, but a breakdown of the components that make up the total time-to-solution on the c5n instance is provided in Fig. 9c. The job runtime is defined as the interval between the job creation time and the S3 time stamp of the computed gradient. The container runtime is the runtime of the Docker container on the master node and includes the time it takes AWS Batch to launch the remaining workers and to establish an ssh connection between all instances/containers, which was the only supported communication protocol for AWS Batch at the time [66]. Currently, AWS Batch requires this process to be managed by the user using a shell script that is run inside each container. The Python runtime in Fig. 9c is defined as the runtime of our application on the main node and includes IO, memory allocation and code generation time. Our timings in Fig. 9c show that the overhead from requesting instances and establishing a cluster, i.e., the difference between the Python and container runtime, is reasonable for a small number of instances, but grows significantly as the number instances is increased. Depending on the runtime of the application, the overhead thus takes up a significant amount of the time-to-solution, but for compute-heavy applications which run for one or multiple hours, this amount of overhead may still be acceptable. For 3D imaging applications that run for multiple hours, it is therefore to be expected that the ratio of application runtime to overhead will improve significantly.

Fig. 9d shows the cost for running our scaling test as a function of the cluster size. The cost is calculated as the instance price (per second) times the runtime of the container on the main node times the number of instances. The cost per gradient grows significantly with the number of instances, as the overhead from establishing an ssh connection to all workers increases with the cluster size. The communication overhead during domain decomposition adds an additional layer of overhead that further increases the cost for an increasing number of instances. This is an important consideration for HPC in the cloud, as the shortest time-to-solution does not necessarily correspond to the cheapest approach. Another important aspect is that AWS Batch multi-node jobs do not support spot instances [66]. Spot instances allow users to access unused EC2 capacities at significantly lower price than at the on-demand price, but AWS Batch multi-node jobs are, for the time being, only supported with on-demand instances.

The scaling and cost analysis in Figs. 9a, 9b, 9c and 9d was carried out on the largest instances of the respective instance types (r5.24xlarge and c5n.18xlarge) to guarantee exclusive access to the compute nodes and network bandwidth. Increasing the number of instances per run therefore not only increases the total number of available cores, but also the amount of memory. However, for computing a single gradient, the required amount of memory is fixed, so increasing the number of instances reduces the required amount of memory per instance, as wavefields are distributed among more workers. In practice, it therefore makes sense to chose the instance type based on the required amount of memory per worker, as memory is generally more expensive than compute. In our specific case, computing the gradient requires 170 GB of memory, which requires either a single r5.12xlarge instance or multiple smaller instances, which not only differ in the amount of memory, but also in the number of available CPU cores. We repeat our previous scaling test, but rather than using the same instance type in all runs, we choose the instance type based on the required amount of memory. Furthermore, for every instance type, we utilize the maximum amount of available cores using multi-threading with OpenMP. The kernel runtimes for an increasing number of instances is shown in Fig. 10a. In each bar, we indicate which instance type was used, as well as the total number of cores across all instances. The corresponding costs for computing each gradient is shown in Fig. 10a. Compared to the previous example, we observe that using 16 small on-demand instances leads to a lower cost than using a single more expensive large instance, but that using a single instance ultimately remains the most cost-effective way of computing a gradient, due to the possibility to utilize spot instances. An additional pricing model offered by AWS are reserved instances, which are EC2 instances that can be purchased at rates similar to spot prices if reserved for a minimum period of one year [67]. However, this pricing model is unsuitable for our event-driven approach, as users cannot save costs by minimizing idle time, since instances are paid for in advance.

In terms of cost, our scaling examples underline the importance of choosing the EC2 instances for the AWS Batch jobs based on the total amount of required memory, rather than based on the amount of CPU cores. Scaling horizontally by using an increasingly large number of instances
expectedly leads to a faster time-to-solution, but results in a significant increase of cost as well (Fig. 9d). As shown in Fig. 10b, this increase in cost can be avoided to some extent by choosing the instance size such that the total amount of memory stays approximately constant, but ultimately the restriction of not supporting spot instances makes multi-node batch jobs not attractive in scenarios where single instances provide sufficient memory to run a given application. In practice, it makes therefore sense to use single node/instance batch jobs and to utilize the full number of available cores on each instance. The largest EC2 instances of each type (e.g., r5.24xlarge, c5n.18xlarge) have two CPU sockets with shared memory, making it possible to run parallel programs using either pure multi-threading or a hybrid MPI-OpenMP approach. In the latter case, programs still run as a single Docker container, but within each container use MPI for communication between CPU sockets, while OpenMP is used for multithreading on each CPU. For our example, we found that computing a single gradient of the BP model with the hybrid MPI-OpenMP approach leads to a 20 percent speedup over the pure OpenMP program (Table 1), which correspondingly leads to 20 percent cost savings as well.

### 4.3 Cost Comparison

One of the most important considerations of high performance computing in the cloud is the aspect of cost. As users are billed for running EC2 instances by the second, it is important to use instances efficiently and to avoid idle resources. In our specific application, gradients for different seismic source locations are computed by a pool of parallel workers, but as discussed earlier, computations do not necessarily complete at the same time. On a conventional cluster, programs with a MapReduce structure, are implemented based on a client-server model, in which the workers compute the gradients in parallel, while the master (the server) collects and sums the results. This means that the process has to wait until all gradients \( g_i \) have been computed, before the gradient can be summed and used to update the image. This inevitably causes workers that finish their computations earlier than others to sit idle. This is problematic when using a cluster of EC2 instances, where the number of instances are fixed, as users have to pay for idle resources. In contrast, the event-driven approach based on Lambda functions and AWS Batch automatically terminates EC2 instances of workers that have completed their gradient calculation, thus preventing resources from sitting idle.

We illustrate the difference between the event-driven approach and using a fixed cluster of EC2 instances by means of a specific example. We consider our previous example of the BP synthetic model and compute the gradient \( g_i \) for 100 random source locations and record the runtimes (Fig. 11a). We note that most gradients take around 250 seconds to compute, but that the runtimes vary due to different domain sizes and varying EC2 capacity. We now model the idle time for computing these gradients on a cluster of EC2 instances as a function of the the number of parallel instances, ranging from 1 instance (fully serial) to 100 instances (fully parallel). For a cluster consisting of a single instance, the cumulative idle time is naturally zero, as the full workload is executed in serial. For more than one instance, we model the amount of time that each instance is utilized, assuming that the workloads are assigned on a first-come-first-served basis. The cumulative idle time \( t_{\text{idle}} \) is then given as the sum of the differences between the runtime of each individual instance \( t_i \) and the instance with the longest runtime:

\[
t_{\text{idle}} = \sum_{i=1}^{n_{\text{EC2}}} \left( \max\{t_i\} - t_i \right).
\]

The cumulative idle time as a function of the cluster size \( n_{\text{EC2}} \) is plotted in Fig. 11b. We note that the cumulative idle time generally increases with the cluster size, as a larger number of instances sit idle while waiting for the final gradient to be computed. On a cluster with 100 instances each gradient is computed by a separate instance, but all workers have to wait until the last worker finishes its computation (after approximately 387 seconds). In this case, the varying time-to-solutions of the individual gradients leads to a cumulative idle time of 248 minutes. Compared to the cumulative computation time of all gradients, which is 397 minutes, this introduces an overhead of more than 60 percent, if the gradients are computed on a cluster with
100 instances. The cumulative idle time is directly proportional to the cost for computing the 100 gradients, which is plotted on the right axis of Fig. 11b. With AWS Batch, the cumulative idle time for computing the 100 gradients is zero, regardless of the number of parallel instances that AWS Batch has access to. Any EC2 instance that is not utilized anymore is automatically shut down by AWS Batch, so no additional cost other than the pure computation time of the gradients is invoked [63]. A follow up case study of this work, found that the ratio of cumulative idle time to cumulative compute time (about 60 percent) also held true for a 3D seismic imaging example, in which the average container runtime was substantially longer than in the 2D example (namely 120 minutes) [63].

While computing the 100 gradients on an EC2 cluster with a small number of instances results in little cumulative idle time, it increases the overall time-to-solution, as a larger number of gradients have to be sequentially computed on each instance (Fig. 11c). With AWS Batch this trade-off does not exist, as the cumulative idle time, and therefore the cost for computing a fixed workload, does not depend on the number of instances. However, it is to be expected that in practice the time-to-solution is somewhat larger for AWS Batch than for a fixed cluster of EC2 instances, as AWS Batch needs to request and launch EC2 instances for every new gradient computation.

4.4 Resilience

In the final experiment of our performance analysis, we analyze the resilience of our workflow and draw a comparison to running an MPI program on a conventional cluster of EC2 instances. Resilience is an important factor in high performance computing, especially for applications like seismic imaging, whose runtime can range from several hours to multiple days. In the cloud, the mean-time-between failures is typically much shorter than on comparable HPC systems [6], making resilience potentially a prohibiting factor. Furthermore, using spot instances further increases the exposure to instance shut downs, as spot instances can be terminated at any point in time with a two minute warning.

Seismic imaging codes that run on conventional HPC clusters typically use MPI to parallelize the sum of the source indices and thus often leverage the User Level Fault Mitigation (ULFM) Standard [69] or the MVAPICH2 implementation of MPI, which includes built-in resilience [70]. Both MVAPICH2 and ULFM enable the continued execution of an MPI program after node/instance failures. In our event-driven approach, resilience in the cloud is naturally provided by using AWS Batch for the gradient computations, as each gradient is computed by a separate container. In case of exceptions, AWS Batch provides the possibility to automatically restart EC2 instances that have crashed, which allows the completion of programs with the initial number of nodes or EC2 instances.

We illustrate the effect of instance restarts by means of our previous example with the BP model (Fig. 5). Once again, we compute the gradient of the LS-RTM objective function for a batch size of 100 and record the runtimes without any instance/node failures. In addition to the default strategy for backpropagation, we compute the gradients using optimal checkpointing, in which case the average runtime per gradient increases from 5 minutes to 45 minutes, as a smaller memory footprint is traded for additional computations.

We then model the time that it takes to compute the 100 gradients for an increasing number of instance failures with and without restarts. We assume that the gradients are computed fully in parallel, i.e., on 100 parallel instances and invoke an increasing number of instance failures at randomly chosen times during the execution of program. Without instance restarts, we assign the workload of the failed instances to the workers of the remaining instances and model how long it takes complete the computations. With restarts, we add a two minute penalty to the failed worker and then restart the computation on the same instance. The two minute penalty represents the average amount of time it takes AWS Batch to restart a terminated EC2 instance and was determined experimentally.

Fig. 12 shows the ratio of the time-to-solution for computing the 100 gradients without events (i.e., without failures) to the modeled time-to-solution with events. This ratio is known as the resilience factor [71] and provides a metric of how instance failures affect the time-to-solution and therefore the cost of running a given application in the cloud:

$$r = \frac{\text{time-to-solution}_{\text{event-free}}}{\text{time-to-solution}_{\text{event}}}.$$  

Ideally, we aim for this factor being as close to 1 as possible, meaning that instance failures do not significantly increase the time-to-solution. Figs. 12a and 12b compare the resilience factors with and without restarts for the two different backpropagation strategies, which represent programs of different runtimes. The resilience factor is plotted as a function of the percentage of instance failures and is the average of 10 realizations, with the standard deviation being depicted by the shaded colors. The plots show that the largest benefit from being able to restart instances with AWS Batch is achieved for long running applications (Fig. 12b). The resilience factor with instance restarts approaches a value of 0.5, since in the worst case, the time-to-solution is doubled if an instance fails shortly before completing its gradient computation. Without being able to restart instances, the gradient computations need to be completed by the remaining workers, so the resilience factor continuously
decreases as the failure percentage increases. For short running applications (Fig. 12a), the overhead of restarting instances diminishes the advantage of instance restarts, unless a significant percentage of instances fail, which, however, is unlikely for programs that run in a matter of minutes. In fact, during our numerical experiments, we did not observe any spot-related shut-downs, as the runtime of our application was only in the order of five minutes. On the other hand, long running programs or applications with a large number of workers are much more likely to encounter instance shut downs and our experiment shows that these programs benefit from the automatic instance restarts of AWS Batch.

5 Discussion
The main advantage of an event-driven approach based on AWS Batch and Lambda functions for seismic imaging in the cloud is the automated management of computational resources by AWS. EC2 instances that are used for carrying out heavy computations, namely for solving large-scale wave equations, are started automatically in response to events, which in our case are Step Functions advancing the serverless workflow to the ComputeGradients state. Expensive EC2 instances are thus only active for the duration it takes to compute one element $g$, of the full or mini-batch gradient and they are automatically terminated afterwards. Summing the gradients and updating the variables (i.e., the seismic image) is performed on cheaper Lambda functions, with billing being again solely based on the execution time of the respective code and the utilized memory. The cost overhead introduced by Step Functions, SQS messages and AWS Batch is negligible compared to the cost of the EC2 instances that are required for the gradient computations, while cost savings from spot instances and eliminating idle EC2 instances lead to significant cost savings, as shown in our examples. With the benefits of spot instances (factor 2-3), avoidance of idle instances and the overhead of spinning clusters (factor 1.5-2), as well as improved resilience, we estimate that our event-driven workflow provides cost savings of up to an order of magnitude in comparison to using fixed clusters of (on-demand) EC2 instances.

A second alternative to running cloud applications on fixed EC2 clusters are in-between approaches based on a combination of EC2 instances, task-based workflow tools and auto-scaling. These approaches potentially benefit from the possibility to avoid containerization by using bare metal instances, while leveraging automatic up- and down-scaling of EC2 instances to save cost. However, workflow tools like AWS Batch or Step Functions are currently not available for EC2 clusters and thus need to be replicated by the user. Furthermore, any cluster-based workflows, even with auto-scaling, require at least a single EC2 instance to be permanently running, while our serverless approach makes it hypothetically possible to suspend tasks or workloads indefinitely without incurring any cost at all. Regarding performance, our analysis showed that Docker containerization did not lead to considerable performance impairments, while latency between EC2 instances assigned by AWS Batch was in fact smaller than latency between user-requested instances. Using batch processing to compute an embarrassingly parallel workload is not only advantageous in the cloud, but also on on-premise HPC systems, as parallel jobs that are broken into multiple smaller and shorter jobs are oftentimes processed faster by HPC schedulers than single large workloads. In addition to the improved flexibility regarding nested parallelization, this makes task-based asynchronous batch processing interesting in the setting of traditional HPC as well.

Another major advantage of our proposed serverless approach is the handling of resilience. Instead of running as a single program, our workflow is broken up into its individual components and expressed through Step Function states. Parallel programs based on MPI rely on not being interrupted by hardware failures during the entire runtime of the code, making this approach susceptible to resilience issues. Breaking a seismic imaging workflow into its individual components, with each component running as an individual (sub-) program and AWS managing their interactions, makes the event-driven approach inherently more resilient, as the runtime of individual workflow components is much shorter than the runtime of the full program, thus minimizing the exposure to instance failures. Computing an embarrassingly parallel workload with AWS Batch, rather than as a MPI-program, provides a natural layer of resilience, as AWS Batch processes each item from its queue separately on an individual EC2 instance and Docker container, but also includes the possibility of automatic instance restarts in the event of failures.

The most prominent disadvantage of the event-driven workflow is that EC2 instances have to be restarted by AWS Batch in every iteration of the workflow. In our performance analysis, we found that the overhead of requesting EC2 instances and starting the Docker container lies in the range of several minutes and depends on how many instances are requested per gradient. However, items that remain momentarily in the batch queue, do not incur any cost until the respective EC2 instance is launched. The overhead introduced by AWS Batch therefore only increases the time-to-solution, but does not affect the cost negatively. Due to the overhead of starting EC2 instances for individual computations, our proposed workflow is therefore applicable if the respective computations (e.g., computing gradients) are both embarrassingly parallel and take a long time to compute; ideally in the range of hours rather than minutes. We therefore expect that the advantages of our workflow will be even more prominent when applied to 3D seismic data sets, where computations are orders of magnitude more expensive than in 2D.

Our application, as expressed through AWS Step Functions, represents the structure of a generic gradient-based optimization algorithm and is therefore applicable to problems other than seismic imaging and full-waveform inversion. The design of our workflow lends itself to problems that exhibit a certain MapReduce structure, namely they consists of a computationally expensive, but embarrassingly parallel Map part, as well as a computationally cheaper to compute Reduce part. On the other hand, applications that rely on dense communications between workers or where the quantities of interest such as gradients or functions values are cheap to compute, are less suitable for this specific design. For example, deep convolutional neural networks
TABLE 2
An Overview how the AWS Services Used in our Workflow Map to Other Cloud Providers

<table>
<thead>
<tr>
<th>Amazon Web Services</th>
<th>Microsoft Azure</th>
<th>Google Cloud</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Compute Cloud</td>
<td>Virtual Machines</td>
<td>Compute Engine</td>
</tr>
<tr>
<td>Simple Storage System</td>
<td>Blob storage</td>
<td>Cloud Storage</td>
</tr>
<tr>
<td>AWS Batch</td>
<td>Azure Batch</td>
<td>Pipelines</td>
</tr>
<tr>
<td>Lambda Functions</td>
<td>Azure Functions</td>
<td>Cloud Functions</td>
</tr>
<tr>
<td>Step Functions</td>
<td>Logic Apps</td>
<td>N/A</td>
</tr>
<tr>
<td>Simple Message Queue</td>
<td>Queue Storage</td>
<td>Cloud Pub/Sub</td>
</tr>
<tr>
<td>Elastic File System</td>
<td>Azure Files</td>
<td>Cloud Filestore</td>
</tr>
</tbody>
</table>

(CNNs) exhibit mathematically a very similar problem structure to seismic inverse problems, but forward and backward evaluations of CNNs are typically much faster than solving forward and adjoint wave equations, even if we consider very deep networks like ResNet [72]. Implementing training algorithms for CNNs as an event-driven workflow as presented here, is therefore excessive for the problem sizes that are currently encountered in deep learning, but might be justified in the future if the dimensionality of neural networks continues to grow.

The event-driven workflow presented in this work was specifically designed for AWS and takes advantage of specialized services for batch computing or event-driven computations that are available on this platform. However, in principle, it is possible to implement our workflow on other cloud platforms as well, as almost all of the utilized services have equivalent versions on Microsoft Azure or the Google Cloud Platform (Table 2) [73], [74]. Services for running parallel containerized workloads in the cloud, as well as event-driven cloud functions, which are the two main components of our workflow, are available on all platforms considered in our comparison. Furthermore, both Microsoft Azure as well as the GCP offer similar Python APIs as AWS for interfacing cloud services. We also speculate that, as cloud technology matures, services between different providers will likely grow more similar to each other. This is based on the presumption that less advanced cloud platforms will imitate services offered by major cloud providers in order to be competitive in the growing cloud market.

Overall, our workflow and performance evaluation demonstrate that cost-competitive HPC in the cloud is possible, but requires a fundamental software re-design of the corresponding application. In our case, the implementation of an event-driven seismic imaging workflow was possible, as we leverage Devito for expressing and solving the underlying wave equations, which accounts for the major workload of seismic imaging. With Devito, we are able to abstract the otherwise complex implementation and performance optimization of wave equation solvers and take advantage of recent advances in automatic code generation. As Devito generates code for solving single PDEs, with the possibility of using MPI-based domain decomposition, we are not only able to leverage AWS Batch for the parallelization over source experiments, but can also take advantage of AWS Batch’s multi-node functionality to shift from data to model parallelism. In contrast, many seismic imaging codes are software monoliths, in which PDE solvers are enmeshed with IO routines, parallelization and manual performance optimization. Adapting codes of this form to the cloud is fundamentally more challenging, as it is not easily possible to isolate individual components such as a PDE solver for a single source location, while replacing the parallelization with cloud services. This illustrates that separation of concerns and abstract user interfaces are a prerequisite for porting HPC codes to the cloud such that the codes are able to take advantage of new technologies like object storage and event-driven computations. With a domain-specific language compiler, automatic code generation, high-throughput batch computing and serverless visual algorithm definitions, our workflow represents a true vertical integration of modern programming paradigms into a framework for HPC in the cloud.

6 CONCLUSION

Porting HPC applications to the cloud using a lift and shift approach based on virtual clusters that emulate on-premise HPC clusters, is problematic as the cloud cannot offer the same performance and reliability as conventional clusters. Applications such as seismic imaging that are computationally expensive and run for a long time, are faced with practical challenges such as cost and resilience issues, which prohibit the cloud from being widely adapted for HPC tasks. However, the cloud offers a range of new technologies such as object storage or event-driven computations, that allow to address computational challenges in HPC in novel ways. In this work, we demonstrate how to adapt these technologies to implement a workflow for seismic imaging in the cloud that does not rely on a conventional cluster, but is instead based on serverless and event-driven computations. These tools are not only necessary to make HPC in the cloud financially viable, but also to improve the resilience of workflows. The code of our application is fully redesigned and uses a variety of AWS services as building blocks for the new workflow, thus taking advantage of AWS being responsible for resilience, job scheduling, and resource allocations. Our performance analysis shows that the resulting workflow exhibits competitive performance and scalability, but most importantly minimizes idle time on EC2 instances and cost and is inherently resilient. Our example therefore demonstrates that successfully porting HPC applications to the cloud is possible, but requires to carefully adapt the corresponding codes to the new environment. This process is heavily dependent on the specific application and involves identifying properties of the underlying scientific problem that can be exploited by new technologies available in the cloud. Most importantly, it requires that codes are modular and designed based on the principle of separation of concerns, thus making this transition possible.

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1 INTRODUCTION

With increasing uptake and use of multimedia technologies, there is an associated increase in data being generated and transmitted (processed) over our network-based systems, often to multiple subscribers. Applications can include video-on-demand, high definition streaming, multimedia social networks (combing text, audio and video) and Internet-of-Things (IoTs). This paradigm of data transfer to mobile edge clouds, to offer low latency network services at the network edges. Most conventional network services are implemented via hardware-based network functions, including firewalls and load balancers, to guarantee service security and performance. However, implementing hardware-based network functions usually incurs both a high capital expenditure (CAPEX) and operating expenditure (OPEX). Network Function Virtualization (NFV) exhibits a potential to reduce CAPEX and OPEX significantly, by deploying software-based network functions in virtual machines (VMs) on edge-clouds. We consider a fundamental problem of NFV-enabled multicasting in a mobile edge cloud, where each multicast request has both service function chain and end-to-end delay requirements. Specifically, each multicast request requires chaining of a sequence of network functions (referred to as a service function chain) from a source to a set of destinations within specified end-to-end delay requirements. We devise an approximation algorithm with a provable approximation ratio for a single multicast request admission if its delay requirement is negligible; otherwise, we propose an efficient heuristic. Furthermore, we also consider admissions of a given set of the delay-aware NFV-enabled multicast requests, for which we devise an efficient heuristic such that the system throughput is maximized, while the implementation cost of admitted requests is minimized. We finally evaluate the performance of the proposed algorithms in a real test-bed, and experimental results show that our algorithms outperform other similar approaches reported in literature.
resource utilization in MEC networks and reduce service cost. It is however challenging to efficiently utilize existing VNF instances or create new VNF instances to maximize the number of multicast requests and minimize overall cost – subject to the computing capacity constraint on each cloudlet in the MEC network and the end-to-end delay requirement of each admitted multicast request. The key challenge is to identify which cloudlets should be used to host VNFs required within a multicast request service chain, i.e., which existing VNF instances can be used for which request? Second, each NFV-enabled multicast request usually has a QoS requirement to guarantee that its traffic reaches the destinations within the specified end-to-end delay requirement. Identifying how to meet the end-to-end delay requirement of each admitted NFV-enabled multicast request is challenging. In this paper, we tackle the aforementioned challenges, by investigating efficient methods that investigate VNF sharing, service chaining, and routing that can meet QoS requirements of NFV-enabled multicast requests in an MEC network.

There are extensive studies on multicasting in conventional networks or software-defined networks, which do not consider service function chain requirements [17], [18], [51]. These solutions however cannot be directly applied to NFV-enabled multicasting. There are also recent investigations on NFV-enabled multicasting. However, these approaches do not consider end-to-end delay requirements [39], and they assume that only one service instance is included in the service function chain [51], or that the VNFs in each service chain are consolidated into a single location [45], [47]. For example, Zhang et al. [51] investigated the NFV-enabled multicast problem by assuming that there are sufficient computing and bandwidth resources in a Software Defined Network (SDN) to accommodate a multicast request. Xu et al. [47] investigated the problem of NFV-enabled multicasting, by devising an approximation algorithm with a provable approximation ratio for realizing a single NFV-enabled multicast request and an online algorithm with a guaranteed competitive ratio for the online NFV-enabled multicasting problem. Ren et al. [39] investigated the NFV-enabled multicasting in an SDN, by assuming that the traffic of each multicast request can be processed by multiple instances of the VNFs in its service chain. These methods are likely to increase the cost/delay of implementing such multicast requests, as placing VNFs into multiple cloudlets can lead to a greater delay to form a service function chain and incur a higher cost.

To the best of our knowledge, we are the first to consider the problem of delay-sensitive NFV-enabled multicasting problem in an MEC network, by designing both approximation algorithms and efficient heuristics. The main contributions of this paper are as follows.

- We study the NFV-enabled multicasting problem in an MEC network, with an aim to minimize the implementation cost of the request while meeting its delay requirement.
- We propose an efficient heuristic for the NFV-enabled multicasting problem. We also devise the very first approximation algorithm with an approximation ratio, if the delay requirement is neglected.
- We also consider a set of NFV-enabled multicast request admissions with the aim to maximize the weighted system throughput. We also propose a heuristic for this problem.
- We evaluate the performance of the proposed algorithms through experimental simulations in synthetic networks and within a real test-bed. Experimental results demonstrate that the proposed algorithms outperform existing reported approaches.

The rest of the paper is organized as follows. Section 2 reviews related work. Section 3 introduces the system model, notations, and problem definition. Section 4 devises an approximation algorithm for the NFV-enabled multicasting problem without end-to-end delay requirements, and proposes an efficient heuristic for the problem with delay requirements using the proposed approximation algorithm as a subroutine. Section 5 devises an efficient heuristic algorithm for the the NFV-enabled multicasting problem with resource constraints on cloudlets. Section 6 evaluates the performance of the proposed algorithms experimentally in a real test-bed, and Section 7 concludes the paper.

2 Related Work

Recently, traffic steering has re-gained much attention due to the challenges introduced by software-defined networking and network function virtualization [5], [6], [15], [16], [18], [23], [27], [33], [46], [47], [50]. Unicasting is one of the primary focus of existing studies. For example, Moens et al. [33] focused on hybrid networks with both hardware and software network functions. Cziva et al. [7] addressed the problem of the placement of virtual functions by minimizing the total number of VNF instances. Yu et al. [29] investigated profit maximization associated with placing VNFs onto a set of locations, and considered the delay requirement of each unicast request. Xu et al. [45] studied the offloading problem of delay-sensitive tasks with network function requirements in a MEC network, by proposing efficient heuristics and an online algorithm with a competitive ratio. Xie et al. [44] investigated the VNF sharing problem with an aim to improve resource utilization, by finding a common link for a set of service chains, so that the deployed service chains can be shared by all users. Kiji et al. [19] proposed a virtual network function placement and routing algorithm for multicast requests with service chain requests, through merging multiple service paths (MSC-M). Although there exist studies that consider the delay requirements of user requests [22], [29], [45], they only considered unicast requests and their solutions cannot be applied to the NFV-enabled multicasting problem, which is a generalization of the NFV-enabled unicasting problem. Chen and Wu [5] devised algorithms for the VNF placement to minimize the cost of implementing NFV-enabled unicast requests by balancing set-up and bandwidth consumption costs.

There are studies on multicasting in conventional networks [2], [14], [24], [25], [34], [43]. Recently, with the emergence of new networking technologies such as mobile edge computing, software-defined networking (SDN) and NFV, multicasting has re-gained the attention by the research community [17], [18]. For example, Huang et al. [18] studied online multicasting in software-defined networks with both node and link capacity constraints. Huang et al. [17] studied the scalability problem of multicasting in SDNs, by proposing
an efficient algorithm to find a branch-aware Steiner Tree for each multicast request. These solutions however cannot be directly applied to the problem of NFV-enabled multicasting in MEC networks, because they did not consider the service chain requirements of multicast requests.

Investigations on NFV-enabled multicasting include [1], [30], [31], [39], [41], [47], [49], [51]. For instance, Zhang et al. [51] investigated the NFV-enabled multicasting problem in an SDN without resource capacity constraints, assuming that data traffic of each multicast request can only be processed by one server. Xu et al. [47], [48] considered the NFV multicasting problem by assuming the traffic of each request can be processed by multiple servers, with the objective to minimize the implementation cost. Approximation and online algorithms for the problems are proposed. They however assumed that the VNFs in each service chain is consolidated into a single data center. Ma et al. [30], [31] proposed an online algorithm for the NFV-enabled multicasting problem without taking into account the end-to-end delay requirement. Soni et al. [41] proposed a scalable multicast group management scheme and a load balancing method for the routing of best-effort traffic and bandwidth-guaranteed traffic. These studies however did not consider end-to-end delay requirements of multicast requests. Alhussein et al. [1] devised exact solutions for the problem of joint VNF placement and routing for multicast requests in 5G core networks, such that the cost of provisioning NFV-enabled multicast services is minimized, by formulating the problem into a mixed integer linear program (MILP). The delay requirement of NFV-enabled requests has not been considered and the MILP-based exact solutions might not be scalable for large problem sizes. Yi et al. [49] considered delay requirements of the NFV-enabled multicasting problem; however VNF sharing is not explored. To guarantee scalability and solution quality, Ren et al. [39] proposed approximation algorithm with an approximation ratio for the problem of embedding a service graph that consists of VNF instances into a substrate network, by assuming that the traffic of each multicast request can be processed by multiple instances of the VNFs in its service chain. The delay requirement of multicast requests however is not considered in the study. Similarly, the delay requirement of multicast requests is not considered [30], [31], and the authors only consider a single multicast request.

3 PRELIMINARIES

In this section, we first introduce the system model, notation and key concepts. We then define the problem being considered more precisely.

3.1 System Model

We consider a mobile edge cloud (MEC) network \( G = (V, E) \) with a set \( V \) of switches, a set of cloudlets and a set \( E \) of links between switches and cloudlets. Each cloudlet is attached to a switch in \( V \) via an optical fiber, and the communication delay between a switch and its attached cloudlet is negligible. Let \( \mathcal{V}_C \) be the set of switches with attached cloudlets. Clearly, \( \mathcal{V}_C \subseteq V \). Cloudlets are usually deployed in shopping malls, airports, or base stations that are within the proximity of mobile users. Due to space limitation of installing cooling equipment in those places, each cloudlet is usually equipped with (a small number of) servers and thus has computing resource capacity to implement VNF instances. We denote by \( C_{\text{unit}} \), the computing capacity of the cloudlet attached to switch node \( v \in \mathcal{V}_C \). In addition, transferring data throughput links in \( E \) incurs a communication latency. Let \( d_e \) be the delay associated with transmitting a unit of data traffic via link \( e \in E \). We assume that there is an SDN controller that both makes traffic steering decisions and manages network function instances that run on a server in the MEC network \( G \). Fig. 1 is an illustrative example of an MEC network.

![Fig. 1. An MEC network G.](image)

3.2 NFV-Enabled Multicast Requests and Service Chains

A delay-aware NFV-enabled multicast request is a request that transfers an amount of data traffic from a source to a set of destinations. The data traffic must be processed by a sequence of VNFs before reaching their destinations, while also meeting delay constraints.

Let \( r_k \) be a delay-aware NFV-enabled multicast request, denoted by a quadruple \( r_k = (s_k, D_k; b_k, SC_k) \), where \( s_k \in V \) is the source, \( D_k \) is the set of destinations with \( D_k \subseteq V \), \( b_k \) is the size of its data traffic, and \( SC_k \) is the service chain of \( r_k \) that consists of a sequence of VNFs. Without loss of generality, we consider that the data traffic \( b_k \) of request \( r_k \) is given (derived from historical information).

Let \( \mathcal{F} \) be the set of VNFs provided by the network service provider in \( G \). A VNF \( f_i \in \mathcal{F} \) can be needed by request \( r_k \) to form its service function chain \( SC_k \). Assume that there are \( L_k \) VNFs in \( SC_k \), where \( 1 \leq l \leq L_k \) for each \( SC_k \) and \( SC_k \subseteq \mathcal{F} \). We further assume that there is a number of already instantiated VNF instances for each type of network function \( f_i \) in cloudlets of \( G \). Due to the resource capacity constraints on cloudlets, we allow the instances of VNF \( f_i \) to be shared among different requests.

To admit request \( r_k \), all data traffic from source \( s_k \) of \( r_k \) needs to be processed through an instance of each VNF \( f_i \in SC_k \) prior to reaching destinations in \( D_k \), as illustrated in Fig. 2. An existing instance must therefore be selected for each VNF \( f_i \in SC_k \), or a new instance of \( f_i \) must be instantiated in a cloudlet of \( G \). Existing or newly created VNF instances of each service chain \( SC_k \) can be placed in multiple cloudlets, because a single cloudlet may not have all the instances of the VNFs in \( SC_k \), or it may lack sufficient computing resources to create new instances for all VNFs in \( SC_k \).

Each multicast request needs a certain amount of computing resource to process its data traffic. Let \( C_{\text{unit}}(f_i) \) be
the number of computing resource needed to process a unit amount of data traffic. If $f_i$ is implemented as a newly created instance, the total number of computing resources that should be assigned to the new instance to process the traffic of request $r_k$ is $C_{\text{unit}}(f_i) \cdot b_k$. Otherwise, an existing instance of $f_i$ should have at least an amount $C_{\text{unit}}(f_i) \cdot b_k$ of available computing resource to process the traffic of $r_k$. Notice that we assume that the accumulative available resources in the cloudlets of $G$ are higher then the total resource demand of a single request $r_k$; however, for a specific cloudlet in $V_{CL}$, it may not have enough resources to meet the demand of $r_k$.

### 3.3 Delay Requirements of Multicast Requests

The end-to-end delay of implementing a multicast request plays a vital role in guaranteeing the quality of services of users. We thus consider that each multicast request has a delay requirement, which specifies the maximum delay it can tolerate for transmitting its data from its specified source to its destinations. For a delay-aware NFV-enabled multicast request, its experienced delay consists of the total processing delay in the selected cloudlets and the total transfer delay from the source to cloudlets and from the cloudlets to the destinations, which are defined in the following.

**Processing Delay.** The processing delay experienced by a multicast request $r_k$ depends on both the amount of data traffic that needs to be processed and the computing resource assigned to process the traffic. Without loss of generality, we assume that the processing delay $d_{k,l}^p$ of each multicast request $r_k$ by VNF $f_l$ is proportional to the amount of traffic it needs to process, i.e.,

$$d_{k,l}^p = \alpha_l \cdot b_k,$$

where $\alpha_l$ is a given proportional factor of VNF $f_l$.

The accumulative processing delay incurred due to the traffic processing by network functions in $SC_k$ of $r_k$ is

$$d_k^p = \sum_{l \in SC_k} d_{k,l}^p.$$

**Transmission Delay.** Let $P_k$ be the set of routing paths from source $s_k$ to destinations in $D_k$, where each path $p_m \in P_k$ denotes a routing path from $s_k$ to a destination $t_m \in D_k$. The transmission delay of each $r_k$ is the maximum end-to-end delay incurred in the paths in $P_k$. We denote by $d_k^t$ the transmission delay of request $r_k$, which can be defined as follows:

$$d_k^t = \max_{p_m \in P_k} \sum_{e \in p_m} d_e \cdot b_k.$$

The delay experienced by multicast request $r_k$ thus is

$$d_k = d_k^p + d_k^t,$$

which needs no greater than the specified delay requirement $D_k$, i.e.,

$$d_k \leq D_k.$$

### 3.4 Cost Models

As the network service provider of an MEC network $G$ charges user requests on a pay-as-you-go basis, the major concern of the service provider is its operational cost, which consists of computing resource usage costs in cloudlets, bandwidth resource usage costs in links, and VNF instance instantiation costs. Let $c(e)$ and $c(v)$ be the usage costs of one unit of bandwidth and computing resources at link $e \in E$ and cloudlet $v \in V_{CL}$, respectively. Denote by $c_l(v)$ the cost of instantiating an instance of network function $f_l$ in cloudlet $v \in V_{CL}$, and let $n_{l,v}'$ be the number of newly created instances for network function $f_l$ in cloudlet $v$. Denote by $n_{l,v}$ the number of existing instances of $f_l$ in $v$ that are used to process the traffic of $r_k$.

The operational cost of admitting a delay-aware NFV-enabled multicast request $r_k$ can be specified as

$$c_k = \sum_{f_l \in SC_k} \sum_{v \in V_{CL}, r_k} \left( (n_{l,v} + n_{l,v}' \cdot c_l(v) \cdot b_k + n_{l,v}', c_l(v)) + \sum_{e \in T_k} c(e) \cdot b_k, \right)$$

where $V_{CL,r_k}$ is the set of cloudlets that are used to implement the instances of VNFs in $SC_k$ of request $r_k$, and $T_k$ is the obtained multicast tree that is used to route the data traffic of $r_k$.

### 3.5 The Directed Steiner Tree [4]

The Steiner tree problem is defined as follows: given a graph $G = (V, E)$ with a cost function $c$ on the edges, and a subset of terminals $X \subset V$, the goal is to find a minimum cost tree that includes all the terminals in $X$. The found minimum cost tree is referred to as the Steiner tree.

### 3.6 Problem Definition

We consider a mobile edge cloud (MEC) network $G = (V, E)$ with a set $V_{CL}$ of cloudlets with $V_{CL} \subset V$, and a set of multicast requests $R$. Given a snapshot of the MEC at a given time instant and a NFV-enabled multicast request $r_k$, understanding how request $r_k$ can be realised across a set of VNFs remains a key challenge. We thus first consider the problem of admitting a single multicast request $r_k$, such that its operational cost is minimized. Further, considering that the accumulated computing resources in an MEC may be insufficient to implement all requests, another question is identifying how to carry out admission control for multicast requests to maximize weighted throughput. In the following we define these two optimization problems precisely.

**Problem 1.** Assuming that each multicast request can be implemented using the computing resources assigned to existing VNF instances, the NFV-enabled multicasting problem with a single multicast request in MEC network $G$ is to...
TABLE 1
Symbols

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G = (V, E)$</td>
<td>a mobile edge cloud (MEC) network with a set $V$ of switches and a set $E$ of links</td>
</tr>
<tr>
<td>$R$</td>
<td>a set of delay-aware NFV-enabled multicast requests</td>
</tr>
<tr>
<td>$V_{CL}$</td>
<td>the set of switches with attached cloudlets, and clearly $V_{CL} \subseteq V$</td>
</tr>
<tr>
<td>$C_v$</td>
<td>the computing capacity of the cloudlet attached to a switch node $v \in V_{CL}$</td>
</tr>
<tr>
<td>$r_k = (s_k, D_k, b_k, SC_k)$</td>
<td>a delay-aware NFV-enabled multicast request, where $s_k \in V$ is the source, $D_k$ is the set of destinations with $D_k \subseteq V$, $b_k$ is the size of its data traffic, and $SC_k$ is the service chain of $r_k$ that consists of a sequence of VNFs.</td>
</tr>
<tr>
<td>$F$ and $f_i$</td>
<td>the set of VNFs provided by the network service provider in $G$ and a VNF $f_i$</td>
</tr>
<tr>
<td>$L_k$</td>
<td>the number of VNFs in $SC_k$</td>
</tr>
<tr>
<td>$C_{unit}(f_i)$</td>
<td>the amount of computing resource needed to process a unit amount of data traffic</td>
</tr>
<tr>
<td>$d_k$</td>
<td>the processing delay of each multicast request $r_k$ by VNF $f_i$</td>
</tr>
<tr>
<td>$a_i$</td>
<td>a given proportional factor of VNF $f_i$</td>
</tr>
<tr>
<td>$d_p$</td>
<td>the accumulative processing delay incurred due to the traffic processing by network functions in $SC_k$ of $r_k$</td>
</tr>
<tr>
<td>$P_k$</td>
<td>the set of routing paths from source $s_k$ to destinations in $D_k$</td>
</tr>
<tr>
<td>$p_{si} \in P_k$</td>
<td>a routing path from $s_k$ to a destination $t_m \in D_k$</td>
</tr>
<tr>
<td>$d_k$</td>
<td>the transmission delay of request $r_k$</td>
</tr>
<tr>
<td>$d_l$ and $D_k$</td>
<td>the delay experienced by multicast request $r_k$ and its delay requirement</td>
</tr>
<tr>
<td>$c(e)$ and $c(v)$</td>
<td>the usage costs of one unit of bandwidth and computing resources at link $e \in E$ and cloudlet $v \in V_{CL}$</td>
</tr>
<tr>
<td>$c_l(v)$</td>
<td>the cost of instantiating an instance of network function $f_l$ in cloudlet $v \in V_{CL}$</td>
</tr>
<tr>
<td>$n_v^0$</td>
<td>the number of newly created instances for network function $f_l$ in cloudlet $v$</td>
</tr>
<tr>
<td>$n_v^i$</td>
<td>the number of existing instances of $f_l$ in $v$ that are used to process the traffic of $r_k$</td>
</tr>
<tr>
<td>$c_l$</td>
<td>the operational cost of admitting a delay-aware NFV-enabled multicast request $r_k$</td>
</tr>
<tr>
<td>$V_{CL} \cap r_k$</td>
<td>the set of cloudlets that are used to implement the instances of VNFs in $SC_k$ of request $r_k$</td>
</tr>
<tr>
<td>$T_{sc}$</td>
<td>the obtained multicast tree that is used to route the data traffic of $r_k$</td>
</tr>
<tr>
<td>$ST$ and $R_{ad}$</td>
<td>the weighted throughput and the set of admitted multicast requests</td>
</tr>
<tr>
<td>$n_k^i$</td>
<td>the number of cloudlets that are used to implement the VNFs in $SC_k$ in the current infeasible solution</td>
</tr>
<tr>
<td>$n_k^m$ and $n_k^m$</td>
<td>the proper number of cloudlets in the feasible solution of algorithm Heu_Delay</td>
</tr>
<tr>
<td>$G' = (V', E')$</td>
<td>the auxiliary graph constructed in algorithm Appro_NoDelay</td>
</tr>
<tr>
<td>$f_{l,v}$ and $f_{l,v}^0$</td>
<td>the pair of virtual VNF nodes for the $i$th VNF instance of $f_l$ in cloudlet $v \in V_{CL}$</td>
</tr>
<tr>
<td>$w(f_{l,v}^0, f_{l,v}^0)$</td>
<td>the weight of edge $(f_{l,v}^0, f_{l,v}^0)$ in the auxiliary graph $G'$.</td>
</tr>
<tr>
<td>$v_{l,i}^f$ and $v_{l,i}^f$</td>
<td>a pair of virtual cloudlets for the $i$th VNF and cloudlet $v \in V'$.</td>
</tr>
<tr>
<td>$w_{v_{l,i}^f}$ and $w_{v_{l,i}^f}$</td>
<td>a virtual source node and a virtual destination node for the virtual network function $f_l$ and cloudlet $v \in V'$.</td>
</tr>
<tr>
<td>$c^*$</td>
<td>the optimal solution for the NFV-enabled multicasting problem</td>
</tr>
<tr>
<td>$L_{max}$</td>
<td>the maximum length of the service chains of the requests in $R$, i.e., $L_{max} = \arg \max_{r \in R}</td>
</tr>
<tr>
<td>$L_{com}$ and $R(L_{com})$</td>
<td>the number of common VNFs that requests have in their service chains VNFs in common of their service chains, and the set of such requests.</td>
</tr>
</tbody>
</table>

route the traffic of request $r_k$ to each destination in $D_k$ by chaining either existing or newly created instances of VNF, such that the operational cost (i.e., Eq. (6)) of implementing $r_k$ is minimized, while meeting the end-to-end delay requirement $D_k$ of $r_k$ and capacity constraint on each cloudlet $v \in V_{CL}$.

Problem 2. Assuming that the computing resource in each cloudlet in the MEC network $G$ has available capacity. For each request in $R$, the network may or may not have enough resources to admit it, the NFV-enabled multicasting problem in an MEC network $G$ for a given set $R$ of NFV-enabled multicast requests is to maximize the system throughput while minimizing the operational cost, subject to computing capacity on each cloudlet, where the system throughput is defined as the total amount of data that is processed and transferred by the system for admitted multicast requests. Let $ST$ be the weighted throughput and $R_{ad}$ the set of admitted multicast requests, then

$$ST = \sum_{r_k \in R_{ad}} b_k.$$ (7)

The NFV-enabled multicasting problems are NP-hard, as its special case – the traditional multicast problem without NFV service chain constraints is NP-hard [8].

For clarity, the symbols used in this paper are summarized in Table 1.

4 ALGORITHMS FOR THE ADMISSION OF A SINGLE NFV-ENABLED MULTICAST REQUEST

In this section, we deal with NFV-enabled multicasting for a single NFV-enabled multicast request admission. We first propose an efficient heuristic for the problem. We then consider a special case of the problem without delay requirements, by devising an approximation algorithm.

4.1 An Efficient Heuristic

The basic idea of the proposed heuristic is based on an observation that a feasible solution to the problem needs to meet the capacity constraints on cloudlets, service function chain requirements, and the end-to-end delay requirement of each multicast requests $r_k$. We thus adopt a two-phase heuristic that progressively considers the mentioned constraints and requirements.

Phase One. We first propose an algorithm to jointly consider the capacity constraint and the service chain requirement, by ignoring the delay requirement of $r_k$. The proposed
algorithm smartly explores existing VNF instances in each cloudlet that can be shared with the VNF instances of \( r_k \). Notice that the solution may not be feasible to the NFV-enabled multicasting problem, because the delay requirement of \( r_k \) is not considered in this phase. For the sake of clarity, we describe the proposed algorithm for the problem without delay requirement in the next subsection, which is referred to as \textit{Appro-NoDelay}. By now, we assume we already obtained the multicast tree for \( r_k \) in \( G \) without considering its delay requirement.

\textbf{Phase Two.} We refine the obtained multicast tree into a feasible solution to meet the delay requirement of \( r_k \). In particular, we observe that a longer delay will be the result if the VNFs of \( SC_k \) are implemented in multiple cloudlets. This is because that if the VNFs are distributed into different cloudlets, the data traffic transmission among two consecutive VNFs has to be performed by inter-cloudlet links, which incurs higher delays than those by intra-cloudlet data transfers. However, putting all VNFs into a single cloudlet may also incur a longer delay, since the selected cloudlet may be far away from the destinations of \( r_k \). This means that a large or a small value for the number of cloudlets of a request may not be proper to meet the delay requirement of \( r_k \). We thus adopt a binary search to narrow down the choices of the proper number of cloudlets for \( r_k \), making the delay requirement of \( r_k \) being met quickly. Specifically, let \( n_k' \) be the number of cloudlets that are used to implement the VNFs in \( SC_k \) in the current infeasible solution, and denote by \( n_k \) the proper number of cloudlets in the feasible solution. We first set

\[
    n_k = \left\lceil \frac{|V_{CL}| + 1}{2} \right\rceil.
\]

The proposed algorithm first tries to re-assign the VNFs in service function chain \( SC_k \) such that they are implemented in exactly \( n_k \) cloudlets. If \( n_k < n_k' \), we identify a number of \((n_k' - n_k)\) cloudlets that implements VNFs of \( SC_k \) in the obtained infeasible solution from the Steiner tree [4] (i.e., multicast tree) in \( G' \) and have the longest average data transfer delay from it to the destinations in \( D_{r_k} \). Let \( F' \) be the set of instances of VNFs in \( SC_k \) that are implemented in the identified cloudlets. The VNFs in \( F' \) are pre-consolidated to the rest \( n_k \) cloudlets in \( V' \) one by one, by selecting a cloudlet with the lowest implementation cost for each \( f_i \in F' \). If the pre-consolidation makes the delay requirement of \( r_k \) being met, the algorithm terminates with a feasible solution. Otherwise, if the experienced delay of \( r_k \) is reduced but still greater than its requirement, we continue the above procedure by searching the appropriate number of cloudlets in the range of \([1, n_k]\). The rationale is that the number of cloudlets in the multicast tree is still too many, and the inter-cloudlet communication leads to the delay requirement violation. The number of cloudlets still needs to be reduced. Instead, if the experienced delay is increased, we try to find the appropriate value for \( n_k \) in the range of \([n_k, |V_{CL}|]\). This means increasing the number of cloudlets for \( r_k \) may reduce the experienced delay of multicast request \( r_k \). On the other hand, if \( n_k > n_k' \), we need to find the additional \( n_k - n_k' \) cloudlets that have the lowest implementation cost for VNFs of \( r_k \), and pre-assign VNFs in \( F' \) to the cloudlets one by one. The above binary search procedure continues until a feasible solution is obtained or the multicast request is rejected. The detailed heuristic is described in Algorithm 1 and its basic idea is shown in Fig. 3. For simplicity, this algorithm is referred to as algorithm \textit{Heu_Delay} in the rest of this paper.

\begin{algorithm}
\caption{Heu_Delay} \label{alg:heu}
\begin{algorithmic}
\REQUIRE \( G = (V,E), V_{CL}, \) computing capacity \( C_v \) for each cloudlet \( v \in V_{CL}, \) and a multicast request \( r_k = (s_k, D_{r_k}; b_{r_k}, SC_k) \) and its delay requirement \( d_{r_k}^{req} \).
\ENSURE The locations for the VNFs of service chain \( SC_k \) of multicast request \( r_k \) and the multicast tree \( T_k \) to transfer its data.
\STATE /*Phase one: find cloudlets and routing paths for \( r_k \) by considering its service chaining requirement and cloudlet capacity constraints.*/
\STATE 1: Find a multicast tree for \( r_k \) without considering its delay requirement \( d_{r_k}^{req} \), by invoking algorithm Appro_NoDelay;
\STATE 2: Find a multicast tree for \( r_k \) without considering its delay requirement \( d_{r_k}^{req} \); by invoking algorithm Appro_NoDelay;
\STATE 3: Let \( n_k' \) be the number of cloudlets that are used to implement VNFs in \( SC_k \) of the found multicast tree;
\STATE 4: /*Phase two: adjust the multicast tree to meet the delay requirement of \( r_k \).*/
\STATE 5: \( n_{min} \leftarrow 1; \)
\STATE 6: \( n_{max} \leftarrow |V_{CL}|; \)
\STATE 7: \textbf{while} \( n_{min} \leq n_{max} \) \textbf{do}
\STATE 8: \( n_k \leftarrow \frac{n_{min} + n_{max}}{2}; \)
\STATE 9: \textbf{if} \( n_k < n_k' \) \textbf{then}
\STATE 10: Identify the number of \( n_k' - n_k \) cloudlets that implements VNFs of \( SC_k \) in the obtained solution from the Steiner tree in \( G' \) and has the top-\((n_k' - n_k)\) highest average data transfer delays from it to the destinations in \( D_{r_k} \);
\STATE 11: Move the VNFs that were implemented in the \( n_k' - n_k \) cloudlets of the infeasible solution to the rest cloudlets one by one.
\STATE 12: \textbf{else}
\STATE 13: Find the additional \( n_k' - n_k \) cloudlets that have the lowest implementation cost for VNFs of \( r_k \), and assign VNFs in \( F' \) to the cloudlets one by one.
\STATE 14: \textbf{if} the experienced delay of \( r_k \) is met \textbf{then}
\STATE 15: return;
\STATE 16: \textbf{else}
\STATE 17: \textbf{if} the experienced delay of \( r_k \) is decreased \textbf{then}
\STATE 18: \( n_{max} \leftarrow n_k'; \)
\STATE 19: \textbf{else}
\STATE 20: \( n_{min} \leftarrow n_k; \)
\end{algorithmic}
\end{algorithm}
4.2 An Approximation Algorithm for the Problem Without Delay Requirements

The proposed approximation algorithm for the problem without delay requirements is to reduce the problem in $G$ to the Steiner tree problem in an auxiliary graph $G'$, via a non-trivial reduction. Since each cloudlet $v \in V_{CL}$ has computing capacity to implement the VNFs of each request, the VNFs in each service function chain $SC_k$ can be implemented in multiple cloudlets or consolidated into a single cloudlet to save the communication cost due to the transmissions between different cloudlets. To ensure that each cloudlet has sufficient computing resource to implement the VNFs in $SC_k$ of each multicast request $r_k$, we adopt a conservative method of reserving $\sum_{f_i \in SC_k} b_k \cdot C_{unit}(f_i)$ resource for $r_k$ in each cloudlet. The cloudlet with an amount of available computing resource that is less than $\sum_{f_i \in SC_k} b_k \cdot C_{unit}(f_i)$ will be removed from the network $G$, where the available resource in idle VNF instances are also accounted.

The Construction of Auxiliary Graph $G' = (V', E')$. We now construct $G'$ based on the sub-network of $G$.

We start by constructing the node set $V'$ of $G'$. Specifically, we first add source node $s_k$ into the auxiliary graph. We also add each node in $V$ into $V'$, i.e., $V' \leftarrow V' \cup V$. Notice that, since $V_{CL} \subset V$, all switch nodes in $V_{CL}$ are added into $V'$ as well. However, only their functionalities of forwarding traffic will be used.

Recall that VNFs in $SC_k$ of multicast request $r_k$ can be assigned to existing VNFs or newly instantiated VNF instances. To determine whether making use of existing VNF instances or creating new ones, we create a widget for each cloudlet $v \in V_{CL}$ and each network function $f_i \in SC_k$ to represent the resource availability of the cloudlet $v$ for $f_i$ by two cases. Case 1: the amount of available computing resource to instantiate new instances of VNFs; Case 2: existing VNF instances of $f_i$ in $v \in V_{CL}$ that are available to process the traffic of $r_k$. There is a widget for each pair of cloudlet and VNF, which actually means a possible placement of a VNF to a cloudlet.

For Case 1, we add a pair of virtual VNF nodes into the widget, to represent each of existing VNF instances of $f_i$ with sufficient computing resource processing the data traffic of $r_k$ in cloudlet $v \in V_{CL}$. Denote by $f'_{i,v}$ and $f''_{i,v}$, the pair of virtual VNF nodes for the $i$th VNF instance of $f_i$ in cloudlet $v \in V_{CL}$. We then add an edge from $f'_{i,v}$ to $f''_{i,v}$ into the widget. The weight of edge $(f'_{i,v}, f''_{i,v})$ is the cost of processing a unit traffic by an existing VNF instance of $f_i$ in cloudlet $v$, i.e., $w(f'_{i,v}, f''_{i,v}) = c(v_{f_i,v})$.

For Case 2, we add a pair of virtual cloudlets for each cloudlet $v \in V_{CL}$ into each widget to denote the amount of available computing resource to instantiate a new instance of $f_i$ in cloudlet $v$, as shown in Fig. 4. Let $v_{i,k}^v$ and $v_{i,k}^{v'}$ be such a pair of virtual cloudlets for the $i$th VNF and cloudlet $v$. To jointly consider the processing and transmission costs, we connect each pair of virtual cloudlets, $v_{i,k}^v$ and $v_{i,k}^{v'}$, i.e., $E' \leftarrow E' \cup \{(v_{i,k}^v, v_{i,k}^{v'})\}$. The weight of edge $(v_{i,k}^v, v_{i,k}^{v'})$ is the sum of the instantiation cost of VNF $f_i$ and the cost of processing a unit traffic by the $i$th VNF in $SC_k$ for each multicast request $r_k$ in cloudlet $v$. That is, $w(v_{i,k}^v, v_{i,k}^{v'}) = \frac{b_k}{C_{unit}(f_i)} + c(v_{f_i,v})$.

We also add a widget source node $ws_{i,v}$ and a widget destination node $wd_{i,v}$ for the widget for network function $f_i$ and cloudlet $v \in V_{CL}$. Node $ws_{i,v}$ is connected to node $v_{i,k}^v$ and the node $f'_i$ for each existing instance of network function $f_i$ that has enough computing resource to process the data traffic of $r_k$. In addition, node $v_{i,k}^v$ and node $f'_i$ for each existing instance of network function $f_i$ are both connected with the widget destination node $wd_{i,v}$. The weights of those edges are set to zeros. It must be mentioned that widget source and destination nodes are used to guarantee that either a new instance for $f_i$ is created or an existing VNF instance of $f_i$ is selected to process the traffic of $r_k$, which will be proved in the algorithm analysis part.

The widgets become part of the auxiliary graph $G'$.

We then connect the widgets and other nodes in the auxiliary graph $G'$ as follows.

- $s_k$ to widget source nodes: There is an edge from source node $s_k$ to each widget source node $ws_{i,v}$ of the widget for the first VNF $f_i$ of $SC_k$ and every $v \in V_{CL}$. The weight of edge $(s_k, ws_{i,v})$ is set as the transmission cost of data traffic of $r_k$.
- Widget destination to widget source nodes: Since the data traffic of $r_k$ may be processed by multiple cloudlets, there is an edge from the widget destination node of each widget for network function $f_i$ to the widget source node of each widget for VNF $f_{i+1}$, for each $l$ with $1 \leq l \leq L_k - 1$, i.e., $E' \leftarrow E' \cup \{(wd_{i,v}, ws_{i+1,v})\}$ for $l$ with $1 \leq l \leq L_k - 1$ and $v, u \in V_{CL}$. The weight of edge $(wd_{i,v}, ws_{i+1,v})$ is the transmission cost of a unit traffic along the shortest path from cloudlet $v$ to cloudlet $u$.
- Widget destinations of $f_{i,v}$ to cloudlet nodes: We finally connect each of the widgets that are created for the last VNF $f_{i,k}$ in $SC_k$ with the cloudlet node. Specifically, there is an edge from node $wd_{i,k,v}$ to cloudlet node $u$ in $V'$, i.e., $E' \leftarrow E' \cup \{(wd_{i,k,v}, u)\}$. The weight of edge $(wd_{i,k,v}, u)$ is the transmission cost of a unit traffic along the shortest path from cloudlet $v$ to cloudlet $u$.

An example of the constructed auxiliary graph is shown in Fig. 5.

Problem Reduction. We now reduce the NFV-enabled multicasting problem without delay requirements in $G$ to the Steiner tree problem in the directed auxiliary graph $G'$. Recall that in the construction of $G'$, the VNF processing
and transmission costs are considered as the weights of edges.

We thus find a directed Steiner tree in $G'$ that spans nodes in $\{s_k\} \cup D_k$. We then transfer the Steiner tree in $G'$ to routing paths for $r_k$ in the original network $G$. Specifically, if a widget for $f_i \in SC_k$ and cloude $v \in V_{CL}$ is included in the Steiner tree, either a newly created VNF instance or an existing one in cloude $v$ will be used to implement $f_i$, depending on which edge of the widget is included in the Steiner tree. Notice that the edges among the widgets in $G'$ correspond to the shortest paths of their endpoints of the edges in $G$. We thus replace each of such edges with its shortest path in $G$.

Algorithm 2. Appro_NoDelay

Input: $G = (V, E)$, $V_{CL}$, computing capacity $C_v$ for each cloude $v \in V_{CL}$, and a multicast request $r_k = (s_k, D_k; b_k, SC_k)$.

Output: The locations for the VNFs of service chain $SC_k$ of multicast request $r_k$ and the multicast tree $T_k$ to transfer its data.

1: Construct an auxiliary directed graph $G' = (V', E')$, as shown in Fig. 5;
2: Find a directed Steiner tree $T$ in $G'$ that spans nodes in $\{s_k\} \cup D_k$, using Charikar’s algorithm [4];
3: For each path from the widget source node to the widget destination node of a widget in $T$, condense the path to a single node;
4: Replace each of all other edges in $T$ with its corresponding shortest path in network $G$; /*The edges among widgets correspond to shortest paths in the original network $G$. */

4.3 Algorithm Analysis

We now analyze the feasibility of the solution obtained and performance of the proposed algorithms.

We first show the feasibility of the solution delivered by Algorithm 2. Intuitively, if a solution to the NFV-enabled multicasting problem, it needs to satisfy the following three conditions:

- **Condition 1**: each VNF $f_i \in SC_k$ will be assigned to one or multiple cloudlets by either creating a new instance or using an existing instance
- **Condition 2**: the traffic of $r_k$ will be processed by VNFs as the specified order in $SC_k$
- **Condition 3**: the processed traffic by the VNFs in $SC_k$ is forwarded to destinations in $D_k$ of $r_k$

For Condition 1, we show that in each of the selected cloudlets for $f_i$, either a new instance is created or an existing instance is selected for it in the following lemma.

**Lemma 1.** If a cloudlet $v \in V_{CL}$ is selected for VNF $f_i \in SC_k$ of multicast request $r_k$, either a newly created instance of $f_i$ or a newly created instance is used to process the traffic of $r_k$.

**Proof.** Following the construction of $G'$, showing the feasibility of the solution is to show that if the Steiner tree found in $G'$ has one path from $w_{SL_v}$ to $wd_{lv}$ of each selected widget, the path will be the only path in the Steiner tree, and no other paths in the widget will be included. Let $W_{lv}$ be the widget that is built for network function $f_i$ in cloude $v \in V_{CL}$. Assume that widget $W_{lv}$ is included into the Steiner tree for the subgraph, and let $p$ be the path. Let $W_{lv}$ be the path from $w_{SL_v}$ to $wd_{lv}$ of $W_{lv}$ in $G'$ that is included in the Steiner tree. We prove by contradiction. Assume that there is another instance (either newly created or existing one) of $f_i$ is used to process the traffic of $r_k$. Let the path $p$ be such an additional instance. This means that edge $(\langle f'_{lv}, f'_{lv} \rangle)$ has to be included in the Steiner tree found in $G'$. Edges $\langle w_{SL_v}, f'_{lv} \rangle$ and $\langle f'_{lv}, wd_{lv} \rangle$ have to included, according to the structure of the widget; otherwise, edge $(\langle f'_{lv}, f'_{lv} \rangle)$ is a stand alone edge that can be removed. Let $p'$ be the path that consisting of edges $\langle w_{SL_v}, f'_{lv} \rangle$, $\langle f'_{lv}, f''_{lv} \rangle$, and $\langle f''_{lv}, wd_{lv} \rangle$, as shown in Fig. 6. Paths $p'$ and $p$ however make it not a tree. Therefore, only one path from $w_{SL_v}$ to $wd_{lv}$ will be included in the Steiner tree for the subgraph of $G'$ that is composed of source node $s_k$ and the widgets, meaning that a newly created or existing instance of $f_i$ will be selected in cloudlet $v \in V_{CL}$. The lemma holds.

We consider Condition 2 in the following lemma.

**Lemma 2.** The traffic of $r_k$ will be processed by the VNF instances in $SC_k$ in the specified order.
Assume that the traffic of $C$ is then $k_{j} = (S_{C} + b_{j}/C_{0} + j/C_{1} C + \ldots$ is the will be forwarded to its destinations in for multicast request $D_{b}$ by (1) selecting either $0$ $G$ is met. Algorithm 1 adopts a binary search of data from its source to $2$. It can of such widgets, if its $2$, we find an approxi-
that requires to transfer an amount $b_{i}$ of data from its source to a set $D_{k}$ of destinations and process its traffic by the VNFs in $S_{C_{k}}$. There is an approximation algorithm, i.e., Algorithm 2, for a special case of the NFV-enabled multicasting problem without delay requirements, which delivers a feasible solution that has an approximation ratio of $i(i - 1)/|D_{k}|^{1/3}$ [4], and the time complexity of $O((L_{k} \cdot |V| \cdot C_{0}/c_{\text{out}(f_{j})}) + |V|^{3} \cdot |D_{k}|^{3})$, where $L_{k}$ is the number of VNFs in the service chain $S_{C_{k}}$ of multicast request $r_{k}$, i.e., $L_{k} = |S_{C_{k}}|$, and $i$ is the level of the directed Steiner tree [4].

**Proof.** From Lemmas 1, 2, and 3, we know that the solution obtained by finding a Steiner tree in $G$ is feasible. In the following, we analyze the approximation ratio and running time of the proposed approximation algorithm.

Assume $c^{*}$ is the optimal solution for the NFV-enabled multicasting problem. In Algorithm 2, we find an approxi-
mate Steiner tree $T'$ in the auxiliary graph $G'$. $T'$ is then converted to routing paths for $r_{k}$ in $G$ by (1) selecting either an existing instance for a network function or a newly created instance of each VNF $f_{j}$ in $S_{C}$ if the widget for $f_{j}$ is included in the Steiner tree, and (2) replacing the edges between selected widgets using their corresponding shortest paths in $G$. In (1), the processing is determined according to which type of VNF instance is selected. In (2), the replaced auxiliary graph edge has the same weight as the total cost of its corresponding shortest path in $G$. Therefore, the cost do not change in the transfer from tree $T'$ to the multicast tree $T$ for multicast request $r_{k}$. Since the approximation ratio of the algorithm in [4] is $i(i - 1)/|D_{k}|^{1/3}$, the approximation of Algorithm 2 is $i(i - 1)/|D_{k}|^{1/3}$ as well.

We now show the time complexity of Algorithm 2. It can be seen that the most time consuming part of the algorithm is the finding of a Steiner tree in the auxiliary graph. The time complexity of Charikar’s algorithm in auxiliary graph $G' = (V', E')$ is $O(|V'|^{13})$ [21]. We can see that there are $O(c_{\text{out}(f_{j})}^{2})$ instances of VNF $f_{j}$ in cloudlet $v \in V_{CL}$. According to the construction of the auxiliary graph, we thus have $O(l_{v}/c_{\text{out}(f_{j})}^{4} + 4) = O(l_{v}/c_{\text{out}(f_{j})}^{4})$ nodes for each widget. In total, we have $L_{k} \cdot |V_{CL}|$ widgets. Therefore, there are $O(L_{k} \cdot |V_{CL}| \cdot C_{0}/c_{\text{out}(f_{j})}^{4} + |V|^{3})$ nodes in auxiliary graph $G'$. The time complexity thus is $O((L_{k} \cdot |V| \cdot C_{0}/c_{\text{out}(f_{j})}^{4} + |V|^{3}) \cdot |D_{k}|^{3})$. \hfill \box

We finally analyze the performance of Algorithm 1 the following theorem.

**Theorem 2.** Given an MEC network $G = (V, E)$ with a set $V_{CL}$ of cloudlets and a multicast request $r_{k} = (s_{k}, D_{k}; b_{k}, S_{C_{k}})$ that requires to transfer an amount $b_{k}$ of data from its source to a set $D_{k}$ of destinations with an end-to-end delay requirement $d_{k}^{\text{req}}$ and process its traffic by the VNFs in $S_{C_{k}}$. There is a heuristic algorithm, i.e., Algorithm 1, for the NFV-enabled multicasting problem for a single multicast request, which delivers a feasible solution in time $O(|V_{CL}| + 1) \cdot |V|^{2} + (L_{k} \cdot |V| \cdot C_{0}/c_{\text{out}(f_{j})}^{4} + |V|^{3}) \cdot |D_{k}|^{3}$, where $L_{k}$ is the number of VNFs in the service chain $S_{C_{k}}$ of multicast request $r_{k}$, i.e., $L_{k} = |S_{C_{k}}|$, and $i$ is the level of the directed Steiner tree [4].

**Proof.** We first show the solution feasibility of the proposed heuristic by showing that the end-to-end delay requirement of $r_{k}$ is met. Algorithm 1 adopts a binary search based heuristic to find the proper number of cloudlets.
each multicast request $r_k$ until the end-to-end delay requirement of $r_k$ is met or it is rejected. Therefore, as long as the request is admitted, its end-to-end delay requirement is met.

We then analyze the time complexity of the proposed heuristic. Clearly, in the worse case, the binary search can make $\lceil \log V_{CL} + 1 \rceil$ iterations. Within each iteration, the most time consuming parts include (1) the identification of cloudlets that involved finding the delays from cloudlets to destinations in $D_k$ via all pair shortest paths, which take $O(|V|^3)$ time, and (2) the assignment of VNFs one by one, taking $O(|SC_k|)$ time. In total, the time complexity of the proposed heuristic is $O(|\log V_{CL} + 1| \cdot |V|^3 \cdot |SC_k| + (L_k \cdot |V| \cdot C_{mut} + |V|)^2 \cdot |D_k|^2) = O(|\log |V| + 1| \cdot |V|^3 + (L_k \cdot |V| \cdot C_{mut} + |V|)^2 \cdot |D_k|^2)$, assuming that $|SC_k|$ is a small constant.

5 Algorithm for Admissions of a Set of NFV-Enabled Multicasting Requests

In this section, we consider a set of multicast request admissions. Given a set of NFV-enabled multicast request, we admit as many as requests in the set such that the weighted system throughput is maximized, while the accumulated implementation cost of all admitted requests is minimized, subject to computing capacities on cloudlets in an MEC.

5.1 Overview

Recall that we proposed both approximate and heuristic solutions for the NFV-enabled multicasting problem for the admission of a single multicast request, a simple method for the NFV-enabled multicasting problem is to consider algorithm $Heu_{Delay}$ as a black-box and admit each request one by one invoking algorithm $Heu_{Delay}$ iteratively. This method however may miss the opportunities of sharing VNFs among the requests, if the consecutively admitted requests do not have common VNFs in their service chains. Further, the constructed auxiliary graph $G'$ in algorithm $Heu_{Delay}$ for a request may no longer useful for the other. This consequently may lead to a prohibitively long time to make decisions of request admissions.

The basic idea behind the proposed algorithm is as follows. We observe that some requests have the same service chain requirements, and the VNFs in their service chains can be shared with high opportunities. Fig. 7 illustrates this idea, from which we can see that requests are classified into different categories, with each category having a set of requests that share a number of VNFs. Specifically, the algorithm first considers the category in which multicast requests the maximum number of common VNFs of their service chains. Then, the requests in this category, we start with the requests with smaller data traffic, and admit the requests one-by-one. This procedure continues until no more requests can be admitted in the category.

5.2 Heuristic Algorithm

We propose an efficient heuristic for the NFV-enabled multicasting problem for a set of requests with different service chain requirements, based on Algorithm 1.

Specifically, the heuristic consists of a number of iterations within each iteration, a set of requests with the same number of VNFs in common are processed. Let $L_{com}$ be the number of common VNFs that requests have in their service chains. Let $L_{max}$ be the maximum length of the service chains of the requests in $R$, i.e., $L_{max} = \arg\max_r \in R |SC_k|$. Initially, $L_{com} = L_{max}$. It decreases by one in each iteration of the algorithm until $L_{com} = 0$.

Within each iteration, we first find the requests that have $L_{com}$ VNFs in common of their service chains. Denote by $R(L_{com})$ the set of such requests. We then rank the requests in $R(L_{com})$ in increasing order of their data traffic. For each request $r_k \in R(L_{com})$, we invoke the proposed approximation algorithm in 2. Notice that the requests in $R(L_{com})$ may have different source nodes and different destination sets. We thus need to adjust the auxiliary graph after the admission of each multicast request, by removing the source node for the previous request, and add the source node of the current request. This means that, before admitting the next multicast request $r_{k+1}$, we make adjustments of the constructed auxiliary graph $G'$ instead of constructing a new one. Specifically, the widgets that are built for the $L_{com}$ VNFs are updated accordingly, if multicast request $r_k$ is admitted. Also, the widgets for the VNFs that are not among the $L_{com}$ of request $r_{k+1}$ is added to the auxiliary graph. This iteration continues until no more requests can be admitted within this category. The steps of this algorithm are detailed in Algorithm 3.

We now analyze the feasibility of Algorithm $Heu_{MultiReq}$ in the following theorem.

Theorem 3. Given an MEC network $G = (V, E)$ with a set $V_{CL}$ of cloudlets, a set $R$ of NFV-enabled multicast requests with each multicast request $r_k = (s_k, D_k; b_k, SC_k)$ that requires to transfer an amount $b_k$ of data from its source to a set $D_k$ of destinations with an end-to-end delay requirement $d_k^{max}$ and process its traffic by the VNFs in $SC_k$. There is an efficient algorithm, Algorithm 3, for the NFV-enabled multicasting problem.

Proof. To show the solution delivered by Algorithm 3 is feasible, we need to show the classification of requests does not affect the solution feasibility of Algorithm 2. Assume that the algorithm currently considers request $r_{k+1}$. If its
previous request $r_k$ is admitted, the widgets of the corresponding cloudlets that implement the VNFs of $r_k$ are then updated, since the resource availabilities of these cloudlets or statuses of their existing VNF instances changed. Otherwise, there is not any change of the widgets in the auxiliary graph. Considering that the feasibility of admitting one request by Algorithm 2 can be shown by Lemma 2, Algorithm 1 delivers a feasible solution when multiple requests are considered.

\begin{algorithm}
\caption{Heu\textunderscore Multi\textunderscore Req}
\begin{algorithmic}[1]
\State $G = (V, E), V_{CL}, C_v$ for each $e \in E, C_v$ for each $v \in V_{CL}$, and a set of multicast requests with each multicast request being denoted by $r_k = (s_k, D_k, b_k, SC_k)$.
\State \textbf{Input:} The system throughput achieved by the admitted requests in $R$.
\State $N_{ad} \leftarrow 0$;
\For{$L_{com} \leftarrow 0, 1, \ldots, L_{max}$}
\State Find the maximum number of requests in $R$ that have $L_{com}$ common VNFs in their service chains, and let $R(L_{com})$ be the set of requests;
\State Rank the multicast requests in $R(L_{com})$ according to their data traffic;
\For{each request $r_k \in R(L_{com})$}
\State $T \leftarrow \emptyset$;
\While{$G$ is $(s_k, D_k)$-connected OR $r_k$ is admitted}
\State Construct auxiliary graph $G' = (V', E')$; by creating $L_k \cdot |V_{CL}|$ widgets, adding all the switch nodes in $V$ of the original network $G$, and interconnecting the added nodes as shown in Fig. 5, or adjust the auxiliary if it is already constructed in the admission of previous requests;
\State Find a Steiner tree $T$ for in auxiliary graph $G'$;
\If{the delay of each branch of $T$ is smaller than $d_k^{eq}$}
\State Admit multicast request $r_k$;
\Else
\State Find the branches of $T$ that violate delay requirement $d_k^{eq}$;
\State For each of such found branch, identify an edge with the maximum delay;
\State Remove the identified edges from graph $G$;
\EndIf
\EndWhile
\If{$T \neq \emptyset$}
\State For each path from the widget source node to the widget destination node of a widget in $T$, condense the path to a single node;
\State The widgets that are built for the $L_{com}$ VNFs are updated according to the resource availabilities after admitting $r_k$;
\If{$k + 1 < |R(L_{com})|$}
\State The widgets for the VNFs that are not among the $L_{com}$ of request $r_{k+1}$ is added to the auxiliary graph;
\EndIf
\EndIf
\EndFor
\EndFor
\EndAlgorithm
\end{algorithm}

6 PERFORMANCE EVALUATION

In this section we evaluate the performance of the proposed algorithms in a real testbed.

6.1 Test-Bed Setup

We build a test-bed consisting of both an underlay network with hardware switches and an overlay network with virtual switches, as shown in Fig. 8. The physical underlay consists of five H3C S5560X-30S-EI switches [12], with the support for VXLAN for virtual tunnel building and SDN capabilities. It has also one server with E5 Gold 5218 CPU, 128G RAM and four PCs with i7-8700 CPU, 16G RAM. Netconf and SNMP protocols are used to manage the switches and the links that interconnect them [3], [35]. We considered a design approach that uses the VXLAN functionality provided by the switch, where VXLAN is a widely used overlay technology [37]. The H3C S5560X-30S-EI switch implements a VXLAN tunnel based on hardware, which can greatly improve performance compared to traditional methods. The overlay mechanism provides connectivity within, and potentially across multiple testbed sites as it can transit any routed layer-3 underlay. We use VXLAN as a point-to-point tunneling mechanism (VXLAN VN1 identifies a single link between two nodes [37]). SDN-capable switches can also perform encapsulation and decapsulation of VXLAN tunnels, each tunnel corresponds to a port in the switch. Using VXLAN, we build an overlay network with a number of Open vSwitch (OVS) [36] nodes and VMs. The overlay network is built following the topology generated using a graph generation tool GT-ITM [10] and the real network topologies AS1755, AS4755. Its OVS nodes and VMs are controlled by a Ryu [40] controller. The proposed algorithms are implemented as Ryu applications.

6.2 Environment Settings

We consider an MEC network consisting of the number of nodes from 50 to 250. The number of servers in each network is set to 10 percent of the network size, and the servers are randomly co-located with the switches. We also use real network topologies, i.e., the GÉANT [9] and an ISP network from [42]. There are nine cloudlets for the GÉANT topology as set in [11] and the number of data centers in the ISP networks are provided by [38]. The computing capacity of cloudlet varies from 40,000 to 120,000 MHz [13] (cloudlets with around tens of servers). Five types of network functions, i.e., Firewall, Proxy, NAT, IDS, and Load Balancing, are considered, and their computing demands are adopted from [11], [32]. The source and destination nodes of each

![Fig. 8. A test-bed with both hardware switches and virtual resources.](image)
multicast request is randomly generated, the ratio of the maximum number $D_{\text{max}}$ of destinations of a multicast request to the network size $|V|$ is randomly drawn in the range of $[0.05, 0.2]$. The data of each request is randomly drawn from [10, 200] Megabyte, and the delay requirement of transferring such data is randomly drawn from $[0.05, 5]$ seconds. Notice that the transfer of larger amount of data can be divided into smaller amounts and transferred by multiple multicast requests. Unless otherwise specified, these parameters will be adopted in the default setting.

We compare the performance of the proposed approximation and heuristic algorithms against the following benchmarks.

- We consider the case where the VNFs of each multicast request may be placed to multiple cloudlets for processing while there exist solutions that consolidate all VNFs of a multicast request into a single location. We thus compare our solutions with such a solution, which is referred to as algorithm Consolidated.

- We evaluate the performance of the proposed approximation and heuristic algorithms against the one in [39] that does not consider the delay requirement of multicast requests, and we use NoDelay to represent the algorithm.

- We also compare the performance of our algorithm against that of a greedy solution that prefers to select existing VNF instances for each multicast request $r_k$. Specifically, it finds the cloudlet that is the closest to source node $s_k$ and has an VNF instance for its first VNF in $SC_k$, if there does not exist such cloudlets, a new VNF instance is created in the closest cloudlet. The procedure continues until all VNFs in $SC_k$ are considered. This greedy algorithm is referred to as algorithm ExistingFirst.

- Another greedy benchmark prefers to create new instances for each of the VNFs in $SC_k$, which is referred to as algorithm NewFirst.

- The fifth benchmark selects the cloudlet that can achieve the lowest processing cost for each VNF in $SC_k$. For simplicity, it is referred to as algorithm LowCost. Specifically, algorithm LowCost finds the cloudlet that is the closest to the source $s_k$ and then places as many VNFs in $SC_k$ to the cloudlet until all existing VNF instances are used or no computing resource available to instantiate new ones. If there are still VNFs in $SC_k$ that have not been assigned, it finds the next cloudlet that is the closest to the found cloudlets.

### 6.3 Performance Evaluation of Algorithms Heu_Delay and Appro_NoDelay

We first evaluate the performance of algorithms Heu_Delay and Appro_NoDelay against that of algorithms Consolidated, NoDelay, ExistingFirst, NewFirst, and LowCost, in terms of the average operational cost, the average end-to-end delay, and the running time, by varying the network size from 50 to 250 while fixing the number of requests at 100. Fig. 9 shows the results of the proposed algorithms.

From Fig. 9a, we can see that Algorithm Heu_Delay achieves a lower operational cost than those of algorithms ExistingFirst, NewFirst, and LowCost. The reason is that Algorithm Heu_Delay jointly considers existing VNF instances and newly instantiated ones. However, the greedy approaches NewFirst, ExistingFirst, and LowCost only prefer new, existing, or low processing cost VNF instances. They unfortunately could miss the opportunities of further reducing the operational cost. Specifically, if the use of existing VNF instances can save the processing cost, NewFirst has a higher cost due to creating new instances. Also, there are some cases when creating new VNF instances can save transmission costs, which can be missed by algorithm ExistingFirst. In addition, it can be seen from Fig. 9a that Algorithm Heu_Delay has a higher operational cost than algorithms Appro_NoDelay and NoDelay. This is because algorithms Appro_NoDelay and NoDelay do not consider the delay requirement of requests, making it choose cloudlets with lower operational costs.

As shown in Fig. 9b, the average delay experienced by each multicast request by Algorithm Heu_Delay is much lower than its comparison counterparts. The reason is that Algorithm Heu_Delay carefully finds a trade-off between the delay and cost of implementing an NFV-enabled request. Also, from Fig. 9c, we can see that the running time of Algorithm Heu_Delay is around 50 seconds for network size 200, which is slightly larger than those of algorithms Appro_NoDelay and NoDelay and smaller than algorithms ExistingFirst, NewFirst, and LowCost. The reason is that Heu_Delay has an additional process of binary search to find a proper number of cloudlets for each request $r_k$. Algorithm NoDelay has a lower running time compared with algorithm Appro_Delay because the delay requirement of requests is not considered, which reduces the solution space.
We then evaluate the performance of algorithms Heu_Delay and Appro_NoDelay against that of algorithms Consolidated, NoDelay, ExistingFirst, NewFirst, and LowCost, in real networks AS1755 and AS4755, by varying the ratio of the number of cloudlets to the number of switches, i.e., $|C_L|/|V|$ from 0.05 to 0.2. Fig. 10 illustrates the results. Figs. 10a and 10d show that algorithms Heu_Delay and Appro_NoDelay achieve lower operational costs than algorithms Consolidated, ExistingFirst, and NewFirst, while algorithms Appro_NoDelay and NoDelay has the highest delay. We can also see that the average cost of implementing a multicast increases first when the ratio $|C_L|/|V|$ increases from 0.05 to 0.1 and then decreases afterwards. The rationale behind is that VNFs of each multicast request may be assigned to more cloudlets with the increase of number of cloudlets, thereby pushing up the transmission cost from its source to the cloudlets and from the cloudlets to its destinations. However, with the further increase of cloudlets, it is more likely that these cloudlets are deployed in locations that are close to the source and destinations of the multicast request. The transmission cost then can be reduced afterwards.

We then investigate the impact of the maximum delay requirement on algorithm performance in the real network AS1755, by varying the maximum delay requirement of each multicast request from 0.8 seconds to 1.8 seconds with an increment of 0.2 seconds. Fig. 11 illustrates that the cost of implementing a multicast request is decreasing with the increase of the maximum delay requirement. The rationale behind is that a higher delay requirement of a request allows the algorithm to select cloudlets with lower costs but further from the source node of the request. Obviously, the experienced delay will be higher, as shown in Fig. 11.

6.4 Performance Evaluation of Algorithm Heu_MultiReq

We now compare the performance of Algorithm Heu_MultiReq against that of algorithms Consolidated, NoDelay,
ExistingFirst, NewFirst, and LowCost, in terms of the system throughput, the total operational cost, the average end-to-end delay, and the running time, by varying the network size from 50 to 250 and fixing the number of requests to 100. Results are shown in Fig. 12, from which we can see that Algorithm Heu_MultiReq achieves around 30, 30, 35 percent higher system throughput than algorithms ExistingFirst, NewFirst, LowCost, and Consolidated when the network size is 200. The rationale behind is that algorithms ExistingFirst, NewFirst, and LowCost prefer existing, newly instantiated, and low processing cost VNF instances for each multicast request, and the cloudlets for those VNF instances may not have sufficient computing resource to implement the request, thereby leading to its rejection. Further, from Figs. 12a and 12b, it can be seen Algorithm NoDelay has a higher end-to-end delay than that of Algorithm

![Graphs showing performance metrics](image-url)
Heu_MultiReq, although it delivers a slight higher system throughput. Similar results can be observed from Fig. 13 when the performance of Algorithm Heu_MultiReq is evaluated against that of algorithms Consolidated, NoDelay, ExistingFirst, NewFirst, and LowCost, in real networks AS1755 and AS4755.

We then investigate the impact of the number of requests on the performance of algorithms Heu_MultiReq, Consolidated, NoDelay, ExistingFirst, NewFirst, and LowCost, in terms of system throughput, average operational cost, average end-to-end delay, and running time, by varying the number of requests from 50 to 300 while fixing the network size to 100. Fig. 14 shows that the system throughput increases first with the growth on the number of requests from 50 to 100, and then keeps stable afterwards, because the cloudlet capacities are saturated. We can also see that the average cost of implementing a multicast increases with the growth of request number. The rationale behind is that each multicast request may be assigned to more cloudlets for processing with the increase of number of requests, considering that the resources in cloudlets are saturated and may not be enough to implement all VNFs of a service chain. This eventually increases the transmission cost for each multicast request.

7 CONCLUSION AND FUTURE WORK

In this paper, we study the problem of delay-aware, NFV-enabled multicasting in a mobile edge cloud network, by exploring the sharing of VNF instances of requests. If cloudlets have sufficient computing resource to process traffic of a multicast request, with no delay requirement, we proposed an approximate solution with a provable approximation ratio; otherwise, we developed an efficient heuristic. We also considered a set of NFV-enable multicast request admissions with the aim to maximize the weighted system throughput, for which we proposed an efficient heuristic. We finally evaluate the performance of the proposed algorithms against state-of-the-arts approaches in a real testbed, and the results show that the performance of our algorithms is promising.

In this paper we considered the sharing of idle VNFs that have been released by other requests. The requests with the same service chain requirements may share resources with high probability. However, requests may have dynamic resource demands, and may share resources with others as long as they have complimentary demands. Understanding how to learn such dynamic complimentary resource demands among requests is challenging. Therefore, we consider the adoption of machine learning methods to classify requests with complimentary demands as our future research study—akin to existing efforts in interference-aware scheduling in cloud-based data centers. Existing efforts that make use of an interference index to characterize these competing/complementary workloads can also be utilized in the proposed environment. Another is to explore the dynamic admission of NFV-enabled delay-aware requests, taking account of uncertainty (variability) of processing and transmission delays. The admission of requests in the current time slot can impact the admission of future requests. Understanding how online learning algorithms can adapt to support such admission control remains another potential research topic.

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REFERENCES


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Safety Enhancement for Real-Time Parallel Applications in Distributed Automotive Embedded Systems: A Stable Stopping Approach

Guoqi Xie*, Senior Member, IEEE, Gang Zeng*, Member, IEEE, and Renfa Li*, Senior Member, IEEE

Abstract—In distributed automotive embedded systems, safety issues run through the entire life cycle, and safety mechanisms for error handling are desirable for risk control. This article focuses on safety enhancement (i.e., safety mechanisms for error handling) for a safety-critical automotive application within its deadline. A stable stopping approach used for safety enhancement for an automotive application is proposed based on the static recovery mechanism provided in ISO 26262. The Stable Stopping-based Safety Enhancement (SSSE) approach is proposed by combining known backward recovery, proposed forward recovery, and proposed forward-and-backward recovery through primary-backup repetition. The stable stopping (i.e., SSSE) approach is a convergence algorithm, which means that when the reliability value reaches a steady state and the algorithm can stop. Experimental results reveal that the exposure level defined in ISO 26262 drops from E3 to E1 after using SSSE, and such improvement enables a safety guarantee of higher level.

Index Terms—Distributed automotive embedded systems, safety enhancement, stable stopping

1 INTRODUCTION

1.1 Background

SAFE driving has been the eternal theme of automobiles since their invention. Various safety components, such as safety belt, airbag, and brake-by-wire, have been developed for automobiles to enhance their safety. The 1st edition of the functional safety standard ISO 26262 for road vehicles was officially released in Nov. 2011 to enable different automotive software developers to follow the same principles of safety development [1], [2], [3]. Currently, the 2nd edition of the ISO 26262 standard has been released in Dec. 2018 to further strengthen safety development [4]. Functional safety refers to the absence of unreasonable risks due to hazards caused by the malfunctioning behavior of Electrical and Electronic (E/E) systems according to the definition in ISO 26262 [1], [4]. Safety issues run through the entire life cycle of automotive development, such that the maximum possible safety value should be known during the early design phase to help control risk in the actual development process. Safety enhancement (i.e., safety mechanisms for error handling) is extremely desirable for risk control.

Risk refers to the probability of occurrence of harm and the severity of that harm [1]; hence, safety can be enhanced by reducing risk through reducing the probability or the severity of harm or both. In ISO 26262, severity refers to the measure of the extent of harm to an individual in a specific situation [1], and it cannot be changed for a specific automotive application because it is determined by the nature of the harm. Therefore, risk can only be reduced by reducing the probability of occurrence of harm. In ISO 26262, the probability of occurrence of harm is represented by exposure [1]. Reliability is often associated with random hardware failures, and it is used to represent the probability of the non-occurrence of harm (i.e., reliability = 1 - exposure) in automotive safety issues [5].

1.2 Motivation

With the increasing distribution and complexity of distributed automotive embedded systems, safety-critical automotive applications such as brake-by-wire application [6], [7] and vehicle cruiser control application [8] are parallel applications, where some tasks can be performed simultaneously on different Electronic Control Units (ECUs) in parallel, thereby reducing end-to-end response time. Recently, Directed Acyclic Graph (DAG) has been used to represent the above parallel automotive applications in some works [7], [8] (details about parallel automotive application modeling can be found in Section 3).

A real-time application must guarantee correct response within a specified time constraint (i.e., deadline) [9]. In automotive embedded systems, many safety-critical applications must be real-time, such as anti-lock brake [10]; If the
anti-lock brake cannot be finished within its deadline, vehicle collisions may happen due to the delay of the brake action output, thereby causing harm or injury to people (including drivers, passengers, and pedestrians) or damage to automobiles and roads. Violating the real-time constraint (i.e., deadline) is one of systematic failures, which may result in malfunctioning behaviors [5]. That is, a safety-critical automotive application must be completed correctly within its deadline, (i.e., guaranteeing the real-time constraint) [5]; otherwise, the probability of occurrence of harm is considered to be 100 percent. Therefore, the essence of safety enhancement is to enhance the reliability of the automotive application while guaranteeing its real-time constraint.

Ref. [5] studied the problem of enhancing the safety of a real-time parallel automotive application by presenting the Reliability Enhancement Technique (RET). RET is a Backward Safety Enhancement (BSE) approach because it tries to migrate each task to another ECU that generates maximum reliability value from the exit task to the entry task (i.e., backward recovery) [details about BSE can be found in Section 4.1]). However, merely using BSE is insufficient to enhance safety due to the following reasons.

1) BSE merely enhances reliability through backward recovery (backward means that the recovery process is from exit to entry tasks), and it does not apply forward recovery (forward means that the recovery process is from entry to exit tasks). For an end-to-end parallel automotive application, applying forward-and-backward recovery to enhance safety could be efficient.

2) BSE is a non-repeated approach, thereby severely limiting the strength of safety enhancement. In fact, safety enhancement can be implemented through repeated recovery, which is an effective fault tolerance measure that uses redundancy. Particularly, the 2nd edition of ISO 26262 introduces the concept of fault tolerance, which means the ability to deliver a specified application in the presence of one or more faults [4].

1.3 Main Contributions
The development life cycle of safety-critical automotive applications includes analysis (concept), design, implementation, and running phases [5]. This study focuses on safety enhancement for a real-time parallel automotive application by using fault tolerance measure during the design phase. The novel contributions of this study include:

1) Considering that BSE in Ref. [5] is merely a backward recovery, we propose the Forward Safety Enhancement (FSE) algorithm (Algorithm 1). Different from BSE that it handles the recovery process from exit to entry tasks, FSE handles the recovery process from entry to exit tasks. FSE is a novel contribution as it tries to forward reallocate each task to another ECU that can generate maximum reliability value without violating given constraints. The forward-and-backward recovery is then proposed to further enhance safety by combining known BSE and proposed FSE algorithms.

2) Considering that BSE and FSE are merely non-repeated algorithms, we proposed the Repeated BSE (RBSE) algorithm (Algorithm 2) and Repeated FSE (RFSE) algorithm (Algorithm 3). Different from BSE and FSE that they handle the recovery process without repetition, RBSE and RFSE handle the recovery process through primary-backup repetition. RBSE (or RFSE) is a novel contribution as it tries to backward (or forward) add a new replica for each task to an available ECU that can generate maximum reliability value among all available ECUs without violating given constraints. The forward-and-backward recovery through primary-backup repetition is then proposed to further enhance safety by combining proposed RBSE and RFSE algorithms.

3) Considering that RBSE and RFSE could be invoked repeatedly until reaching a stable safety value, the Stable Stopping-based Safety Enhancement (SSSE) approach is proposed by combining the above four algorithms. In other words, SSSE is basically a combination of known BSE algorithm and proposed FSE, RBSE, and RFSE algorithms. The stable stopping (i.e., SSSE) approach is a convergence algorithm, which means that when the reliability value reaches a steady state, the algorithm can stop. SSSE is a novel contribution because it is a new combined approach. There are four algorithms to combine the SSSE approach, where we propose there new algorithms.

2 RELATED RESEARCH
The objective of this study is to enhance the safety for real-time parallel automotive applications through considering two safety properties, namely, reliability and response time. Therefore, this study mainly reviews existing research on the reliability and response time of DAG-based applications.

1) Bi-Criteria (Bi-Objective) Optimization Between Response Time and Reliability. Simultaneously minimizing response time and maximizing reliability (i.e., minimizing exposure) is a bi-criteria optimization problem [11], [12], [13]. Ref. [11] provided the NP-hard complexity results and the optimal mapping algorithm for the bi-objective optimization problem under different variants of multiprocessors, including homogeneous and heterogeneous speeds. Ref. [12] proposed meta-heuristic algorithm to solve the above bi-objective optimization problem for a parallel application while meeting the user-defined budget. Ref. [13] proposed a bi-objective algorithm for a parallel application based on Wind Driven Optimization (WDO) to implement the trade-off between minimizing response time and maximizing reliability.

2) Response Time Optimization With Reliability Constraint. Ref. [5] presented an approach to minimize the response time of an automotive application while guaranteeing the reliability constraint without primary-backup repetition. Refs. [14], [15] presented the MaxRe and RR algorithms to minimize the resource cost of a DAG-based application while guaranteeing the reliability constraint through primary-backup repetition. To effectively assure the reliability requirement, Ref. [16] proposed reliability pre-allocation technique based on geometric mean, which makes the preallocated reliability values closer to the center. Particularly,
non-repeated and repeated approaches are both proposed in Refs. [14], [15], [16].

(3) Reliability Optimization With Real-Time Constraint. Ref. [5] first studied the problem of enhancing the safety for an automotive application while guaranteeing its real-time constraint by presenting the BSE algorithm. As explained in Section 1, BSE is merely a backward recovery approach toward safety enhancement and is a non-repeated approach, thereby severely limiting the strength of safety enhancement. In other words, BSE is insufficient to enhance safety. This study aims to propose a novel safety enhancement approach by introducing backward-and-forward recovery and primary-backup repetition for parallel automotive applications.

3 MODELS

Readers can refer to Tables 1 and 2 for main abbreviation and notation, respectively, used in this study.

3.1 Application Model

Fig. 1 shows a simple execution process of the brake-by-wire application [6], which is represented by a DAG, in a Controller Area Network (CAN)-based distributed automotive embedded system. For this parallel application, ECU \( u_1 \) receives the data from the sensor to trigger the entry task \( n_1 \), which is executed in \( u_1 \). \( n_1 \) finishes its execution and sends message \( m_{1,2} \) to \( n_2 \) executed in ECU \( u_4 \). Notice that \( m_{1,2} \) is transmitted in the CAN bus.

Let \( U = \{u_1, u_2, \ldots, u_{|U|}\} \) be a heterogeneous ECU set in the system. In this study, we uniformly use \( |X| \) to represent the size of the set \( X \). Meanwhile, a motivational automotive application represented by a DAG \( G = (N, W, M, C) \) is shown in Fig. 2.

1) Let \( n_i \) be the \( i \)th task of \( G \). \( \text{pred}(n_i) \) and \( \text{succ}(n_i) \) represent the immediate predecessor task set and immediate successor task set of \( n_i \), respectively. For instance, there are \( \text{pred}(n_8) = \{n_2, n_3, n_6\} \) and \( \text{succ}(n_8) = \{n_{10}\} \) in Fig. 2. The entry and exit tasks are denoted by \( n_{\text{entry}} \) and \( n_{\text{exit}} \), respectively. For the application in Fig. 2, there are \( n_{\text{entry}} = n_1 \) and \( n_{\text{exit}} = n_{10} \). The task model should support time-triggered and event-triggered paradigms according to the timing analysis of AUTO-SAR standard [17], and the event-triggered task model (i.e., a task is released only if it receives the data from all its predecessor tasks) is considered in this study. Let \( D(G) \) be the deadline of automotive application \( G \), and assume that the period of \( G \) is not less than \( D(G) \). Meanwhile, the application and all the tasks share the same deadline and period.

2) \( W \) represents a \( |N| \times |U| \) matrix, where \( w_{i,k} \) denotes the Worst Case Execution Time (WCET) of \( n_i \) executed in ECU \( u_k \). \( w_{i,k} \) is infinity (i.e., \( w_{i,k} = +\infty \)) if \( n_i \) cannot be allocated to \( u_k \) because some ECUs cannot only execute specific tasks. The WCETs of each task in three ECUs \( \{u_1, u_2, u_3\} \) are listed in Table 3. The

### Table 1

**Main Abbreviations in This Article**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>RET</td>
<td>Reliability Enhancement Technique</td>
</tr>
<tr>
<td>BSE</td>
<td>Backward Safety Enhancement</td>
</tr>
<tr>
<td>RBSE</td>
<td>Repeated BSE</td>
</tr>
<tr>
<td>FSE</td>
<td>Forward Safety Enhancement</td>
</tr>
<tr>
<td>RFSE</td>
<td>Repeated FSE</td>
</tr>
<tr>
<td>SSSSE</td>
<td>Stable Stopping-based Safety Enhancement</td>
</tr>
<tr>
<td>WCET</td>
<td>Worst Case Execution Time</td>
</tr>
<tr>
<td>WCRT</td>
<td>Worst Case Response Time</td>
</tr>
<tr>
<td>ECU</td>
<td>Electronic Control Unit</td>
</tr>
<tr>
<td>DAG</td>
<td>Directed Acyclic Graph</td>
</tr>
<tr>
<td>EST</td>
<td>Earliest Start Time</td>
</tr>
<tr>
<td>EFT</td>
<td>Earliest Finish Time</td>
</tr>
<tr>
<td>LFT</td>
<td>Latest Finish Time</td>
</tr>
<tr>
<td>AST</td>
<td>Actual Start Time</td>
</tr>
<tr>
<td>AFT</td>
<td>Actual Finish Time</td>
</tr>
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### Table 2

**Main Notations in This Article**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_i )</td>
<td>A computing task in an automotive application</td>
</tr>
<tr>
<td>( m_{i,j} )</td>
<td>A CAN message from tasks ( n_i ) to ( n_j )</td>
</tr>
<tr>
<td>( w_{i,k} )</td>
<td>WCET of task ( n_i ) executed in ECU ( u_k )</td>
</tr>
<tr>
<td>( c_{i,j} )</td>
<td>WCRT of message ( m_{i,j} )</td>
</tr>
<tr>
<td>( \lambda_k )</td>
<td>Failure rate for ECU ( u_k )</td>
</tr>
<tr>
<td>( R(n_i, u_k) )</td>
<td>Reliability of task ( n_i ) executed in ECU ( u_k )</td>
</tr>
<tr>
<td>( u_{\text{allocated}}(n_k) )</td>
<td>Assigned ECU for task ( n_k )</td>
</tr>
<tr>
<td>( R(G) )</td>
<td>Reliability of application ( G )</td>
</tr>
<tr>
<td>( LB(G) )</td>
<td>Lower bound of application ( G )</td>
</tr>
<tr>
<td>( D(G) )</td>
<td>Real-time constraint of application ( G )</td>
</tr>
<tr>
<td>( EST(n_i, u_k) )</td>
<td>Earliest start time of task ( n_i ) executed in ECU ( u_k )</td>
</tr>
<tr>
<td>( EFT(n_i, u_k) )</td>
<td>Earliest finish time of task ( n_i ) executed in ECU ( u_k )</td>
</tr>
<tr>
<td>( LFT(n_i, u_k) )</td>
<td>Latest finish time of task ( n_i ) executed in ECU ( u_k )</td>
</tr>
<tr>
<td>( AST(n_i) )</td>
<td>Actual start time of task ( n_i )</td>
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<tr>
<td>( AFT(n_i) )</td>
<td>Actual finish time of task ( n_i )</td>
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<td>( avaBT(n_i, u_k) )</td>
<td>Available begin time of task ( n_i ) executed in ECU ( u_k )</td>
</tr>
<tr>
<td>( avaET(n_i, u_k) )</td>
<td>Available end time of task ( n_i ) executed in ECU ( u_k )</td>
</tr>
</tbody>
</table>
TABLE 3
WCETs of Tasks on Different ECUs
of the Motivational Automotive
Application in Fig. 2 [5]

<table>
<thead>
<tr>
<th>Task</th>
<th>(u_1)</th>
<th>(u_2)</th>
<th>(u_3)</th>
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</thead>
<tbody>
<tr>
<td>(n_1)</td>
<td>14</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>(n_2)</td>
<td>13</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>(n_3)</td>
<td>11</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>(n_4)</td>
<td>13</td>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>(n_5)</td>
<td>12</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>(n_6)</td>
<td>13</td>
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<td>9</td>
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<td>(n_7)</td>
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<td>15</td>
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<tr>
<td>(n_8)</td>
<td>5</td>
<td>11</td>
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</tr>
<tr>
<td>(n_9)</td>
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<td>12</td>
<td>20</td>
</tr>
<tr>
<td>(n_{10})</td>
<td>21</td>
<td>7</td>
<td>16</td>
</tr>
</tbody>
</table>

weight 9 of \(n_1, u_3\) in Table 3 represents the WCET of \(n_1\) in \(u_3\) (i.e., \(w_{13} = 9\)).

3) Let \(c_{ij}\) be the Worst Case Response Time (WCRT) of message \(m_{ij}\), which is a CAN message from tasks \(n_i\) to \(n_j\). Considering that the WCRT analysis generally involves a tight WCRT upper bound within a pseudo-polynomial computational time, the WCRTs in this study are the theoretical upper bounds [18].

4) The non-preemptive scheduling for ECUs is adopted to keep consistent with CAN buses. The motivational automotive application in Fig. 2 is used as an explanation of the proposed algorithms. For simplicity, the units of all parameters are ignored in the example.

3.2 Reliability Model

Random hardware failures occur unpredictably during the lifetime of a hardware element, but random hardware failure rates can be reasonably predicted because random hardware faults (including permanent and transient faults) occur based on a probabilistic distribution. [1]. This study considers transient faults (e.g., single bit faults in case of an Error Correcting Code (ECC) with single error correction, double error detection capability as pointed in the 2nd edition of the ISO 26262 standard [4]), which usually obey the Poisson distribution [5], [14], [15], [19], [20]. The reliability of \(n_i\) executed in \(u_k\) in its WCET is calculated by

\[
R(n_i, u_k) = e^{-\lambda w_{ik}},
\]

where \(\lambda\) is the failure rate of an ECU.

In general, the CAN link’s failure rate (about \(10^{-9}\)) is much less than ECU’s failure rate (about \(10^{-6}\)) due to the usage of CRC and ACK in CAN link [21], [22]. Therefore, communication faults can be disregarded when considering the ECU faults. Thus, the reliability of the application is the product of all its tasks [5], [14], [15], [19], [20]:

\[
R(G) = \prod_{n_i \in N} R(n_i) = \prod_{n_i \in N} R(n_i, u_{allo(n_i)}),
\]

where \(u_{allo(n_i)}\) represents the allocated ECU of \(n_i\).

3.3 Lower Bound of Application

Definition 1 (Lower Bound). The lower bound means the minimum response time of an application without any constraints.

Scheduling tasks with minimum response time in multi-processors (ECUs) is known to be an NP-hard optimization problem [23], [24]. Obtaining approximate lower bound is a fast process. The Heterogeneous Earliest Finish Time (HEFT) [23] and Optimistic Cost Table (OCT) [24] algorithms are two typical list scheduling algorithm to obtain approximate lower bounds. This study adopts HEFT to get the lower bound of the application.

There are two phases in HEFT: 1) tasks in the application are sorted in ascending order by the upward rank values (task prioritization phase); 2) each task is allocated to the ECU that has the minimum EFT combining the insertion strategy according to the task prioritization standard (task allocation phase). Ref. [23] provided enough details about how to obtain the lower bound of the motivational application in Fig. 2 by using HEFT.

1) The task prioritization is based on the descending order of \(\text{rank}_\text{ET}\), which is calculated by [23]

\[
\text{rank}_\text{ET}(n_i) = \bar{w} + \max_{u_j \in \text{succ}(n_i)} \{c_{ij} + \text{rank}_\text{ET}(n_j)\},
\]

where \(\bar{w}\) represents the average WCET of task \(n_i\). In the motivational example of Fig. 2, the task prioritization is organized by \(n_1\) (\(\text{rank}_\text{ET}(n_1) = 108\)), \(n_3\) (\(\text{rank}_\text{ET}(n_3) = 80\)), \(n_4\) (\(\text{rank}_\text{ET}(n_4) = 80\)), \(n_2\) (\(\text{rank}_\text{ET}(n_2) = 77\)), \(n_5\) (\(\text{rank}_\text{ET}(n_5) = 69\)), \(n_6\) (\(\text{rank}_\text{ET}(n_6) = 63.3\)), \(n_9\) (\(\text{rank}_\text{ET}(n_9) = 44.3\)), \(n_7\) (\(\text{rank}_\text{ET}(n_7) = 42.7\)), \(n_8\) (\(\text{rank}_\text{ET}(n_8) = 35.7\)), and \(n_{10}\) (\(\text{rank}_\text{ET}(n_{10}) = 14.7\)) using the HEFT algorithm [23].

2) The task allocation can get the lower bound of the application. Let \(EFT(n_i, u_k)\) be the Earliest Finish Time (EFT) of task \(n_i\) executed in ECU \(u_k\). The minimum EFT of the exit task should be the lower bound of the application:

\[
LB(G) = \min_{u_k \in U} \{EFT(n_{\text{exit}}, u_k)\}.
\]

Therefore, we can iteratively get the minimum EFT of each task from the entry to the exit tasks. We can get the EFT of each task based on

\[
EFT(n_i, u_k) = EST(n_i, u_k) + w_{ik},
\]

where \(EST(n_i, u_k)\) represents the Earliest Start Time (EST) of task \(n_i\) executed in ECU \(u_k\):

\[
\begin{cases}
EST(u_{\text{entry}}, u_k) = 0; \\
EST(n_i, u_k) = \max \{\text{avaBT}(n_i, u_k), \max_{n_j \in \text{pred}(n_i)} \{AFT(n_j) + c_{i,j}^{p,k}\}\},
\end{cases}
\]

(3)

\(\text{avaBT}(n_i, u_k)\) is the available begin time of \(u_k\). \(n_i\) is an immediate predecessor task of \(n_i\). \(AFT(n_k)\) is the Actual Finish Time (AFT) of task \(n_k\). \(n_i\) has different \(EST(n_i, u_k)\) values depending on the allocations of predecessor tasks because \(c_{i,k}^{p,k}\) is not a fixed value.

\[
c_{i,k}^{p,k} = \begin{cases} 0, & p \neq k; \\
\infty, & p = k. 
\end{cases}
\]

Table 4 shows the details of obtaining lower bound of the motivational example. First, \(n_1\) is allocated to \(u_3\) (denoted with bold text) as it has the minimum EFT of 9. Then, \(n_3\) is
allocated to \( u_3 \) (denoted with bold text) as it has the minimum EFT of 28. The EFTs of \( n_1, n_2, n_5, n_6, \) and \( n_8 \) are shown in Table 4. Finally, the lower bound of application is 80, as the AFT of the exit task (i.e., \( n_{10} \)) is 80. On the basis of Table 4, Fig. 3 shows the HEFT-generated lower bound (\( LB(G) = 80 \)) of the motivational automotive application \( G \) in Fig. 2, whereas the arrows between tasks represent their communication.

The real-time constraint \( D(G) \) must be larger than or equal to \( LB(G) \). For the motivational automotive application in Fig. 2, we let the real-time constraint be \( D(G) = 100 \), as shown in Fig. 3.

### 3.4 Problem Statement and Two Constraints

Considering that the essence of safety enhancement is to enhance the reliability of the automotive application while guaranteeing its real-time constraint as explained in Section 1.2, the problem to be solved in this study is to enhance the reliability of the parallel automotive application \( G \)

\[
R(G) = \prod_{n_i \in N} R(n_i),
\]

towards enhancing the safety by task allocations, under its real-time constraint

\[
RT(G) \leq D(G),
\]

where \( RT(G) \) represents the response time of application \( G \). The problem statement in fact involves two constraints.

**Definition 2 (Constraint 1).** The precedence constraints between the current task (i.e., \( n_i \)) and its immediate predecessor and immediate successor tasks, namely, the Actual Start Time (AST) and AFT of \( n_i \) must adhere to the data dependencies with other tasks.

1) The backward sequence of tasks is sorted by the descending order of the AFT values generated by HEFT. For example, the backward sequence of tasks in Fig. 3 is \( n_{10}, n_9, n_8, n_7, n_6, n_2, n_5, n_4, n_3, n_1 \), and \( n_1 \).

2) The exit task \( n_{10} \) is first considered. \( n_{10} \) has moved its end time to 100 (i.e., \( D(G) = 100 \)) in fixed ECU \( u_2 \) without violating Constraint 2, as shown in Fig. 4. We assume that \( n_{10}, n_9, n_8, \) and \( n_7 \) have been re-allocated using BSE, as shown in Fig. 4. Note that \( n_{10}, n_9, n_8, \) and \( n_7 \) are only rescheduled on the same ECU without migration. ECU migration is avoided due to one of the following reasons: 1) the current task \( n_i \) cannot be migrated to another ECU otherwise Constraint 1 will be violated; 2) the current task \( n_i \) can be migrated to the other ECU without violating Constraint 1, but the new ECU cannot obtain a higher reliability value than the current ECU for \( n_i \).

3) The fifth task \( n_5 \) is prepared to be re-allocated. \( n_6 \) (denoted with red color) can be migrated from \( u_2 \) to \( u_3 \) because it can generate maximum reliability for \( n_5 \) without violating Constraint 1, as shown in Fig. 4.

4) The following tasks use the same principle as the aforementioned tasks. As shown in Fig. 5, \( n_2, n_5, n_4, \) and \( n_1 \) are only moved in fixed ECUs without migration, whereas \( n_3 \) (denoted with red color) is migrated from \( u_3 \) to \( u_2 \). Finally, the reliability of the automotive application is enhanced from 0.9477163 (Fig. 3).
to 0.95294318 (Fig. 5) by using RET to migrate \( n_6 \) and \( n_3 \). Fig. 5 shows that \textit{Constraint 1} and \textit{Constraint 2} are not violated. In other words, the following facts are found: 1) each task adheres to the data dependencies with other tasks; 2) the application is finished within the deadline.

### 4.2 Proposed FSE Algorithm

Fig. 5 shows that the begin time of the entry task \( n_1 \) is not started at 0 but at 8 after using BSE. An intuitive feeling is that the begin time of \( n_1 \) can be set to 0 through making some possible migrations of \( n_1 \) and its successor tasks to enhance safety without violating \textit{Constraint 1} and \textit{Constraint 2}. The above process is explored from the entry task to the exit task (i.e., forward recovery).

1. The forward sequence of tasks is sorted by the ascending order of AST values generated by BSE. For example, the forward sequence of tasks in Fig. 5 is \( n_1, n_3, n_4, n_5, n_2, n_6, n_7, n_8, n_9, \) and \( n_{10} \).

2. The entry task \( n_1 \) is first considered. \( n_1 \) has moved its start time and end time to 0 and 8, respectively, in fixed ECU \( u_3 \) without migration. The reason is that if \( n_1 \) can be migrated, it has already been migrated with the BSE algorithm, as shown in Fig. 5.

3. \( n_1, n_3, n_4, \) and \( n_5 \) have moved their individual begin times and end times to smaller values in fixed ECUs without migration, as shown in Fig. 6.

4. The fifth task \( n_2 \) is prepared to be re-assigned. \( n_2 \) can be migrated from \( u_3 \) to \( u_1 \) because it can generate maximum reliability for \( n_2 \) without violating \textit{Constraint 1}, as shown in Fig. 6. \( n_2 \) cannot be migrated by using BSE. However, \( n_2 \) can be migrated when using FSE further. In the following, the migration of \( n_2 \) will be explained.

To determine whether the current task \( n_i \) can be migrated, its \( EST \) and Latest Finish Time (LFT) in each ECU are required to obtain in advance to ensure that \textit{Constraint 1} and \textit{Constraint 2} are not violated. The \( EST \) of \( n_i \) in \( u_k \) has been shown in Eq. (3), whereas the LFT of \( n_i \) in \( u_k \) is calculated by

\[
\begin{align*}
LFT(n_i, u_k) &= D(G); \\
LFT(n_i, u_k) &= \min\{ avaET(n_i, u_k), \min_{n_j \in \text{successor}(n_i)} (\text{AST}(n_j) - c_{i,j}) \},
\end{align*}
\]

where \( c_{i,j} \) is the available end time of \( u_k \) for \( n_j \). \( n_i \) has different \( LFT(n_i, u_k) \) values depending on the allocations of successor tasks because \( c_{i,j} \) is not a fixed value.

Take \( n_2 \) as an example. The \( avaBT(n_2, u_k) \) and \( avaET(n_2, u_k) \) values are as follows:

\[
\begin{align*}
avaBT(n_2, u_1) &= 0 \\
avaBT(n_2, u_2) &= 42 \\
avaBT(n_2, u_3) &= 19
\end{align*}
\]

Then, the \( EST(n_2, u_k) \) and \( LFT(n_2, u_k) \) values are as follows:

\[
\begin{align*}
EST(n_2, u_1) &= 27 \\
EST(n_2, u_2) &= 42 \\
EST(n_2, u_3) &= 19
\end{align*}
\]

After \textit{Constraint 1} and \textit{Constraint 2} are determined, the \( R(n_i) = R(n_i, u_{\text{allocated}}) = \max_{w_i,k \leq (LFT(n_i, u_k) - EST(n_i, u_k))} R(n_i, u_k) \), where \( w_i,k \leq (LFT(n_i, u_k) - EST(n_i, u_k)) \) should be satisfied to guarantee \textit{Constraint 1} is not violated. Note that \( u_{\text{allocated}} \) can be the currently allocated ECU \( u_{\text{allocated}} \) to be moved in or a reallocated ECU \( u_{\text{new}} \) to be migrated to.

Continuing with \( n_2 \) as an example. The \( R(n_2, u_k) \) values are as follows:

\[
\begin{align*}
R(n_2, u_1) &= 0.99094128 \\
R(n_2, u_2) &= \text{NULL} \\
R(n_2, u_3) &= 0.99282586
\end{align*}
\]

5) The following tasks use the same principle as the aforementioned tasks. As shown in Fig. 7, \( n_6, n_7, n_8, n_9, \) and \( n_{10} \) are moved in fixed ECUs without migration. Finally, the reliability of the automotive application is enhanced from...
0.95294318 (Fig. 5) to 0.95475549 (Fig. 7) by using FSE to migrate $n_2$. Fig. 7 shows that Constraint 1 and Constraint 2 are not still violated.

The FSE algorithm is proposed on the basis of the aforementioned analysis, as shown in Algorithm 1.

**Algorithm 1. The FSE Algorithm**

**Input:** $U = \{u_1, u_2, \ldots, u_{|U|}\}$, $G$, $D(G)$, task mapping generated by the previous algorithm.

**Output:** $R(G)$, $RT(G)$, and task mapping

1. The tasks’ forward sequence $forward\_seq$ is also sorted by the ascending order of the AST values generated by the previous algorithm;
2. while (there are tasks in $forward\_seq$) do
   3. $n_i \leftarrow forward\_seq.out();$
   4. $u_{cur}(i)$ indicates the currently allocated ECU.
   5. Move the $n_i$ in ECU $u_{cur}(i)$ by $AST(n_i) \leftarrow EST(n_i, u_{cur}(i))$
      and $AFT(n_i) \leftarrow (AST(n_i) + w_{cur}(i));$
   6. for (each ECU $u_k \in U$) do
      7. Calculate $EST(n_i, u_k)$ using Eq. (3);
      8. Calculate $LFT(n_i, u_k)$ using Eq. (4);
      9. if $(((LFT(n_i, u_k) - EST(n_i, u_k)) < w_{i,k})$ then
         10. continue;
      11. end if
      12. end for
      13. Select the ECU $u_{new}(i)$ with the maximum reliability value
         under $R(n_i, u_{new}(i)) > R(n_i, u_{new}(i))$
      14. if ($u_{new}(i)$ is not NULL) then
         15. Migrate the $n_i$ to ECU $u_{new}(i)$ by $AST(n_i) \leftarrow EST(n_i, u_{new}(i))$
            and $AFT(n_i) \leftarrow (AST(n_i) + w_{new}(i));$
         16. end if
   17. end while
   18. Calculate the reliability $R(G)$ using Eq. (2);

Overall, FSE tries to forward reallocate each task to another ECU that can generate maximum reliability value without violating Constraint 1 and Constraint 2. The details are explained as follows. The time complexity of FSE is $O(|N|^2 \times |U|)$ and is explained below. (1) Traversing all tasks requires $O(|N|)$ time (Lines 2–18). (2) Calculating $EST(n_i, u_k)$ and $LFT(n_i, u_k)$ requires $O(|N| \times |U|)$ time (Lines 6–11). Therefore, the time complexity of the FSE algorithm is the same as those of HEFT and BSE.

5 REPEATED SAFETY ENHANCEMENT

5.1 Proposed RBSE Algorithm

Fig. 7 shows that the end time of the exit task $n_{10}$ is not ended at 100 but at 92 after using FSE. An intuitive feeling is that the end time of $n_{10}$ can be set to 100 through making some possible migrations of tasks by BSE again. Setting the end time of $n_{10}$ to 100 is necessary, but making possible migrations of tasks by BSE again is unfeasible. The reason is that through the backward-and-forward recovery of BSE and FSE in Section 4, the migration of tasks has reached the extreme. Continually invoking BSE or FSE recovery cannot generate new migrations. Fortunately, primary-backup repetition is an effective fault tolerant measure (i.e., recovery through repetition) to implement safety enhancement.

Passive repetition and active repetition are two types of primary-backup repetition paradigms [14], [15]. Passive repetition aims to reschedule the task on a backup ECU when this task fails in the primary ECU. Active repetition will simultaneously execute $\varepsilon$ replicas of the task in $\varepsilon$ ECUs. Each ECU only execute one replica for the same task, such that the maximum number of replicas is the number of ECU. Homogenous redundancy and heterogeneous redundancy are two types of redundancy paradigms [4]. Homogeneous redundancy means the duplication of homogeneous elements, whereas heterogeneous redundancy means the combination of hardware devices and software tasks. In this study, active repetition and homogeneous redundancy are adopted because such combination implements independence among tasks. In addition, as pointed out in the 2nd edition of ISO 26626, homogeneous redundancy during the design phase focuses primarily on controlling the effects of transient faults or random faults in the hardware (i.e., ECU in this study), on which a similar software is executed (e.g., temporal redundant execution of software). The aforementioned features are consistent with the automotive application and reliability models of this study.

Given that the reliability of $n_i$ in $u_k$ is $R(n_i, u_k) = e^{-\lambda u_k}\nu_i,$ according to Eq. (1), the failure probability of $n_i$ in $u_k$ without repetition is

$\text{Fail}(n_i, u_k) = 1 - R(n_i, u_k) = 1 - e^{-\lambda u_k}\nu_i.$

Assume that there are $num_i$ ($num_i \leq |U|$) replicas for $n_i$, the failure probability of $n_i$ through active repetition is

$\text{Fail}(n_i) = \prod_{\beta=1}^{num_i} \text{Fail}\left(n_i^\beta, u_{allo}(n_i^\beta)\right) = \prod_{\beta=1}^{num_i} \left(1 - R\left(n_i^\beta, u_{allo}(n_i^\beta)\right)\right),$

where $u_{allo}(n_i^\beta)$ represents the allocated ECU of replica $n_i^\beta$.

Therefore, the reliability of $n_i$ is

$R(n_i) = 1 - \text{Fail}(n_i) = 1 - \prod_{\beta=1}^{num_i} \left(1 - R\left(n_i^\beta, u_{allo}(n_i^\beta)\right)\right).$ (6)

(1) The backward sequence of tasks is sorted by the descending order of the AFT values. For example, the backward sequence of the tasks in Fig. 7 is $n_{10}, n_8, n_7, n_9, n_6, n_4, n_2, n_3, n_5, n_{10}$ and $n_1$.
(2) The exit task $n_{10}$ is first considered. $n_{10}$ just moves its begin time and end time to 93 and 100, respectively, in fixed ECU $u_2$ without any repetition. Repetitions on $u_1$ or $u_3$ will violate Constraint 1. The details are explained below. The EST and LFT of $n_{10}$ are
The LFT values of \( n_{10} \) on \( u_1 \) and \( u_2 \) are
\[
LFT(n_{10}, u_1) - EST(n_{10}, u_1) = 100 - 85 = 15 < w_{10,1} = 21,
\]
and
\[
LFT(n_{10}, u_3) - EST(n_{10}, u_3) = 100 - 85 = 15 < w_{10,3} = 16;
\]
hence, neither \( u_1 \) nor \( u_3 \) can be added with replicas.

The EST and LFT values of \( n_8 \) are
\[
\begin{align*}
EST(n_8, u_1) &= 69 \\
EST(n_8, u_2) &= 65 \\
EST(n_8, u_3) &= 69 \\
LFT(n_8, u_1) &= 82 \\
LFT(n_8, u_2) &= 93 \\
LFT(n_8, u_3) &= 82.
\end{align*}
\]

The LFT values of \( n_8 \) on \( u_2 \) and \( u_3 \) are
\[
LFT(n_8, u_2) - EST(n_8, u_2) = 93 - 65 = 28 > w_{8,2} = 11,
\]
and
\[
LFT(n_8, u_3) - EST(n_8, u_3) = 82 - 69 = 13 < w_{10,3} = 14;
\]
hence, a replica can be added to \( u_3 \), whereas a replica cannot be added to \( u_2 \) for \( n_{10} \), as shown in Fig. 8.

The RBSE algorithm is proposed on the basis of the aforementioned analysis, as shown in Algorithm 2.

Overall, RBSE tries to backward add a new replica for each task to an available ECU that can generate maximum reliability value among all available ECUs without violating the Constraint 1 and Constraint 2. The time complexity of RBSE is also \( O(|N|^2 \times |U|) \), which is the same as those of HEFT and BSE.

**Algorithm 2. The RBSE Algorithm**

**Input:** \( U = \{u_1, u_2, \ldots, u_{|U|}\} \), \( G, D(G) \), task mapping generated by the previous algorithm

**Output:** \( R(G), RT(G) \), and task mapping

1. The tasks’ backward sequence \( backward\_seq \) is also sorted by the descending order of the AFT values generated by the previous algorithm;
2. while (there are tasks in \( backward\_seq \)) do
3. \( n_i \leftarrow backward\_seq.out() \);
4. \( U_{cur} \) indicates that the ECU set has been allocated;
5. for (each ECU \( u_{cur(i)} \in U_{cur}() \) do
6. Move the \( n_i \) in ECU \( u_{cur(i)} \) by \( AFT(n_i) \leftarrow LFT(n_i, u_{cur(i)}) \) and \( AST(n_i) \leftarrow (AFT(n_i) - w_{i,k}) \);
7. end for
8. for (each ECU \( u \in (U - U_{cur}) \) do
9. Calculate \( EST(n_i, u) \) using Eq. (3);
10. Calculate \( LFT(n_i, u) \) using Eq. (4);
11. if ((\( LFT(n_i, u) - EST(n_i, u) \) < \( w_{i,k} \)) then
12. continue;
13. end if
14. Calculate \( R(n_i, u) \) using Eq. (1);
15. end for
16. Select the ECU \( u_{new(i)} \) with the maximum reliability value under \( R(n_i, u_{new(i)}) > R(n_i, u_{cur(i)}) \)
17. if (\( u_{new(i)} \) is not NULL) then
18. Add the replica of \( n_i \) to ECU \( u_{new(i)} \) by \( AFT(n_i) \leftarrow LFT(n_i, u_{new(i)}) \) and \( AST(n_i) \leftarrow (AFT(n_i) - w_{i,new(i)}) \);
19. end if
20. Calculate the reliability \( R(n_i) \) using Eq. (6);
21. end while
22. Calculate the reliability \( R(G) \) using Eq. (2);

### 5.2 Proposed RFSE Algorithm

Similar to the non-repeated FSE, a repeated FSE can be implemented by adding a possible replica in an ECU for each task.

1. Similar to FSE, the forward sequence of tasks is also sorted by the ascending order of the AST values generated by RBSE. For example, the forward sequence of tasks in Fig. 9 is \( n_1, n_3, n_5, n_4, n_2, n_6, n_7, n_9, n_{10} \), and \( n_{10} \).
2. The entry task \( n_1 \) is first handled. The begin time and end time of \( n_1 \) are just moved to 0 and 8, respectively, in fixed ECU \( n_2 \). No additional replicas are added for \( n_3, n_5, n_4, n_2 \) until \( n_6 \); a new replica for \( n_6 \)
is added to $u_1$. That is, $n_6$ has a total of three replicas, as shown in Fig. 10.

3) The tasks $n_1, n_3, n_5, n_4, n_2$, and $n_6$ have been handled in Fig. 10. Similar to already handled tasks, the remaining tasks $n_7, n_9, n_8$, and $n_{10}$ will be handled; however, no additional replicas are added for $n_7, n_9, n_8$, and $n_{10}$ as shown in Fig. 11. Finally, the reliability of the automotive application is enhanced from 0.97584149 (Fig. 9) to 0.97586917 (Fig. 11) by using RFSE to add replicas for $n_6$.

Finally, the RFSE algorithm is proposed and is shown in Algorithm 3.

**Algorithm 3. The RFSE Algorithm**

**Input:** $U = \{u_1, u_2, \ldots, u_{|U|}\}$, $G$, $D(G)$, task mapping generated by the previous algorithm

**Output:** $R(G)$, $RT(G)$, $rep(G)$, and task mapping

1. The tasks' forward sequence $forward_{seq}$ is also sorted by the ascending order of the AST values generated by the previous algorithm;
2. while (there are tasks in $forward_{seq}$) do
3. $n_i \leftarrow forward_{seq}.out()$;
4. $U_{cur}$ indicates that the ECU set has been allocated;
5. for (each ECU $u_{cur(i)} \in (U_{cur})$) do
6. Move the $n_i$ in ECU $u_{cur(i)}$ by $AST(n_i) \leftarrow EST(n_i, u_{cur(i)})$ and $AFT(n_i) \leftarrow (AST(n_i) + w_{i,k})$;
7. end for
8. for (each ECU $u_k \in (U - U_{cur})$) do
9. Calculate $EST(n_i, u_k)$ using Eq. (3);
10. Calculate $LFT(n_i, u_k)$ using Eq. (4);
11. if $((LFT(n_i, u_k) - EST(n_i, u_k)) < w_{i,k})$ then
12. continue;
13. end if
14. Calculate $R(n_i, u_k)$ using Eq. (1);
15. end for
16. Select the ECU $u_{new(i)}$ with the maximum reliability value under $R(n_i, u_{new(i)}) > R(n_i, u_{cur(i)})$;
17. if ($u_{new(i)}$ is not NULL) then
18. Add the replica of $n_i$ to ECU $u_{new(i)}$ by $AST(n_i) \leftarrow EST(n_i, u_{new(i)})$ and $AFT(n_i) \leftarrow (AFT(n_i) + w_{i,new(i)})$;
19. end if
20. Calculate the reliability $R(n_i)$ using Eq. (6);
21. end while
22. Calculate the reliability $R(G)$ using Eq. (2);

**5.3 Proposed SSSE Approach**

After a round of backward-and-forward recovery using RBSE and RFSE, the second round, the third round, and so on can continue until the reliability value reaches a stable and fixed value. We have explained that continually doing non-repeated BSE or FSE recovery can not enhance safety in Section 5.1. Fortunately, invoking repeated RBSE and RFSE recovery could enhance safety by adding a possible new replica to an ECU for each task in each round instead of task migration. The flow chart of SSSE is shown in Fig. 12.

Table 5 shows the reliability enhancement process of the motivational automotive application in Fig. 2 by using related algorithms. BSE, FSE, RBSE (first round), RFSE (first round) gradually enhance the reliability value from 0.94771638 to 0.97586917. 0.97586917 is a stable and fixed value because the same result is obtained using RFSE (first round), RBSE (second round), and RFSE (second round). As the motivational automotive application only includes three ECUs, the second round does not reflect the difference from the first round. The final response time $RT(G)$ is 98, and the number of replicas $rep(G)$ is 16 for the motivational automotive application in Fig. 2. SSSE is proposed by combining the existing HEFT and BSE algorithms and the proposed FSE, RBSE, RFSE algorithms. SSSE can invoke RBSE and RFSE repeatedly until reaching a stable value. Notice that RBSE and RFSE will not always enhance the reliability due to the limited ECUs and strict Constraint 1 and Constraint 2. Therefore, SSSE can stop in the while loop.

It is worth pointing out that SSSE is consistent with the static recovery mechanism in the automotive functional safety standard ISO 26262, where backward recovery, forward recovery, and recovery through repetition have been recommended as static recovery mechanism as pointed in Section 7.4.12, Part 6 of the 2nd edition of ISO 26262 [4].

---

**Table 5**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$R(G)$</th>
<th>$RT(G)$</th>
<th>$D(G)$</th>
<th>$rep(G)$</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEFT</td>
<td>0.94771638</td>
<td>80</td>
<td>100</td>
<td>10</td>
<td>Fig. 3</td>
</tr>
<tr>
<td>BSE</td>
<td>0.95294318</td>
<td>92</td>
<td>100</td>
<td>10</td>
<td>Fig. 5</td>
</tr>
<tr>
<td>FSE</td>
<td>0.95475549</td>
<td>92</td>
<td>100</td>
<td>10</td>
<td>Fig. 7</td>
</tr>
<tr>
<td>RBSE (first round)</td>
<td>0.97584149</td>
<td>98</td>
<td>100</td>
<td>15</td>
<td>Fig. 9</td>
</tr>
<tr>
<td>RFSE (first round)</td>
<td>0.97586917</td>
<td>98</td>
<td>100</td>
<td>16</td>
<td>Fig. 11</td>
</tr>
<tr>
<td>RBSE (second round)</td>
<td>0.97586917</td>
<td>98</td>
<td>100</td>
<td>16</td>
<td>Fig. 11</td>
</tr>
<tr>
<td>RFSE (second round)</td>
<td>0.97586917</td>
<td>98</td>
<td>100</td>
<td>16</td>
<td>Fig. 11</td>
</tr>
</tbody>
</table>
SSSE approach proposed in this study is actually a recovery through repetition based on backward recovery and forward recovery of the static recovery mechanism. Therefore, the special idea of SSSE is the organic combination of the aforementioned three recoveries under the Constraint 1 and Constraint 2. Therefore, the SSSE approach complies with the automotive functional safety standard from a practical perspective. In other words, the stable stopping (i.e., SSSE) approach is a convergence algorithm, which means that when the reliability value reaches a steady state, the algorithm can stop. There is no denying that the proposed SSSE approach has burden or disadvantages in terms of the ECU they have due to the use of repetition.

1) If repetition is not adopted, a task only causes one Message Receiving Interrupt (MRI) to one of its successor task in the ECU receiving its message. For instance, task \( n_8 \) is a successor task of task \( n_6 \) in the motivational automotive application of Fig. 2; after task \( n_6 \) is finished, it only needs to send one message to \( n_8 \), such that \( n_6 \) only causes one MRI to \( n_8 \).

2) If repetition is adopted, a task only causes multiple MRIs to its one successor task in the ECU receiving its message. For instance, after using the SSSE approach, \( n_6 \) causes four message MRIs to \( n_8 \). The details are shown in Fig. 11, where \( n_6 \) has three replicas of \( n_6 \), \( n_6 \), and \( n_6 \), while \( n_8 \) has two replicas of \( n_8 \) and \( n_8 \). In Fig. 11, \( n_6 \) cause four MRIs (denoted with red arrows) to \( n_8 \).

Too many unnecessary MRIs bring considerable ECU load due to the execution of Interrupt Service Routine (ISR) and interrupt-triggered switch overhead between tasks. These disadvantages are not negligible for safety-critical distributed automotive embedded systems, especially for ECUs running at relatively low clock speeds and with small memory space in automotive ECUs [25], [26], [27]. Therefore, although the SSSE approach enhances the reliability of the automotive application, it also brings considerable ECU load that would affect the quality of task scheduling.

6 EXPERIMENTS

The state-of-the-art BSE algorithm is selected to compare with the proposed algorithms. The metrics are response time \( RT(G) \), reliability \( R(G) \), and replica number \( rep(G) \).

As this study focuses on the early design phase, the application parameters are known on the basis of their real deployment. The parameter values of a real automotive are adopted. The failure rate falls in the range of \( 10^{-6} / \mu s - 16 \times 10^{-6} / \mu s \). The WCETs of the tasks and the WCRTs of the messages fall under the range of \( 100 \mu s - 400 \mu s \) generated by uniform distribution. The ECU number is 16.

According to the related description in ISO 26262, fault tolerance is merely a static recovery mechanism during the design phase to maintain the safe state of automotive application, and does not involve the final operational results [4]. Considering that fault tolerance does not address the context where the application is used to switch off the system and the context where a safe state can be directly reached by switching off the application, and not all imaginable faults can be tolerated as pointed out in the 2nd edition of ISO 26262 [4], this study focuses on safety prevention using safety enhancement technique during the design phase, and does not determine the final execution result during the running phase. Therefore, the automotive applications in this study will be tested by simulation to implement safety enhancement in a static recovery manner.

### 6.1 ASIL Determination in ISO 26262

We introduce the Automotive Safety Integrity Level (ASIL) in advance because it is related to the safety evaluation of an automotive application. In ISO 26262, ASIL is a risk classification scheme to define the safety requirements. The ASIL is established by performing a Hazard Analysis and Risk Assessment (HARA) of a potential hazard by evaluating at the severity, exposure and controllability of the automotive operating scenario. An ASIL is the combination of severity, exposure, controllability [1], [28]. There are four ASILs identified by the ISO 26262: ASIL A, ASIL B, ASIL C, ASIL D [1], [4].

<table>
<thead>
<tr>
<th>Severity</th>
<th>Exposure</th>
<th>Controllability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
</tr>
<tr>
<td>S1</td>
<td>E1</td>
<td>QM</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td>QM</td>
</tr>
<tr>
<td></td>
<td>E3</td>
<td>QM</td>
</tr>
<tr>
<td></td>
<td>E4</td>
<td>QM</td>
</tr>
<tr>
<td>S2</td>
<td>E1</td>
<td>QM</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td>QM</td>
</tr>
<tr>
<td></td>
<td>E3</td>
<td>QM</td>
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<tr>
<td></td>
<td>E4</td>
<td>A</td>
</tr>
<tr>
<td>S3</td>
<td>E1</td>
<td>QM</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td>QM</td>
</tr>
<tr>
<td></td>
<td>E3</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>E4</td>
<td>B</td>
</tr>
</tbody>
</table>

ASIL Determination Formed by a Combination of Severity, Exposure, and Controllability Provided in ISO 26262 [4]

Severity means an estimate of the extent of harm that may occur in a potentially hazardous event; there are four severity levels: S0 (no injuries), S1 (light and moderate injuries), S2 (severe and life-threatening injuries), and S3 (fatal injuries). Exposure means the state of being in an operational situation where it is hazardous if the situation is coincident with the failure mode under the analysis; there are five exposure levels: E0 (incredibly), E1 (very low probability), E2 (low probability), E3 (medium probability), and E4 (high probability). Controllability means the ability to avoid specified harm or damage through the timely reactions of the persons involved; there are four controllability levels: C0 (controllable in general), C1 (simply controllable), C2 (normally controllable), and C3 (uncontrollable). Notice that the controllability is related to the driver’s driving state rather than the system.

ISO 26262 provides the ASIL determination formed by a combination of severity, exposure, and controllability, as shown in Table 6 [4]. Each ASIL is the combination of severity, exposure, and controllability values. There is a subtle difference about ASIL determination between the 1st and 2nd editions of the ISO 26262 standard. In the 1st edition,
the combination of S3, E1, and C3 is ASIL A. However, in the 2nd edition, this combination may be Quality Management (QM) if several unlikely situations are combined and the change may result in a lower probability of exposure than E1. QM indicates that software can be developed according to the quality management process to manage the identified risk, and no safety-related design needs to be considered. QM is not an ASIL but may be specified in the HARA [1], [4]. Severity has been decided after HARA and cannot be changed. Controllability is a fixed value during the design phase. Therefore, to enhance safety for a real-time parallel automotive application, the feasible measure is to drop the exposure of the application according to Table 6.

### 6.2 Real-Life Parallel Automotive Application

The real-life parallel automotive application with 31 tasks from Ref. [5] are adopted. This application contains six blocks: engine controller \((n_1-n_{12})\), automatic gear box \((n_{13}-n_{14})\), anti-locking brake \((n_{15}-n_{17})\), wheel angle sensor \((n_{18}-n_{19})\), suspension controller \((n_{20}-n_{24})\), and body work \((n_{25}-n_{31})\).

The HEFT-generated lower bound of the application is 630 μs. As the real-time constraint must be larger than or equal to the lower bound explained in Section 3.3, The real-time constraint is changed from 630 μs to 1430 μs with 200 μs increments. The results are shown in Fig. 13.

1) Fig. 13(a) shows that the response time values using all the algorithms are less than or equal to corresponding real-time constraints. As the HEFT-generated response time is the lower bound, it is fixed at 630 μs. Except that the real-time constraint is equal to the lower bound (i.e., 630 μs), Fig. 13(a) shows the BSE and FSE-generated response time values are less than the real-time constraints; such results indicate that BSE and FSE still leave a larger optimization space for RBSE, RFSE, and SSSE.

2) Fig. 13(b) shows that a long response time could lead to a high reliability, and the results basically confirm that minimizing response time and maximizing reliability is a bi-criteria optima problem. In addition, Fig. 13(b) clearly shows that the reliability values are divided into three gradients: 1) HEFT; 2) BSE and FSE; and 3) RBSE, RFSE, and SSSE. As expected, SSSE generates the maximum reliability values, followed by RFSE, RBSE, FSE, BSE, and HEFT. HEFT has the lowest reliability value of 0.94920261. By backward-and-forward recovery using BSE and FSE, the reliability value is increased to 0.962–0.980. Through further primary-backup repetition using RBSE and RFSE, the reliability values are increased to 0.974–0.998. With the continuous involvement of RBSE and RFSE (i.e., using SSSE), the reliability values can reach a stable state in each case. SSSE only increases a small amount of reliability compared with RFSE because SSSE only invokes RBSE and RFSE twice at most. Even so, as long as the slack response time increases, safety enhancement can be maximized as much as possible.

### Automotive functional safety standard ISO 26262

Automotive functional safety standard ISO 26262 provides the duration/probability of exposure levels of E1 (very low probability with reliability goal of \(< 0.99\)), E2 (low probability with reliability goal 0.99), E3 (medium probability with reliability goal > 0.9 and \(< 0.99\)), and E4 (high probability with reliability goal \(< 0.9\)) (see Table 7).

Take Fig. 15(b) as an example, the maximum reliability value using BSE is 0.97987526 (E3), whereas using SSSE reaches 0.99877734 (E1). That is, the exposure level drops from E3 to E1 after using SSSE. Such exposure drop (i.e., reliability enhancement, safety enhancement) enables a safety guarantee of higher level. For example, Table 6 shows the ASIL determination formed by a combination of severity, exposure, and controllability in the 2nd edition of ISO 26262 [4]. According to the ASIL determination in Table 6, there are two combinations of ASIL C. We consider one combination, where the severity of the application is S3, the exposure of the application is E3, and the controllability of the driver is C3. Severity and controllability are fixed values during the design phase as explained in Section 3.2. Therefore, to achieve QM availability for the ASIL C application, one possible measure is to drop the exposure of the application from E3 to E1 according to Table 6.

### Table 7

<table>
<thead>
<tr>
<th>Exposure level</th>
<th>Probability of exposure</th>
<th>Reliability goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>Very low probability</td>
<td>Not specified</td>
</tr>
<tr>
<td>E2</td>
<td>Low probability</td>
<td>(&lt; 1%)</td>
</tr>
<tr>
<td>E3</td>
<td>Medium probability</td>
<td>([1%, 10%])</td>
</tr>
<tr>
<td>E4</td>
<td>High probability</td>
<td>(\geq 0.9)</td>
</tr>
</tbody>
</table>

Fig. 13. Values of the real-life parallel automotive application in different real-time constraints.
Even though SSSE has very little reliability enhancement over RBSE and RFSE in Fig. 13(b), the results are still useful for safety-sensitive distributed automotive embedded systems. The classes of probability of exposure in ISO 26262 shown in Table 7 are informative, not prescriptive, and leave a great deal of discretion to whoever is building each component system and ultimately to the automakers and suppliers [29]. For instance, the probability of exposure E1 in ISO 26262 is not specified, and its corresponding reliability goal at least exceeds 0.99, as shown in Table 7. Therefore, different automakers and suppliers can choose different reliability goals according to their product and market orientations as long as the values belong to the same exposure. For instance, it is feasible that the reliability goal for E3 is 0.99877730 because this value exceeds 0.99, and ISO 26262 requires high accuracy in reliability value. When the real-time constraint is 1430 μs shown in Fig. 13(b), SSSE drops to E1, whereas RBSE and RFSE only drops to E2. This result sufficiently reflects the advantages of SSSE.

3) Fig. 13(c) clearly shows that the HEFT-, BSE-, and FSE-generated replicas are fixed at 31 because they do not implement primary-backup repetition. The replicas produced by RBSE, RFSE, and SSSE progressively increase; for example, when the real-time constraint is 630 μs, the numbers of RBSE-, RFSE-, and SSSE-generated replicas are 44, 49, and 54, respectively. As the reliability value increases with the increased real-time constraint, the number of replicas required also rises through primary-backup repetition; for example, when the real-time constraint is 1430 μs, the numbers of RBSE, RFSE, and SSSE-generated replicas reach 60, 88, and 122, respectively. Although primary-backup repetition is a practical fault tolerance measure to enhance safety, it has some weaknesses. That is, RBSE, RFSE, and SSSE, especially SSSE increase the burden on ECUs due to the additional replicas and affect the control efficiency of ECUs due to the time-consuming extra code. These problems should be considered the weaknesses of fault tolerance measure.

4) In each case, the time required to calculate the result using SSSE is very short and is within 1 s. The reason is that SSSE only invokes RBSE and RFSE twice at most.

6.3 Synthetic Parallel Application

In addition to using a real-life parallel automotive application to confirm the advantage of the proposed algorithms, an additional synthetic application with 100 tasks are adopted to analyze the results. This synthetic application has the same parameter values as those of a real-life parallel automotive application. A randomly generated application can be obtained by using a task graph generator [30]. The following parameters are set in this experiment: the communication-to-computation ratio is 1, the shape parameter is 1, and the heterogeneity factor is 0.5. The heterogeneity factor values are in the 0-1 scope in the task graph generator, where 0.1 and 1 are the lowest and highest heterogeneity factors, respectively. The HEFT-generated lower bound of the application is 2642 μs. As the real-time constraint must be larger than or equal to the lower bound explained in Section 3.3, the real-time constraint is changed from 2642 μs to 5042 μs with 600 μs increments. The results are shown in Fig. 14.

1) Similar to Fig. 13(a), Fig. 14(a) shows that the response time values using all the algorithms are also less than or equal to the corresponding real-time constraints. Overall, Fig. 13(a) and Fig. 14(a) have the same regular pattern.

2) Similar to Fig. 13(b), Fig. 14(b) also clearly divides the reliability values into three gradients: 1) HEFT; 2) BSE and FSE; and 3) RBSE, RFSE, and SSSE. HEFT has the lowest reliability value of 0.88677322, which is much lower than 0.94920261 shown in Fig. 13(b). The reason is that the reliability value of the application is the product of all tasks, such that the application reliability value generated by 100 tasks is naturally lower than that generated by 31 tasks (the real-life parallel automotive application contains 31 tasks). Overall, Fig. 13(b) and Fig. 14(b) present the same regular pattern for all the algorithms.

3) Fig. 14(c) shows the same regular pattern as Fig. 13(c). By synthesizing the data in Figs. 13(c) and 14(c), the following facts are confirmed. 1) HEFT, BSE, and FSE have fixed numbers of replicas |N| due to the absence of primary-backup repetition, and |N| represents the size of task set of the application. 2) The numbers of RBSE-generated replicas are between |N| and 2 × |N| because each task is repeated once at most. 3) The numbers of RFSE-generated replicas are between |N| and 3 × |N| because each task is repeated at most once on the basis of RBSE; 3) SSSE-generated replicas are theoretically between |N| and |U| × |N|, where |U| represents the size of processor set, but approximately 4 × |N| in the actual situations.
earlier, the main weaknesses of RBSE, RFSE, and SSSE are that they increase the burden on ECUs due to the additional replicas and affect the control efficiency of ECUs due to the time-consuming extra code.

In each case, the time required to calculate the result using SSSE is still within 1 s because SSSE only invokes RBSE and RFSE five times at most for an automotive application with 100 tasks.

7 CONCLUSION

The SSSE approach for a real-time parallel automotive application to enhance safety to a stable value was proposed in this study. SSSE consists of existing HEFT and BSE algorithms and pretested FSE, RBSE, and RFSE algorithms. SSSE enhances the safety by using stable stopping approach on the basis of the forward-and-backward recovery through primary-backup repetition. SSSE can make the exposure level drops from E3 to E1 towards safety guarantee for higher level. SSSE is actually a recovery through repetition based on backward recovery and forward recovery of the static recovery mechanism pointed out in the 2nd edition of ISO 26262. It is believed that the stable stopping approach can serve as a guideline in the safety design of automotive applications. The future work could further study the safety enhancement of distributed automotive embedded systems by considering multiple automotive applications with different deadlines.

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Customizable Scale-Out Key-Value Stores

Ali Anwar, Yue Cheng, Hai Huang, Jingoo Han, Hyogi Sim, Dongyoon Lee, Fred Douglis, Fellow, IEEE, and Ali R. Butt

Abstract—Enterprise KV stores are often not well suited for HPC applications, and thus cumbersome end-to-end KV design customization is required to meet the needs of modern HPC applications. To this end, in this article we present BESPOKV, an adaptive, extensible, and scale-out KV store framework. BESPOKV decouples the KV store design into the control plane for distributed management and the data plane for local data store. For the control plane, BESPOKV provides pre-built modules, called controllers, supporting common distributed functionalities (e.g., replication, consistency, and topology) and their various combinations. This decoupling allows BESPOKV to take a user-provided single-server KV store, called a datalet, and transparently enables a scalable and fault-tolerant distributed KV store service. The resulting distributed stores are also adaptive to consistency or topology requirement changes and can be easily extended for new types of services. Such specializations enable innovative uses of KV stores in HPC applications, especially for emerging applications that utilize KV-friendly workloads. We evaluate BESPOKV in a local testbed as well as in a public cloud setting. Experiments show that BESPOKV-enabled distributed KV stores scale horizontally to a large number of nodes, and performs comparably and sometimes 1.2× to 2.6× better than the state-of-the-art systems.

Index Terms—Key-value stores, KV HPC stores, scale-out KV stores, application tailored storage

1 INTRODUCTION

The underlying storage and I/O fabric of modern high-performance computing (HPC) systems is in an ever-increasing role in supporting today’s HPC applications. Examples of this include dynamic control [1], coupling applications [2], [3], and storing intermediate results [4], among others. Relatively simple data schemas and indexing enable KV stores to achieve high performance and high scalability, and allow them to serve as a cache for quickly answering various queries, where user experience satisfaction often determines the success of the application. Consequently, a variety of distributed KV stores have been developed, mainly in two forms: natively-distributed and proxy-based KV stores.

The natively-distributed KV stores [5], [6], [7], [8], [9], shown in Fig. 1a, are designed with distributed services (e.g., topology, consistency, replication, and fault tolerance) in mind from the beginning, and are often specialized for one specific setting. For example, HyperDex [10] supports Master-Slave topology and Strong Consistency (MS+SC). Facebook relies on its own distributed Memcached [8] with Master-Slave topology and Eventual Consistency (MS+EC). Amazon employs Dynamo [6] with Active-Active1 topology and Eventual Consistency (AA+EC).

The key limitation of natively-distributed KV stores lie in their inflexible monolithic design where distributed features are deeply baked with backend data stores. Such a design allows the developers to highly optimize the KV store performance. However, such optimizations are not portable to any other KV store. The rigid design implies that these KV stores are not adaptive to ever-changing user demands for different backend, topology, consistency, or other services. For instance, Social Artisan [11] and Behance [12] moved from MongoDB to Cassandra for scalability and maintenance reasons [13]. Conversely, Flowdock [14] migrated from Cassandra to MongoDB due to stability issues. Unfortunately, this migration process is very frustrating and time/money-consuming as requires data remodeling and extra migration resources [13].

Alternatively, proxy-based distributed KV stores leverage a proxy layer to add distributed services into existing backend data stores. For example, Mcrouter [15], and Twemproxy [16] can be used as a proxy to enable a basic form of distributed Memcached [17] with partitioning, as shown in Fig. 1b. Twemproxy supports additional Redis [18] backend

1. Active-Active is also called multi-master in database literature.
as well. Recently, Netflix Dynomite [19] extended Twemproxy to support high availability and cross-datacenter replication, as illustrated in Fig. 1c.

Unlike monolithic natively-distributed KV stores, the use of a separate proxy layer enables support for multiple backends. Each single-server KV store such as Memcached [17], Redis [18], LevelDB [20], and Masstree [21] has own its merit, so the ability to choose one or mix is an ample reward. However, existing proxy-based KV stores are still limited to a single topology and consistency: e.g., Dynomite supports AA +EC only. We see that existing solutions have not yet extracted the full potential of proxy-based distributed KV stores. Table 1 summarizes the limitations of existing proxy-based KV solutions such as Dynomite and Twemproxy.

This paper presents BESPOKV, a flexible, ready-to-use, adaptive, and extensible distributed KV store framework. Fig. 1d illustrates BESPOKV’s distributed KV store architecture. BESPOKV takes as input a single-server KV store, which we call datalet, and transparently enables a distributed KV store service, supporting a variety of cluster topologies, consistency models, replication options, and fault tolerance (Section 3).

For the control plane, BESPOKV provides a set of distributed management units, referred to as controlets. To the best of our knowledge, BESPOKV is the first system supporting multiple consistency techniques, multiple network topologies, dynamic topology/consistency adaptation, automatic failover, and programmability, all at the same time.

Table 2 shows the benefit of the proposed BESPOKV framework. Here, four snippet implementations of the core functions for a simple KV store are presented in pseudocode. To implement everything from scratch (a) Vanilla, a developer creates her own concurrency control functionality (Lock(), Unlock()), and consistency and quorum management logic (Sync(), Quorum()). Using a distributed lock server (b) Lockserver-based, the developer can avoid implementing synchronization functions. Similarly, using Vsync library (c) Vsync-based for consistency management further reduces engineering effort. However, there are two limitations. First, developers still need to familiarize themselves with a large collection of system/library interfaces to use them appropriately in the application code. Second, such approaches often provide only a single technique for replication or consistency: e.g., Vsync uses only a virtual synchrony to replicate data. In contrast, using BESPOKV (option (d)), developers only need to implement a non-distributed version of the KV store (datalet), and then BESPOKV transparently scales it out to a variety of distributed environments with different requirements.
We demonstrate that achieving the above goals is the major focus. Challenges arise when designing BESPOKV to meet the competing goals of compatibility, versatility, modularity, and performance.

Compatibility. BESPOKV strives to transparently make a non-distributed KV store into a distributed one. It should be easy to use, such that a developer simply "drops" the non-distributed version of the store into BESPOKV; in turn, BESPOKV will automatically clone and convert the store into various types of highly scalable and reliable distributed clusters.

Versatility. Due to the diversity of data storage and retrieval requirements, almost all the points on the cluster topology (MS, AA, etc.), consistency (strong, eventual, etc.), replication, and fault tolerance spectrum are valid. However, existing systems only support a fixed single design point, which limits flexibility and adaptability. Therefore, BESPOKV architecture should be versatile enough to cover various design options, and be flexible to support reconfiguration.

Different storage applications implement their distributed management and protocols with preference on diverse dimensions such as cluster topology and consistency. To support applications with tradeoffs among these different dimensions through a generic framework, one should ensure that each configurable dimension has a clear boundary and well defined interface. Hence, different dimensions can be seamlessly combined with each other to form a highly versatile choice of options for application developers. Moreover, the distributed network architecture should be flexible enough to support these wide range of options. Section 3 presents this aspect of BESPOKV’s versatile architecture.

Modularity. Building various design options using different implementations is simply a matter of putting in more engineering effort and not as challenging. In fact, such a na"ıve monolithic redesign approach would essentially be similar to the current approach of per-application implementations. Instead, BESPOKV should be designed in a modular fashion, which makes it possible to reuse a previously developed component. For instance, a controllet supporting MS+SC or AA+EC can be reused for multiple backend data stores. Furthermore, the modules in BESPOKV should be expandable to meet the ever-growing needs for advanced features.

Performance. Achieving the above goals is the major focus of our work. However, BESPOKV should be realized without
sacrificing performance. Design choices for protocol handling, network architecture, and diverse components should be carefully made with efficiency in mind.

## 3 BESPOKV Design

In this section, we describe the design of BESPOKV and how it provides compatibility, versatility, modularity, and high performance for supporting distributed KV stores. Fig. 2 shows the overall architecture of BESPOKV comprising five modules: datalet, controlet, coordinator, client library, and optional components. A collection of datalets form the data plane, the rest of the modules makes up the control plane.

**Datalet** is supplied by the user and responsible for storing data within a single node. Datalet should provide the basic I/O interfaces (e.g., `Put` and `Get`) for the KV stores to be implemented. We refer to this interface as the datalet API. For example, a user can develop a simplest form of in-memory hash table. Users can also mix and match datalets with each datalet using a different data structure.

**Controlet** is supplied by BESPOKV and provides a datalet with distributed management services to realize and enable the distributed KV stores associated with the datalet. The controlet processes client requests and routes the requests to the associated entities: e.g., to a datalet for storing data. BESPOKV provides a default set of controlets, and allows advanced users to extend and design new controlets as needed for realizing a service that may require specialized handling in the controlet.

BESPOKV allows an arbitrary mapping between a controlet and a datalet. A controlet may handle $N \geq 1$ instances of datalets, depending on the processing capacity of the controlet and its datalets, and can leverage physical resource (datacenter) heterogeneity [24], [25] for better overall utilization. For instance, a controlet running on a high-capacity node may manage more datalet nodes than a controlet running on a low-capacity node. For simplicity, we use one-to-one controlet–datalet mappings in the rest of the paper.

**Coordinator** provides three main functions. (1) It maintains the metadata regarding the whole cluster topology and provides a query service as a metadata server. (2) It tracks the liveness of the cluster by exchanging periodic heartbeat messages with the controlets. (3) It coordinates failover in case of a node failure. The coordinator can run on separate node or alongside other controlets.

BESPOKV implements the coordinator on top of ZooKeeper [26] for better resilience. Similar to designing specialized controlets, advanced users have the option to design customized coordinators if needed. It is also possible to design a new coordinator as a special form of controlet from scratch using the BESPOKV-provided controlet programming abstraction as shown in Section 3.2. Nonetheless, because it is widely used across many KV stores, BESPOKV includes the coordinator as a default module in the control plane.

**Client library** is provided by BESPOKV and used by the client applications to utilize the services created by BESPOKV. The library provides a flexible means for mapping data to controlets. The client application uses the library interface to consult with the coordinator and fetch data partitioning and mapping information, which is then used to route requests to appropriate controlets. BESPOKV allows different developers to choose their own partitioning techniques such as consistent hashing and range-based partitioning.

**Optional Components.** BESPOKV provides two optional components facilitating the controlet development: 1) a distributed lock manager (DLM) for a locking service, and 2) a Shared Log for an ordering service. One can build such a distributed management service as a special form of controlets from the scratch, but given its common use in distributed KV store development, BESPOKV imports existing solutions (e.g., Redlock [27] for DLM, and ZLog [28], [29], [30] for Shared Log) and provides interface libraries (Section 3.2, Table 4).

### 3.1 Data Plane

A collection of datalets running on different distributed nodes form the data plane for BESPOKV. A single-server datalet is completely unaware of other datalets.

**Datalet Development.** BESPOKV supports multiple backends. Users can make use of off-the-shelf single-serve data stores such as Redis [18], SSDB [31], and Masstree [21]. In addition, BESPOKV provides datalet templates based on commonly used data structures: currently, a hash-table-based `HT`, a log-based `ILog`, and a tree-based `MT`. For the ease of development, BESPOKV furnishes an asynchronous event-driven network programming framework in which developers can design new datalets, starting from existing templates. We evaluate the reduced engineering effort in Section 8.

**APIs and Protocol Parsers.** For compatibility and modularity, BESPOKV provides a clean set of datalet APIs (between controlet and datalet) and client APIs (between client app and client library). Table 3 presents example datalet and client APIs. As these APIs are consistent with existing I/O interfaces of existing KV stores. Datalet developers can adopt them in a straightforward manner to enable distributed services. This is much easier than library-based replication solutions such as Vsync [22] where developers should learn complex new APIs.
TABLE 3

<table>
<thead>
<tr>
<th>Dataset API provided by application developers</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>Put(key, val)</code> Write the <code>(key, val)</code> pair to the dataset</td>
</tr>
<tr>
<td><code>val=Get(key)</code> Read <code>val</code> of key from the dataset</td>
</tr>
<tr>
<td><code>Del(key)</code> Delete <code>(key, val)</code> pair from the dataset</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Client API provided by BESPOKV</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>CreateTable(T)</code> Create a table <code>T</code> to insert data</td>
</tr>
<tr>
<td><code>Put(key, val, T)</code> Write the <code>(key, val)</code> pair to table <code>T</code></td>
</tr>
<tr>
<td><code>val=Get(key, T)</code> Read <code>val</code> of key from table <code>T</code></td>
</tr>
<tr>
<td><code>Del(key, T)</code> Delete <code>(key, val)</code> pair from table <code>T</code></td>
</tr>
<tr>
<td><code>DeleteTable(T)</code> Delete table <code>T</code></td>
</tr>
</tbody>
</table>

Dataset and Client APIs are for using pre-built controlets.

To offer compatibility and be able to understand application protocols to process incoming requests properly, BESPOKV’s communication substrate supports two options. (1) It provides a BESPOKV-defined protocol using Google Protocol Buffers [32]. This option is suitable for new datalets and is preferred due to its ease of use and better programmability. (2) BESPOKV allows developers to provide a parser for their own protocols. This option is mainly available for porting existing datalets such as Redis or SSDB.

3.2 Control Plane

BESPOKV provides a set of pre-built controlets that provide datalets with common distributed management. Given a datalet, BESPOKV makes distributed KV stores immediately ready-to-use. Developers can also extend these pre-built controlets or design new ones from scratch for advanced services.

Pre-Built Controlets. BESPOKV identifies four core components for distributed management, and provides pre-built controlets that support common design options in existing distributed KV stores. The choice is based on our comprehensive study of existing systems that revealed three key observations: (1) cluster topology, consistency model, replication, and fault tolerance generally define distributed features of KV stores; (2) for the topology, MS and AA are common; and (3) for the consistency model, SC and EC are popular. Detailed descriptions of exemplary controlets supporting MS+SC, MS+EC, AA+SC, and AA+EC options follow in Section 4.

Controlet Development. To support advanced users and new kinds of services, BESPOKV provides an asynchronous event-driven network programming framework for controlet development as well. For each event (e.g., `Put` request, timeout, etc.), developer can define event handlers to instruct how the controlet should process the event to enable versatile distributed management services in the control plane. The aforementioned pre-built controlets indeed consist of a set of pre-defined event handlers for common distributed services.

Discussion. Load imbalance due to hot keys (i.e., hotspots) can be solved by integrating a small metadata cache at BESPOKV’s client library to keep track of hot keys [33]; once the popularity of hot keys exceeds a certain pre-defined threshold, client library replicates this key on a shadow server that is rehashed by adding a suffix to the key. In fact, our proxy-based architecture naturally fits for adding a controlet-side small cache or data migration/replication for load balancing purpose [34], [35], [36], [37], [38].

Control Plane Configuration. To configure the system, each controlet takes as input (1) a JSON configuration file that specifies the basic system deployment parameters such as topology, consistency model, the number of replicas, and coordinator address; and (2) a data host file containing the list of datalets to be managed. BESPOKV loads the runtime configuration information at the coordinator, which serves as the query point for the client library and controlets to periodically retrieve configuration updates. Any change in configuration at runtime (e.g., topology/consistency switch) results in replacing old controlets with new ones. We describe dynamic adaptation mechanisms in Section 5 in detail.

Controlet Programming Abstraction. BESPOKV uses asynchronous event-driven programming model to achieve high throughput. For each event (e.g., incoming network input, timer, etc.), developers are asked to define event handlers to process the event. There are two types of events in BESPOKV: basic and extended events. Basic events represent pre-defined conditions. Developers can create their own extended events by using basic or existing extended events.

Other Controlet APIs. BESPOKV provides a set of libraries and APIs with common features for controlet development, shown in Table 4.

4 BESPOKV-BASED DISTRIBUTED KV STORES

BESPOKV, to be specific its control plane, transparently turns a user-provided single-server datalet to a scalable, fault-tolerant distributed KV store. Using hash-based `htT` datalet and consistent hashing for the client library as an example, this section presents support for MS+SC, MS+EC, AA+SC, AA+EC and four examples to enable new forms of distributed services by combining existing controlets or extending ones.2

2. Please note that these examples present just one way to implement each combination. Controlet developers can easily implement their own versions.
Upon a node failure, the coordinator launches a new controlet–datalet pair. The new controlet then performs data recovery from another datalet. As AA+EC by asynchronously forwarding Put requests to other datalets (step 4).

**Failover.** Upon a node failure, the coordinator launches a new controlet–datalet pair, and then the new controlet recovers the requests from another datalet. For MS+EC, the new pair is added as a slave. If the master node fails, the coordinator promotes one of the slave nodes to master after a leader election process. The coordinator then updates the cluster topology metadata so that future incoming writes can be routed to the new master, similar to the case of head failure in MS+SC.

**4.3 Active-Active & Strong Consistency**

Supporting AA and SC is expensive in general. AA allows multiple nodes to handle Put requests and SC requires global ordering (serialization) between them. Thus, CR-like optimization is not applicable under AA. For simplicity and comparison purposes, the current BESPOKV’s AA+SC controlet takes the distributed locking based implementation, using the DLM library (Section 3.2). For performance improvement, optimistic concurrency control [42] and inconsistent replication [9] can be added. Instead of using DLM, one can also enable SC using a Shared Log to maintain a global and sequential order of concurrent requests, which we used for AA+EC later in a relaxed manner.

**Example.** Fig. 4b shows a DLM-based AA+SC example. Clients’ Put requests are routed to any controlet (step 1 and step 2). Concurrent Puts from another client (step 2 in our example) are synchronized via the distributed locking service. The first receiving controlet acquires a write lock (step 3) on the key and updates all the relevant datalets (step 4 & 5), releases the lock (step 6), and finally acknowledges to the client (step 7). For a Get request, the controlet that receives the request acquires a read lock on that key, reads the value from the local datalet, releases the lock, and then sends a response back to the client.

**Failover.** Like the previous cases, when a node fails the coordinator launches a new controlet–datalet pair. The new controlet then performs data recovery from another datalet. As AA+SC uses locking, ensuring SC for the new node and adding it as an active node are trivial because all writes are synchronized using locks. However, deadlock freedom should

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3. This way at least one datalet is written straight away as in Cassandra [7]. An alternative design choice is to forward the request to more than one datalet and then acknowledge back. However, this decision solely depends on the type of eventual consistency that is desired.
be guaranteed. Thus, \textsc{bespo} enforces that locks are released after a configurable period of time. If a controlet fails after acquiring a lock, the lock is auto-released after it expires. Note that if a lock is auto-released, but a controlet has not failed and was simply unresponsive for a while, it is terminated to ensure proper continuation of operations. Also, one of the master nodes cleans up the in-flight requests.

### 4.4 Active-Active & Eventual Consistency

For an AA topology, relaxed data consistency is more widely used in practice for performance as in Dynamo [6], Cassandra [7], and Dynomite [19]. In particular, these systems use gossip-based protocols and provide a weaker data consistency model, e.g., acknowledging back to the client if a \texttt{Put} request is written to one node, \( N \) nodes, or a quorum [43].

In order to ensure EC, when multiple masters receive concurrent \texttt{Put} requests, AA should be able to resolve conflicts and agree on the global order of them, unlike MS where one master gets all the writes. In this sense, Dynomite does not support (a strict form of) EC when conflicting \texttt{Put} requests arrive within a time period less than the latency of replication [44].

To address this issue, \textsc{bespo}’s AA+EC controlet uses a Shared Log to keep track of the request ordering. From the Shared Log, asynchronous propagation of writes occur to support EC. One disadvantage of this approach is that we need to scale the Shared Log setup as \textsc{bespo} scales. Alternative approach is to add anti-entropy/reconciliation [45].

**Example.** Fig. 4c depicts how \textsc{bespo} supports AA+EC. In AA, clients can route \texttt{Get} / \texttt{Put} to any of the master controlets (step 1a). On a \texttt{Put}, the receiving controlet (in our example the leftmost one) writes to the Shared Log first (step 2a), commits the request on its local datalet (step 3a), and then responds back to the client (step 4a). All the controlets asynchronously fetch the request (step 5). \texttt{Get}s can be handled by any of the corresponding controlets by retrieving the data from their local datalets. The duration to keep the requests in Shared Log is configurable.

**Failover.** For AA+EC, the failover is handled like with MS+EC, except that leader election is not needed in this case.

### 5 Dynamic Adaptation to Consistency and Topology Model Changes

Separating the control and data planes bring another benefit: \textsc{bespo}’-enabled distributed KV stores can seamlessly adapt to consistency and topology model changes at runtime by switching the controlets while keeping the datalets unchanged. At a high level, upon a consistency and/or topology change request, Coordinator launches a new set of controlets that will provide new services. Two old and new controlets are mapped to one datalet during the transition phase. The old controlet provides the old service with no downtime, and forwards some requests to the new controlet so that it can prepare the new service. When the transition completes, the new controlet takes over the old one. The transition protocol differs per each case. \textsc{bespo} supports any transition between four aforementioned topology and consistency combinations, among which we describe two interesting cases in detail. Section 9.4 presents the experimental results on this aspect.

#### 5.1 Transition From MS+EC to MS+SC

To make a transition from EC to SC, the master node needs to make sure that all the \texttt{Put} requests 1) that have arrived before the transition starts and 2) that arrive during transition are fully propagated to the slave nodes. For the former, the old master keeps flushing out any pending propagation. For the latter, the old master forwards an incoming \texttt{Put} request to the new master controlet which uses chain replication for SC, instead of propagating it asynchronously. When there is no more pending propagation left in the old controlet, the transition is over. SC guarantees will be enforced after the transition has completed. During the transition, any node may respond to \texttt{Get} requests, providing EC guarantee. This means that a \texttt{Get} request, even after the reconfiguration was requested, may experience EC until the transition is over. As controlet developers are responsible for developing the transition functionality for the various consistency/topology modes. A controlet developer can choose an alternative route to fence all writes as soon as the reconfiguration is requested so that all reads observe the same and latest applied value.

Fig. 4 shows transition from MS+EC to MS+SC.\(^4\) Client 1 sends a \texttt{Put} request (Step 1a) to the old master controlet C1. A concurrent \texttt{Get} request (Step 1b) from Client 2 gets serviced as it used to be. The old master forwards \texttt{Put} request (Step 2) to the new master controlet which guarantees SC.

\(^4\) Reverse transition from MS+SC to MS+EC is trivial as the new master just needs to start using async. propagation instead of chain replication.
XTENSIONS TO

Put request (Step 1) is forward to the new
BESPO

Get

Put

Scan

Put

requests (range query) in Section 9.2, and
request fetches
request (Step 3) is serviced by the new
BESPO

Get

template is used and
Put

requests and is in
U

requests are not affected. Fig. 5b shows an exam-
Fig. 5. Transition: MS+EC to MS+SC and AA+EC to MS+EC.

When the new master completes its chain replication pro-
cess, it acknowledges the old master, which in turn acknowl-
edges Client 1. When the transition completes, a Put request
(Step 3) is routed to the new master controlet.

5.2 Transition From AA+EC to MS+EC

In AA+EC, any active node can get a Put request. To
maintain a global ordering between concurrent Puts, an
active node relies on the Shared Log that propagates Puts
to the other nodes on its behalf. On the other hand, in MS +EC, only the master node gets Put requests and is in
charge of propagating them to the slaves. Therefore, the
key operation in the transition from AA+EC to MS+EC is
to move the role of propagating Puts from the Shared Log
to the new master. To this end, when the transition starts,
the new master node takes the in-flight Puts that have not
been propagated yet from the Shared Log and starts prop-
agating them by itself. When an old active controlet
receives a Put request during transition, it does not con-
consider with Shared Log, but forwards the request to the new
master node which will eventually propagate the request.
The Get requests are not affected. Fig. 5b shows an exam-
ple where a Put request (Step 1) is forward to the new
master (Step 2) during transition. When the transition
completes, a Put request (Step 3) is serviced by the new
master. The transition from MS+EC to AA+EC can be sup-
ported by the reverse step order.

6 Extensions to KV Stores

BESPOKV is immediately ready-to-use for popular distrib-
uted KV store use cases. If desired, BESPOKV’s control plane
can be extended to enable new forms of distributed services
by combining existing controlets or extending ones. This
section demonstrates four examples. We evaluated perfor-
mance of Scan requests (range query) in Section 9.2, and
the next two per-request consistency and polyglot persis-
tence in Section 9.5.

6.1 Range Query

We support range query or scan operations as follows. For
datalets, the MassTree-based iMT template is used and
extended to expose a range query API such as GetRange
(Start, End). The client library supports range-based
partitioning, e.g., dividing the name space by alphabetical
order (e.g., A-C on one node, D-F on another node, and so
on). The controlet divides a client request into sub-requests
and forwards the sub-range query requests to correspond-
ing datalets that store the specified range.

6.2 Per-Request Consistency

We extend the client library GET API to support consistency/topology specification on a per-request basis. For
instance, under MS+SC, if the user specifies a lower value of
consistency level, GETs can go to any of the replicas, thus
only eventual consistency is guaranteed.

6.3 Polyglot Persistence

A use case for KV store is to support businesses that may be
divided into different components, and each component
requires its own private data storage. BESPOKV supports such
polyglot persistence [46] by launching custom controlets for
cross-app lazy synchronization (eventual consistency).

6.4 Other Topologies

BESPOKV also supports an AA-MS hybrid topology by config-
uring an MS topology for each shard on top of the logical AA
overlay. Similarly, a P2P-like topology can also be enabled
by allowing clients to send a request to any controlet, which
then routes the request to the actual controlet that manages
the requested data. In this case, a controlet needs to maintain
a routing map similar to a finger table [47] to determine the
location of keys.

Variants of AA. BESPOKV can support Adding routing flex-
ibility to the AA topology is straightforward. Clients can
simply send a request to any of the controlets (or use some
load balancing techniques such as round robin), which then
routes the request to the actual controlet–datalet that holds
the requested data. The extra logic we need to add into con-
троlets is a routing map similar to a finger-table [47] to deter-
mine the location of keys.

6.5 Other Consistency Models

Our Shared Log-based asynchronous fetches for eventual
consistency can be easily configured to support bounded
staleness. Developers simply specify a T-sec polling period,
so that clients are guaranteed not to see the stale data for
more than T sec. Similarly, causal consistency can also be
supported if the controlet serving the Get request fetches
all the pending data from the shared log and communicates
it to other controlets before replying back to the client.

7 BESPOKV’S USE CASES

7.1 Hierarchical and Heterogeneous Storage of HPC

HPC big data problems require efficient and scalable storage
systems, but load balancing I/O servers at scale remains
a challenge. Statistical analysis [48] and Markov chain
model [49] have been used to predict shared resource usage.
A KV store can be used to collect runtime statistics from
HPC storage systems for accurate prediction. However,
eXtent KV stores are designed for one type of storage archi-
tecture (in-memory, SSD, NVM, etc.), leading to suboptimal
performance.

BESPOKV supports the use of different datalets to store
replicas of a KV pair, where each of these datalet can be
tuned for different memory and storage architecture. By
doing so, BESPOKV unifies multiple data abstraction together
and enables multifaceted view on shared data with configur-
able consistency and topology. Fig. 6 shows an example of
how BESPOKV unifies three different data abstractions – a log-structure merge-tree, Masstree, and log, and transparently provides master-slave topology (MS) and eventual consistency (EC). Data is replicated asynchronously in batch mode from master to slaves. In this design, it is possible to run applications with different properties (e.g., write-intensive and read-intensive apps) together.

There are two advantages of this design architecture. First, different applications can choose datalet that best suits their need. As a typical use case, monitoring data collection is write-intensive workload, and prefers a scalable solution that is able to persist all data on persistent storage. Whereas, analytical models incur read intensive workload which could benefit from high read throughput. Second, replicas in different datalets are not evicted simultaneously. For instance, a replica of a KV pair may evict from in-memory based datalet due to size restriction but another replica may stay longer in NVM/SSD based datalet or stay forever in log based datalet that uses HDD.

7.2 Building Burst Buffer File Systems

Burst buffer file systems are becoming an indispensable framework to quickly absorb application I/O requests in exascale computing [50], [51], [52], [53]. Many burst buffer file systems adopt KV stores to manage file system metadata. BESPOKV allows to develop similar file systems with less development effort. In particular, the dynamic and flexible nature of BESPOKV well suits with ephemeral burst buffer file systems [50]. An ephemeral burst buffer file system has to be dynamically constructed and destroyed within compute nodes assigned to a corresponding job. In such a scenario, BESPOKV can quickly initialize the distributed KV store for storing file system metadata.

Furthermore, BESPOKV also allows to dynamically tune the file system behavior. For instance, it is often preferred to relax the strong POSIX consistency semantics for certain HPC workloads (e.g., checkpointing) to maximize the parallel I/O performance [54]. BESPOKV can simplify the development of such a file system, because it natively supports an instantiation of the distributed KV store with desired consistency and reliability levels.

7.3 Accelerating the File System Metadata Performance

KV store is also widely adopted to enhance the performance of file system metadata operations in HPC systems. Metadata performance is one of the major limitations in HPC parallel file systems. A popular approach to address this limitation is to stack up a special file system atop the parallel file system [55], [56], [57]. The stacked file system then quickly absorbs the metadata operations by exploiting a distributed KV store. BESPOKV can accelerate the development of such a stacked file system (evaluated in Section 9.2). Specifically, BESPOKV allows to explore various datalets in backend, and also dynamically tune the file system behavior to comply with the desired performance, consistency and reliability levels.

7.4 Resource and Process Management

KV store has also been used to aid the resource and process management in HPC systems [2], [58]. BESPOKV can help develop an advanced job launching system, because it can adapt to different topology and consistency models on the fly. For example, the simple MS topology may be sufficient for handling jobs on a single cluster, but the AA topology may become more suitable when jobs spans multiple clusters (evaluated in Sections 9.2 and 9.4).

8 BESPOKV IMPLEMENTATION

Current implementation of BESPOKV consists of 69k lines of C++/Python code without counting comments or blank lines. Except controlets, BESPOKV consists of five components. (1) Control Core implements the control plane backbone with support for event and message handling, (2) Client library helps clients route requests to appropriate controlet, and is extended from libmc [59], a in-memory KV store client library. (3) Coordinator uses ZooKeeper [26] to store topology metadata of the whole cluster and coordinates leader elections during failover. It includes a Python-written failover manager that directly controls the data recovery as well as handling BESPOKV process failover. (4) Lock server APIs implement two lock server options—ZooKeeper-based [60] and Redlock-based [27]. (5) Shared Log handler is implemented using ZLog [28], based on CORFU.

The BESPOKV prototype has four pre-built controlets as described in Section 4. All controlet shares the sample event-handling controlet template of 150 LoC. In addition, BESPOKV supports multiple backend datalets with protocol parsers. Using the common datalet template of 966 LoC, we implemented three new datalets with a Protobuf-based [32] parser: tHT, an in-memory hash table; tLog, a persistent log-structured store that uses tHT as the in-memory index; and tMT, a Masstree-based [61] store. In addition, BESPOKV are compatible with existing single-server KV stores SSDB [31] and Redis [18] that use a simple text-based protocol parser. With protocol parsers, we refer them SSDB and tRedis, respectively. Docker based BESPOKV is partially supported right now. We plan to use Kubernetes [62] to simplify deployment in near future.

Using the template-based design approach, we note that for developers (with few years of C/C++ programming experience) non familiar to BESPOKV it took almost three and six person-days time to develop datalet and controlet, respectively. This underscores BESPOKV’s ability to ease development of distributed KV stores.

9 EVALUATION

Our evaluation answers the following questions:

- Are BESPOKV-enabled distributed KV stores scalable (Section 9.2), adaptive to topology and consistency changes (Section 9.4), and extensible (Section 9.5)?
How well μKV handles a node failure? (Section 10)

9.1 Experimental Setup

Testbeds and Configuration. We perform our evaluation on Google Cloud Engine (GCE) and a local testbed. For larger scale experiments (Sections 9.2, 9.3, 9.4, 9.5, and 9.6), we make use of VMs provisioned from the us-east1-b Zone in GCE. Each controller–datalet pair runs on an n1-standard-4 VM instance type, which has 4 virtual CPUs and 15 GB memory. Workloads are generated on a separate cluster comprising nodes of n1-highcpu-8 VM type with 8 virtual CPUs to saturate the cloud network and server-side CPUs. A 1 Gbps network interconnect was used.

For performance stress test (Section 9.7) and fault tolerance experiments (Section 10), we use a local testbed consisting of 12 physical machines, each equipped with 8 2.0 GHz Intel Xeon cores, 64 GB memory, with a 10 Gbps network interconnect. The coordinator is a single process (backed-up using ZooKeeper [26] with a standby process as follower) configured to exchange heartbeat messages every 5 sec with controllers. We deploy the DLM, Shared Log, Coordinator and ZooKeeper on separate set of nodes. μKV’s coordinator communicate with ZooKeeper for storing metadata.

Workloads. We use two workloads obtained from typical HPC services: job launch, and I/O forwarding and three workloads from the Yahoo! Cloud Serving Benchmark (YCSB) [63].

We use approach similar to [2] to generate HPC workloads. The job launch workload is obtained by monitoring the messages between the server and client during a MPI job launch. Control messages from the distributed servers are treated as Get whereas results from the compute nodes back to the servers as Put. The I/O forwarding workload is generated by running SeaweedFS [64], a distributed file system which supports KV store for metadata management. The clients first create 10,000 files, and then performs reads or writes (with 50 percent probability) on each file. We collect the log of the metadata server. We extend these workloads several times until reaching 10M requests with the goal to reflect the time serialization property of the obtained messages.

For YCSB we use an update-intensive workload (Get: Put ratio of 50%:50%), a read-mostly workload (95 percent Get), and a scan-intensive workload (95 percent Scan and 5 percent Put). All workloads consist of 10 million unique KV tuples, each with 16 B key and 32 B value, unless mentioned otherwise. Each benchmark process generates 10 million operations following a balanced uniform KV popularity distribution and a skewed Zipfian distribution (where Zipfian constant = 0.99). The reported throughput is measured in terms of thousand queries per second (kQPS) as an arithmetic mean of three runs.

9.2 Scalability

In this test, we evaluate the scalability of the μKV-enabled distributed KV store using four datalets: tHT and tLog, as examples of newly developed datalets; and tSSDB and tMT, as representatives of existing persistent KV stores. Fig. 7 shows the scalability of μKV-enabled distributed tHT. We measure the throughput when scaling out tHT from 3 to 48 nodes on GCE. The number of replicas is set to three. We present results for all four topology and consistency combinations: MS+SC, MS+EC, AA+SC, and AA+EC. For all cases, μKV scales tHT out linearly as the number of nodes increases for both read-intensive (95 percent Get) and write-intensive (50 percent Get) workloads. For SC, MS+5C using chain replication scales well, while AA+SC performs worse as expected in locking based implementation. For EC, the results show that our EC support scales well for both MS+EC and AA+EC. Performance comparison to existing distributed KV stores will follow in Section 9.7.

Fig. 8 shows similar trend for HPC oriented workloads.
locked with MS+EC. 2091 QPS on Zipfian 95 percent BESPO in MS topology with eventual consistency. The perfor-
270 QPS under Zipfian 95 percent and Redis 500 QPS for 95 percent on eight 3-replica shards across 24 nodes on GCE,
BESPO/C24 latency is 0.67 ms. We get an average of Get QPS for BESPO/C24 750 and Log QPS for 95 percent
300 workloads. We also evaluate the average 200 Get performance for all the tests we use 3 shards with a Zipfian workload of
sistency levels and topology configurations (Section 5). In
Fig. 9.

Fig. 9. BESPOKV scales tSSDB, tLog, and tMT with MS+EC.

disk. It is as expected that the throughput of Scans (range queries) is much lower than point queries. A 48 node tMT cluster gives 18k QPS on Zipfian 95 percent Scan, while Uniform yields slightly higher throughput (21k). Interestingly, this test covers a potential use case of BESPOKV+tLog for flash storage disaggregation, where users can exploit the scale-out capacity of an array of fast SSD (flash) devices/nodes with low-latency datacenter network [65], [66].

### 9.3 Impact of Varying Replication Factor

Next, we analyze the impact of varying the replication fac-
tor on performance. Fig. 10 shows the average throughput of an 8-shard cluster when varying the number of replicas from 1 to 3.

For cluster configurations with EC under read-intensive (uniform and Zipfian 95 percent Get) workloads, a larger replication factor results in higher performance. This is because there are more nodes that can serve Get requests. The effect is more significant for uniform workload, as the load is more balanced.

On the other hand, for MS+SC, scaling the number of replicas does not improve performance. Performance stops scaling above 2 replicas under read-intensive workloads, and it actually degrades by a factor of 2 to 3 for write-intensive workloads. This is because BESPOKV uses chain replication to support MS+SC, and all Putts are going to the head controlets, which need to do more work as the length of the chain increases.

Increasing the number of replicas does increase the performance for AA+SC under read-intensive workloads but with limited improvement. This is due to the high cost of distributed locking. Zipfian workloads severely increase the lock contention at the lock server, leading to the observed performance drop.

### 9.4 Adaptability

We evaluate BESPOKV’s adaptability in switching online consist-
ency levels and topology configurations (Section 5). In all the tests we use 3 shards with a Zipfian workload of
95 percent Get. As shown in Fig. 11, the transition is scheduled to be triggered at 20 sec. The throughput drops to the lowest point for all three cases. This is because clients switch connection to the new controlets. Performance stabilizes in 5 sec, because all the in-flight requests are handled during this process. We observe similar trends for other possible transitions that can be enabled by BESPOKV. This demonstrates BESPOKV’s flexibility and adaptability in switching between different key designs & configurations. This also shows that BESPOKV is able to complete switching in extremely short time compared to existing solutions because BESPOKV does not require data migration or down time.

### 9.5 Extensibility and New Services

As sketched in Section 4, BESPOKV can be extended to support new forms of distributed services. This section evaluates two examples: per-request consistency and polyglot persistence.

We evaluate the per-request consistency service (Section 6.2) under MS+SC and a Zipfian workload with a 25:75 percent ratio of SC:EC as the desired consistency. We observed the performance to be between MS+SC and MS +EC as shown in Fig. 7; for example, with 24 nodes, we obtain ~300k QPS for 95 percent Get and ~270k QPS for 50 percent Get workloads. We also evaluate the average latency of each request. With a weaker consistency require-
ment, the GET latency is 0.67 ms. We get an average of 1.02 ms latency with default strong consistency.

We test polyglot persistence (Section 6.3) by storing each replica in a different type of datalet. We use tHT, tLog and tMT in MS topology with eventual consistency. The perform-
ce of the resulting configuration under Uniform workload is very similar to the numbers in Figs. 7 and 9; for example, with 24 nodes, we obtain 375k QPS for 95 percent Get and 200k QPS for the 50 percent Get workload.

### 9.6 Comparison to Proxy-Based Systems

This section shows that BESPOKV can support new topologies and consistency models for existing single-server KV store, and them compares BESPOKV with two state-of-the-art Proxy-based KV stores. We test BESPOKV+Redis (tRedis) running in MS+SC, MS+EC and AA+EC modes, reusing SSDB’s text-based protocol parser for Redis. We measure the throughput of tRedis on eight 3-replica shards across 24 nodes on GCE, and compare it with Dynomite [19] supporting AA+EC only, and Twemproxy [16] supporting MS+EC only. We perform each test at three different periods of time to capture inter-
ference caused by cloud-based multi tenancy.

Fig. 12 shows the throughput. BESPOKV enables new MS +SC (~500k QPS under Zipfian 95 percent Get,) and AA+EC (~750k QPS under Zipfian 95 percent Get) configurations
to increase the throughput in units of kQPS.\footnote{Twemproxy itself does not provide any consistency support.} For AA+EC, \bespokv outperforms Cassandra and Voldemort. For read-intensive workload, \bespokv's throughput gain over Cassandra and Voldemort is 4.5× and 1.6×, respectively. For write-intensive workload, \bespokv’s throughput gain is 4.4× over Cassandra and 2.75× over Voldemort. In this experiment Cassandra was configured to use persistent storage. However even using tLog as a datalet for \bespokv(also uses persistent storage) we observed a throughput gain of 2.6× and 1.2× over Cassandra and Voldemort, respectively. We suspect that this is because Cassandra uses compaction in its storage engine which significantly effects the write performance and increases the read latency due to use of extra CPU and disk usage\footnote{Uniform workloads show similar trend, hence are omitted.}. Voldemort uses the same design and both are based on Amazon’s Dynamo paper\cite{6}. Furthermore, our findings are consistent with Dynomite in terms of the performance comparison with Cassandra\cite{69}.

As an extra data point, we also see interesting tradeoffs when experimenting with different configurations supported by \bespokv. For instance, MS+EC achieves performance comparable to AA+EC under 95 percent Get workload since both configurations serve Get\textquotesingle s from all replicas. AA+EC achieves 47 percent higher throughput than MS+EC under 50 percent Get workload, because AA+EC serves Put\textquotesingle s from all replicas. For AA+SC, lock contention at the DLM caps the performance for both read- and write-intensive workloads. As a result, MS+SC performs 3.2× better than AA+SC for read-intensive workload and 2× better for the write-intensive workload.

### 9.7 Comparison to Natively-Distributed Systems

In this experiment, we compare \bespokv-enabled KV stores with two widely used natively-distributed (off-the-shelf) KV stores: Cassandra\cite{7} and LinkedIn’s Voldemort\cite{67}. These experiments were conducted on our 12-node local testbed in order to avoid confounding issues arising from sharing a virtualized platform. We launch the storage servers on six nodes and YCSB clients on the other four nodes to saturate the server side. The coordinator, lock server (only for AA +SC), ZLog (only for AA+EC), and ZooKeeper are launched on separate nodes. We use tHT as a datalet to show high efficiency of \bespokv-enabled KV stores.

For Cassandra, we specify consistency level of \textit{one} to make consistency requirements less stringent. Cassandra’s replication mechanism follows the AA topology with EC\cite{68}. For Voldemort we use a \textit{server}-side routing policy, \textit{all-routing} as the routing strategy, a replication factor of three, \textit{one} as the number of reads or writes that can succeed without client getting an exception, and persistence set to \textit{memory}.

Fig. 13 shows the latency and throughput for all tested systems/configurations when varying the number of clients with reasonable performance. As expected, MS+SC is more expensive than MS+EC. Twemproxy is just a proxy to route requests using consistent hashing to a pool of backend servers. Hence, Twemproxy+Redis in supporting MS+EC performs slightly better than \bespokv in supporting MS+EC.\footnote{The small error bars in Fig. 13 show that inherent multi-tenancy effect of cloud-based environment is almost negligible.} However, we observed the same performance for Dynomite +Redis in supporting AA+EC configuration for Redis as \bespokv in supporting AA+EC. The small error bars in Fig. 12 show that inherent multi-tenancy effect of cloud-based environment is almost negligible.

### 9.8 DPDK Optimization

We recently added support for DPDK based communication between clients, controllers, and datalets in to \bespokv. In this experiment, we show performance of socket versus DPDK based communication. We deployed a single shard on our local testbed (Section 9.1) and measured latency and throughput using YCSB. Each node in our local setup is equipped with Intel ethernet controller X540-AT2. We used Intel’s DPDK framework version 17.05. Fig. 14 shows that DPDK reduces latency by up to 65 percent. We also observe 3× improvement in throughput compared to socket based communication. Another interesting finding is that DPDK based communication results in more stable performance.

### 10 Failover & Data Recovery

We also evaluate how \bespokv performs in case of a node failure, and compare it with Redis’s replication used by Dynomite for failover recovery. In this set of tests, we use 3 shards (each with 3 replicas) to clearly reflect the impact of a failure on throughput. The workload consists of 1 million

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5. Twemproxy itself does not provide any consistency support.

6. Uniform workloads show similar trend, hence are omitted.
For MS+SC, we bring down the head node BESPOW Put, and the tail node for the read-intensive workload (95 percent Get, as shown in top half of the figure), to maximize the performance disruption on the respective workloads. For MS+EC, we take down the master node for the write-intensive workload and a random slave node for the read-intensive workload.

We observe that for MS+SC, Put throughput goes down by about 1/3 when the head node crashes at 20 sec, as we have 3 shards. The coordinator detects the node failure from the lack of heartbeat message before assigning the master role to the second node in the chain. The coordinator then launches a new controlet–datalet pair in recovery mode, and inserts the pair to the end of the chain once data recovery completes at around 35 sec. Meanwhile the throughput stabilizes. MS+EC failover shows a similar trend. The top half of Fig. 15a shows the impact of node failure on Get performance under MS topology. For MS+EC, killing the tail brings down Get throughput by 1/3. Once failure is detected, the coordinator makes the 2nd-from-last node in the chain the new tail, and updates the topology metadata. Once clients see the update, they reroute the corresponding Gets to the new tail. Hence, the throughput goes back to normal in 5 sec. MS+EC behaves differently as Gets are served by any of the 3 replicas. Thus, the slave failure drops throughput by only 1/9.

**AA Topology.** In BESPOKV’s AA and Dynomite (with Redis) failover test, we randomly kill a node at 20 sec and record the overall throughput. As shown in Fig. 15b, the throughput is slightly impacted in all cases, because both BESPOKV AA and Dynomite serve reads and writes from all replicas. Dynomite leverages Redis’ master-slave replication to recover data directly from the surviving nodes. We observe trend similar to Dynomite as BESPOKV also uses datalet’s callback functions to import and export the data. Please note that users can choose to add more replicas to increase the overall performance so that in case of a single node failure or a topology/consistency switch the performance drop is not significant enough to affect the HPC application.

**11 RELATED WORK**

Dynomite [19] adds fault tolerance and consistency support for simple data stores such as Redis. Dynomite only supports eventual consistency with AA topology. It also requires the single-server applications to support distributed management functions such as Redis’ streaming data recovery/migration mechanism. BESPOKV’s datalet is completely oblivious of the upper-level distributed management, which offers improved flexibility and programmability.

Pileus [70] is a cloud storage system that offers a range of consistency-level SLAs. Some storage systems offer tunable consistency, e.g., ManhattanDB [71]. FlexKV [72] is another flexible key-value store that can be configured to act as a non-persistent/durable store and operates consistently/consistently. Morphus [73] provides support towards reconfigurations for NoSQL stores in an online manner. MOS [74], [75] and hatS [76], [77] provide flexible and elastic resource-level partitioning for serving heterogeneous object store workloads. ClusterOn [78] proposes to offer generic distributed systems management for a range of distributed storage systems. MBal [36], [37] provides fine-grained service-level differentiation via flexible data partitioning. To the best of our knowledge, BESPOKV is the first generic framework that offers a broad range of consistency/topology options for both users and KV store application developers.

Vsync [22] is a library for building replicated cloud services. BESPOKV embeds single-node KV store application code and automatically scales it with a rich choice of services. Going one step further, BESPOKV can be an ideal platform to leverage Vsync to further enrich flexibility. EventWave [79] elastically scales inelastic cloud programs. PADS [80] provides policy architecture to build distributed applications. Similarly, mOS [81] provides reusable networking stack to allows developers to focus on the core application logic instead of dealing with low-level packet processing. BESPOKV focuses on a specific domain with a well-defined limited set of events–KV store applications.

**12 CONCLUSION**

We have presented the design and implementation of BESPOKV, a framework, which takes a single-server data store and transparently enables a scalable, fault-tolerant distributed KV store service. BESPOKV’s decoupled control and data plane architecture, configurability, and extensibility enable new solutions for emerging HPC systems and workloads. BESPOKV can be easily extended to offer advanced features such as range query, per-request consistency, polyglot persistence, and more. To the best of our knowledge, BESPOKV is first to support a seamless on-the-fly topology/consistency adaptation. As examples, we present a novel mechanism to make transitions from MS+EC to MS+SC, and from AA+EC to MS+EC. We also present several use cases to show effectiveness of BESPOKV to support HPC applications. Evaluation shows that BESPOKV is flexible, adaptive to new user requirements, achieves high performance, and scales horizontally. BESPOKV has been open-sourced and is available at https://github.com/ttdg/bespokv

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Energy-Efficient Parallel Real-Time Scheduling on Clustered Multi-Core

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Abstract—Energy-efficiency is a critical requirement for computation-intensive real-time applications on multi-core embedded systems. Multi-core processors enable intra-task parallelism, and in this work, we study energy-efficient real-time scheduling of constrained deadline sporadic parallel tasks, where each task is represented as a directed acyclic graph (DAG). We consider a clustered multi-core platform where processors within the same cluster run at the same speed at any given time. A new concept named speed-profile is proposed to model per-task and per-cluster energy-consumption variations during run-time to minimize the expected long-term energy consumption. To our knowledge, no existing work considers energy-aware real-time scheduling of DAG tasks with constrained deadlines, nor on a clustered multi-core platform. The proposed energy-aware real-time scheduler is implemented upon an ODROID XU-3 board to evaluate and demonstrate its feasibility and practicality. To complement our system experiments in large-scale, we have also conducted simulations that demonstrate a CPU energy saving of up to 67 percent through our proposed approach compared to existing methods.

Index Terms—Parallel task, real-time scheduling, energy minimization, cluster-based platform, heterogeneous platform

1 INTRODUCTION

MULTI-CORE processors appear as an enabling platform for embedded systems applications that require real-time guarantees, energy efficiency, and high performance. Intra-task parallelism (a task can be executed on multiple cores simultaneously) enables us to exploit the capability of the multi-core platform, and facilitates a balanced distribution of the tasks among the processors. Such a balanced distribution leads to energy efficiency [1]. Directed Acyclic Graph (DAG) task model [2] is one of the most generalized workload model for representing deterministic intra-task parallelism. Recently, quite some effort has been spent on developing real-time scheduling strategies and schedulability analysis of DAG tasks, few to mention [2], [3], [4], [5], [6], [7], [8].

There are several real-world application that uses the DAG model. For example, the work in [3] studies problems related to scheduling parallel real-time tasks, modeled as DAG, on multiprocessor architectures. In a homogeneous computing environment, a low-complexity compile-time algorithm for scheduling DAG tasks is proposed in [9].

Another example would be systems that control asynchronous devices, such as the local-area network adapters that implement real-time communication protocols. Since many of those applications are battery-powered, considering energy-efficient approaches for designing such a platform is crucial. Thanks to the fact that modern generation processors support dynamic voltage and frequency scaling (DVFS), where each processor can adjust the voltage and frequency at runtime to minimize power consumption, per-core energy minimization becomes possible during runtime. Despite the hardness of the problem [10], a significant amount of work has considered power minimization for non-parallel tasks on a multi-core platform (refer to [11] for a survey). Regarding parallel tasks, Guo et al. studied energy-efficient real-time scheduling for DAG tasks as an early research effort [12]. They adopted the federated scheduling and task decomposition framework [2] for minimizing system energy consumption via per-core speed modulation. As the only step (that we are aware of) towards energy-aware scheduling of real-time DAG tasks, they targeted an exciting problem and laid some of the foundations of this work. However, the attention of [12] is restricted to implicit deadline tasks with a system model of per-core DVFS.

Unfortunately, per-core DVFS becomes inefficient as it increases the hardware cost [13]. For balancing the energy efficiency and the hardware cost, there is an ongoing trend to group processors into islands, where processors in the same island execute at the same speed. For example, a big.LITTLE platform (e.g., ODROID XU-3 [14]) consists of high performance (but power-hungry) cores integrated into ‘big’ clusters and low-power cores into ‘LITTLE’ clusters. Such a platform executes several real-life applications with heavy computational demands (e.g., video streaming [15]) in an energy-efficient manner. Apart from the energy consumption issue, a multi-core platform enables task execution with...
high-performance demand and tight deadlines, essential for computation-intensive real-time systems, e.g., autonomous vehicles [16].

Despite the urgent need, to our knowledge, no work has been done that considered the energy-efficient scheduling of DAG tasks in clustered multi-core platforms, where cores form a group of frequency/voltage clusters. Such kind of system balances the energy efficiency and hardware cost compared to the traditional (with individual frequency scaling feature) multi-core models. The scheduling problem becomes highly challenging on such platforms because:

(i) The relationship between the execution time, frequency, and the energy consumption is nonlinear, making it highly challenging to minimize energy consumption while guaranteeing real-time correctness, i.e., none of the tasks miss their deadline.

(ii) Existing solution (e.g., [12]) relies on the assumption that each processor can freely adjust its speed. That solution performs poorly as the assumption is no longer valid under a more realistic platform model considered in this paper.

(iii) The speed of a cluster becomes unpredictable when shared by multiple tasks with sporadic release patterns.

Contribution. In this paper, we propose a novel technique for energy-efficient scheduling of constrained deadline DAG tasks in a cluster-based multi-core system. To the best of our knowledge, no work has investigated the energy-efficient scheduling of DAG tasks on such a cluster-based platform. It is also the first work that has addressed the power awareness issue considering constrained deadline DAG tasks. Specifically, we make the following contributions:

- We consider a more practical cluster-based system model where the cores must execute at the same speed at any time instant within each cluster.
- To better handle constrained deadlines, one need to capture the gaps between deadlines and upcoming releases, as well as handling sporadic releases. Considering a continuous frequency scheme, we first propose a novel concept of speed-profile to present the energy-consumption behavior for each task as well as each cluster, such that they could guide task partitioning in an energy-efficient manner. An efficient greedy algorithm is proposed to partition DAG tasks according to the constructed speed-profiles.
- We propose an approach to creating the speed-profile to adapt to the discrete frequency scheme. Also, we extend our approach to apply to the heterogeneous platform.
- To evaluate the effectiveness of our proposed technique, we implement it on the ODROID XU-3 board, a representative multi-core platform for embedded systems [14]. The experiments report that our approach can save energy consumption by 18 percent compared to a reference approach. For larger-scale evaluation, we perform simulations using synthetic workloads and compare our technique with two existing baselines [12], [17]. The simulation results demonstrate that our method can reduce energy consumption by up to 66 percent compared to the existing ones under the cluster-based platform setting.

Organization. The rest of the paper is organized as follows. Section 2 presents the workload, power, and platform models, and the problem statement. Section 3, describes the importance of creating a speed-profile for an individual task and the whole cluster. Section 4 discusses the approaches to create the speed-profile (considering both the continuous and discrete frequency mode) for each task. In this section, we also propose a greedy algorithm to allocate multiple tasks in the same cluster. Sections 5 and 6 presents the experimental and simulation results. Section 7 discusses related work including a detailed comparison with our existing work [12], [18]. Section 8 concludes this paper.

2 SYSTEM MODEL, PROBLEM STATEMENT, AND BACKGROUND

2.1 System Model and Problem Statement

Workload Model. We consider a set of sporadic parallel task denoted by $\tau = \{\tau_1, \ldots, \tau_n\}$, where each $\tau_i (1 \leq i \leq n)$ is represented as a DAG with a minimum inter-arrival separation (i.e., period) of $T_i$ time units, and a relative deadline of $D_i (\leq T_i)$ time units. An implicit deadline task has the same relative deadline and period, i.e., $D_i = T_i$. As a DAG task, the execution part of task $\tau_i$ contains a total of $N_i$ nodes, each denoted by $N_i^j (1 \leq j \leq N_i)$. A directed edge from $N_i^j$ to $N_i^{j+k} (N_i^j \rightarrow N_i^{j+k})$ implies that execution of $N_i^j$ can start if $N_i^{j+k}$ finishes for every instance (precedence constraints). In this case, $N_i^j$ is called a parent of $N_i^{j+k}$ ($N_i^j$ is a child of $N_i^{j+k}$). A node may have multiple parents or children. The degree of parallelism, $M_i$, of $\tau_i$ is the number of nodes that can be simultaneously executed. $C_i$ denotes the execution requirement of node $N_i^j$. $C_i := \sum_{j=1}^{N_i^j} c_j$ denotes the worst case execution requirement (WCET) of $\tau_i$.

A critical path is a directed path with the maximum total execution requirements among all other paths in a DAG. $L_i$ is the sum of the execution requirements of all the nodes that lie on a critical path. It is the minimum make-span of $\tau_i$, i.e., in order to make $\tau_i$ schedulable, at least $L_i$ time units are required even when number of cores is unlimited. Since at least $L_i$ time units are required for $\tau_i$, the condition $T_i \geq L_i$ (implicit deadline tasks) and $D_i \geq L_i$ (constrained deadline tasks) must hold for $\tau_i$ to be schedulable. A schedule is said to be feasible if upon satisfying the precedence constraints, all the sub-tasks (nodes) receive enough execution from their arrival times, i.e., $C_i$ within $T_i$ (implicit deadline) or $D_i$ (constrained deadline) time units. These terms are illustrated in Fig. 2a.

Platform Model. We consider a clustered multi-core platform, where processors within the same cluster run at the same speed (frequency and supply voltage) at any given time. Such additional restriction comparing to traditional multi-core platform makes the model more realistic in many scenarios. For example, our experiment is conducted on the ODROID XU-3 platform with one ‘LITTLE’ cluster of four energy-efficient ARM Cortex-A7 and one ‘big’ cluster of four performance-efficient ARM Cortex-A15. Note that we do not restrict the hardware-dependent energy parameters (e.g., $\alpha, \beta$ and $\gamma$ in the power model discussed below) to be identical across different clusters—these parameters can be derived using any curve-fitting method, e.g., [19].
Energy Model. Assuming frequency (speed) of a processor at a specific instant \( t \) is \( s(t) \) (in short, denoted as \( s \) ), then its power consumption \( P(s) \) can be calculated as

\[
P(s) = P_s + P_d(s) = \beta + \alpha s^\gamma.
\]

Here, \( P_s \) and \( P_d(s) \) respectively denote the static and dynamic power consumption. Whenever a processor remains on, it introduces \( P_s \) in the system (due to leakage current). Switching activities introduce \( P_d(s) \) which is frequency dependent and represented as \( \alpha s^\gamma \). Here, the \( \alpha > 0 \) depends on the effective switching capacitance [20]; \( \gamma \in [2, 3] \) is a fixed parameter determined by the hardware; \( \beta > 0 \) represents the static part of power consumption. From this model, the energy consumption over any given period \([b, f]\) is calculated as \( E(b, f) = \int_b^f P(s(t)) \, dt \).

Our motivation behind selecting this power model comes from the fact that it complies with many existing works in the community, few to mention [10], [12], [18], [20], [21], [22], [23]. Beside this, recently this model was shown to be highly realistic by showing its similarity with actual power consumption [21]. Fig. 1 shows comparison between the original power consumption results from [24] and the power model in Equation (1).

Assumptions. In this paper, we make the following assumptions: (i) we focus on CPU power consumption, and (ii) Dynamic power management (DPM) is not considered. Appendix B, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TPDS.2020.2985701, provides the details behind these assumptions, their impacts, and some hints to overcome the drawbacks.

Problem Statement. Considering a constrained deadline sporadic DAG task-set on a clustered multi-core platform, we focus on finding a correct scheduling strategy, while the CPU power consumption is minimized.

2.2 Background and Existing Concepts

In this section, we describe some existing concepts and techniques for handling real-time parallel task scheduling, and that constitute an initial step for our proposed work.

Task Decomposition. The well-known task decomposition technique [2] transforms a parallel task \( t_i \) into a set of sequential tasks as demonstrated in Fig. 2b. Upon task decomposition, each node \( N_i \) in \( t_i \) is converted into an individual sub-task with its scheduling window (defined by its own release time and deadline) and execution requirement (\( e_i \)). The allocation of release time and deadline respect all the dependencies (represented by edges in the DAG). Considering that a task is allowed to execute on an unlimited number of cores, starting from the beginning, a vertical line is drawn at every time instant where a node \( N_i \) starts or ends. So the DAG is partitioned into several segments which may contain single/multiple thread(s). Threads assigned to the same segment share equal amount of execution length; e.g., \( N_1, N_2, \) and \( N_3 \) all have 2-time units assigned to the 3rd segment, as demonstrated in Fig. 2b.

Segment Extension. The deadline for each node via task decomposition may be unnecessarily restrictive, e.g., the decomposition of the DAG in Fig. 2a will restrict \( N_3 \) within the 2nd and 3rd segment. To eliminate such unnecessary restriction and allow \( N_3 \) to execute in the 4th segment, segment extension should be applied, e.g., the green rectangle for \( N_3 \) in the 4th segment in Fig. 2b.

Intra-Task Processor Merging. After applying task decomposition and segment extension upon a DAG task \( t_i \), some of these cores (where \( t_i \) is allocated) can be very lightly loaded. Those core cause massive leakage power consumption in the long run and should be avoided when necessary. Intra-task merging [12] seeks to merge those cores to gain overall energy efficiency by reducing the total number of active cores. For example, in Fig. 2b, the third core (executing \( N_3 \)) is lightly loaded, and thus it is better to merge all the execution into the second core and shut it off completely. Such a reduction on the number of active cores minimizes leakage power consumption (see Equation (1) and Fig. 2 in [12]) as well as the total number of clusters.

3 Speed-Profile for Task and Cluster

This section discusses how different tasks share a cluster where all processors in a cluster execute at the same speed. When multiple tasks share a cluster, they may not align well due to sporadic releases and different periods. In a cluster-based platform, the processor having the maximum speed dominates the others in the same cluster. Hence, existing energy-saving techniques may perform poorly in a cluster-based platform. To tackle this problem, we propose a new concept called speed-profile. We provide the definition of speed-profile and its motivation in Section 3.1. Section 3.2 describes how speed-profiles are handled when two tasks are partitioned into the same cluster.

3.1 Speed-Profile for Each DAG

Interesting energy-saving techniques (e.g., segment extension) have been proposed in [12] for the implicit deadline tasks. For the constrained deadline tasks, this technique becomes incompetent because of the non-negligible idle gaps between the task deadline and its next release. For example, consider the task \( t_i \) in Fig. 2b with \( D_i = 10 \) and \( T_i = 12 \). Segment extension can stretch \( N_3 \) to the end of the 4th segment but cannot utilize the idle time of 2 units. Besides, the sub-
optimal solution provided in [12] becomes non-convex (in a convex function, we can find the global maximum or minimum, for some variables of this function, which does not hold for a non-convex function) in a cluster-based platform (see Lemma 1).

Lemma 1. In a cluster-based platform, the convex optimization problem constructed in [12] becomes non-convex.

Proof. The following set of constraints ensure the real-time correctness for each node $N_i \in \tau_i$, i.e., $N_i$ receives enough time to finish execution within its scheduling window

$$\forall l : N_i^l \in \tau_i \therefore \sum_{j=b_i^l}^{d_i^l} t_j^l s_{i,j} \geq c_{i}^l.$$  \tag{2}

We introduce the following inequalities to bound the total length for all segments in task $\tau_i$

$$\sum_{j=1}^{l} t_j^l \leq T_i.$$  \tag{3}

Any value of execution speed and segment length ensures real-time correctness if Equations (2) and (3) are respected. However, the work in [12] considered that the execution speed of a node, $N_i^l$, is constant within its scheduling window (from $b_i^l$ to $d_i^l$), and can be represented by a function of nodes execution requirement and its scheduling window. Also, the work in [12] considered that a single DAG executes at a time, and, hence the execution speed of a node is not affected by the execution speed of other nodes (of other tasks). In this work, we consider the cluster-based platform, and the execution speed of a node depends on the execution speed of other nodes (of other tasks) in the same cluster. As a result, we cannot express the execution speed of a node as a function of its execution requirement, resulting in quadratic inequality constraints (Equation (2)). This makes the optimization problem non-convex.

Due to the characteristics of a clustered platform, at each instant, all cores in a cluster must execute at the speed of the fastest one. If these tasks are not well aligned (concerning their execution speed), the cluster as a whole will perform poorly (w.r.t. energy efficiency). Assigning tasks with similar speed shape on the same cluster may not be an energy efficient option (due to their sporadic releases pattern). As a result, we may not be able to reduce the cluster speed at all.

To address this issue, we propose a novel concept of speed-profile to capture the energy consumption behavior of all possible alignment scenarios.

Definition 1. The Speed-profile of a task describes the percentage/likelihood of all possible speeds that the task may execute at over a period. It is a random variable $S$ with an associated probability function (PF) $f_S(s) = \mathbb{P}(S = s)$, where $s$ is a speed from the finite set of possible speeds, and $f_S(s)$ represents the portions of the time (likelihoods) when it is running at speed $s$.

Example 2. Let us consider a task $\tau_i$ with $T_i = 15$ executing at a speed of 0.6 for 5 time units (not necessarily to be continual), and at a speed of 0.5 for the rest of the time. The speed-profile of the task is thus $S_i = \begin{pmatrix} 0.6 & 0.5 \\ 5/15 & 10/15 \end{pmatrix} = \begin{pmatrix} 0.6 & 0.5 \\ 0.33 & 0.67 \end{pmatrix}$. At any specific time, $t$, there is about 33 percent probability that the cores are running at the speed of 0.6 unit and about 67 percent probability that the cores are running at the speed of 0.5 unit.

It is evident that from another task’s point of view, the speed-profile provides probabilistic information on how the task of interest would restrict the lower bound to the speed of the cluster over time. As the alignment of releases between any two tasks is unknown, we assume in the future analysis that any phase difference is of equal chance over the long run.

Remark 1. The speed-profile $S_i$ of a given task $\tau_i$ remains the same for an initial phase (release offset) $\phi_i \geq 0$. Regarding inter-task combinations, we assume uniform distribution for the phase of any task; i.e., $\phi_i \sim U[0, T_i]$.

Section 4.1 details the calculation for task speed-profile. Here, we describe the calculation of the cluster speed-profile when two tasks are combined on to the same cluster.
3.2 Speed-Profile for the Cluster

As stated earlier, the property of the clustered platform and sporadic arrival pattern of a task makes the exact speed of the cluster unpredictable at a specific time instant (see Fig. 3 and Example 1). As a result, when two tasks $t_i$ and $t_j$ (with speed-profiles) are being considered allocating to the same cluster, we need to construct the merged speed-profile of the cluster (executing them both). To perform such calculation, we introduce a special $\odot$ operator that takes the maximum of the two profiles on a probability basis.

Definition 2. The special operator $\odot$ operates on two (or more) random variables $X$ and $Y$. During this operation, each entry $X_{ij} \in X$ is compared with each entry $Y_{ij} \in Y$ and the value $Z_{ij}$ is calculated as $Z_{ij} = \max(X_{ij}, Y_{ij})$, with a combined (multiplied) probability. If there are multiple occurrences of an entry, all of them are merged into a single entry, and their associated probability are summed together.

Example 3. Let $S_i = \begin{pmatrix} 6 & 5 \\ 0.4 & 0.6 \end{pmatrix}$, $S_j = \begin{pmatrix} 6 & 2 \\ 0.4 & 0.6 \end{pmatrix}$. Then

$$S_i \odot S_j = \begin{pmatrix} 6 \cdot 0.4 & 5 \cdot 0.6 \\ 6 \cdot 0.24 & 6 \cdot 0.36 \end{pmatrix} = \begin{pmatrix} 2.4 & 3 \\ 1.44 & 2.16 \end{pmatrix}.$$

Note that we allocate two different DAGs (with same/different periods) to the same cluster. The speed-profile indicates how long a DAG executes at different speeds within its deadline, i.e., the probability that a DAG executes at a specific speed. The task's period becomes irrelevant as speed-profile is a probability-based measurement. Once $t_i$ and $t_j$ are allocated to the same cluster, we use $S_{ij}$ to denote the speed-profile of the cluster (see Example 3).

In summary, energy minimization in a cluster-based platform is challenging because of sporadic release pattern and the idle gaps between a task deadline and its period. To tackle these problems, we have introduced the concept of speed-profile for both an individual task and a cluster where multiple tasks can be allocated.

4 TASK PARTITIONING ALGORITHM

The ultimate goal of the paper is to partition all DAGs into clusters, such that overall platform energy consumption is minimized. Recall that on a clustered multiprocessor platform, at a given instant, all processors in the same cluster must execute at the same speed. Due to this property of a cluster-based platform, if two tasks that are not well-aligned (in terms of execution speed) are allocated to the same cluster, it will result in reduced energy efficiency. So, we have proposed the concept of speed-profiles (refer to Section 3) which is a tool to measure the potential long-term energy saving of a cluster when partitioning any pair of DAGs into this cluster.

So far we have discussed the importance of the concept of speed-profile but did not mention how to create them given a DAG task, which is the focus on Section 4.1. Then, Section 4.4 describes the task-to-cluster partitioning algorithm.

4.1 Creating the Speed-Profile of a Task

Given a DAG task $t_i$, we provide two approaches to create the speed-profile $S_i$.

1. Although the appearance of the proposed operator is identical to [25], the calculation is quite different. This is due to the “larger value dominating” nature of the platform model considered in this paper.

Approach A: Considering the Maximum Speed from all the Cores. Upon applying the task decomposition, segment extension, and intra-task processor merging techniques (Section 2), some vital information (e.g., the speed of a core at a specific time and number of cores required) becomes available. This information plays a role to calculating the speed-profile $S_i$ of task $t_i$. At any time instant $t$, we consider the maximum speed from all the cores available. It ensures the sufficient speed so that even the heaviest node can finish execution within its scheduling window (defined after task decomposition). We consider constrained deadline (i.e., $D_i \leq T_i$), so the task must have to finish by $D_i$ and rest of the time $(T_i - D_i)$ it remains idle. For each segment $j \in t_i$, (summation of the length of these segments is equal to $D_i$), we create a pair $(s_{i,j}, p_{i,j})$. For the $j^{th}$ segment, $s_{i,j}$ and $p_{i,j}$ respectively denote the maximum execution speed and the probability that the cluster will run at this speed. Let, $M$ cores are allocated to $t_i$. At $j^{th}$ segment, we calculate $s_{i,j}$ and $p_{i,j}$ as follows:

$$s_{i,j} = \max_{k \leq M} \{s_{i,j,k}\}, p_{i,j} = \frac{t_{i,j}}{T_i}.$$

Here, $s_{i,j,k}$ denotes the speed of $k^{th}$ core at $j^{th}$ segment and $t_{i,j}$ is the length of $j^{th}$ segment. The speed-profile $S_i$ will be

$$S_i = \begin{pmatrix} s_{i,1} & s_{i,2} & \cdots & s_{i,z} \\ p_{i,1} & p_{i,2} & \cdots & p_{i,z} \\ 0 & (T_i - D_i)/T_i \end{pmatrix}.$$

The last pair reflects the fact that the core remains idle for the $(T_i - D_i)$ time units at the end of each period.

Example 4. Consider a task $t_i$ with $T_i = 15$, $D_i = 12$ and $C_i = 6.5$. Let, the task is partitioned into three segments of length 5, 7 and 3 time units respectively, where the processor is executing at a maximum speed of 0.6 in the first segment, speed of 0.5 in the second segment, and remain idle in the third segment. The speed-profile is

$$S_i = \begin{pmatrix} 0.6 & 0.5 & 0 \\ 0.33 & 0.47 & 0.2 \end{pmatrix}.$$

Note that, if a cluster contains a single task $t_i$, then $S_i$ also represents the cluster speed-profile. If $t_i$ and $t_j$ (or more tasks) are executing on the same cluster, then the technique described in Section 3.2 needs to be applied before making any choices. The greedy choosing approach for task partitioning is detailed in Section 4.4.

Approach B: A Single Speed Throughout. Theorem 4 of [12] shares a valuable insight: The total energy consumption (assuming processor remains on) is minimized in any scheduling window when execution speed remains uniform (the same) throughout the interval. Motivated by it, we propose another approach of selecting a single speed for a DAG task (job) during the whole duration from its release until its deadline.

2. Note that [12] considered that the speed remains constant within a scheduling slot for each processor. Also, they assumed per core speed scaling and calculated the speed within each scheduling slot through a convex optimization method. This paper considers the clustered platform where the objective function becomes non-convex (see Lemma 1) and thus the existing approach is inefficient.
In this approach, we consider the maximum workload (or the execution requirement) from all the cores available and determine the aggregated workload. Upon dividing the aggregated workload by the deadline, we get the desired single speed. Let $M$ cores be allocated to task $t_i$. At $j$th segment, the execution requirement of the $k$th core is denoted by $w_{i,j,k}$, which is calculated as follows:

$$w_{i,j,k} = s_{i,j,k} \times t_j^k.$$  

We determine the maximum execution requirement as follows:

$$w_{i,j} = \max_{k \leq M} \{w_{i,j,k}\}.$$  

Let $Z$ denotes the total number of segments in $t_i$. The maximum total workload $w_i$ and the desired single speed $s_i$ is calculated using the following equations:

$$w_i = \sum_{j=1}^{Z} w_{i,j}, \quad s_i = \frac{w_i}{D_i}. \quad (4)$$  

Other than the idle pair $(0, (T_i - D_i)/T_i)$, we consider a single speed throughout the deadline so only a single pair $(s_i, p_i)$ is required, where $s_i = w_i/D_i$ and $p_i = D_i/T_i$.

**Example 5.** Consider the task described in Example 4 ($T_i = 15$, $D_i = 12$ and $C_i = 6.5$). It must finish 6.5 unit of workloads within 12-time units. Using this approach its speed-profile is

$$S_i = \left( \begin{array}{c} 0.54 \& 0 \\ 0.8 \& 0.2 \end{array} \right).$$

**Lemma 2.** If a task $t_i$ executes according to the speed-profile $S_i$, it guarantees real-time correctness.

**Proof.** It has been observed in [12] that the following constraint guarantees the real-time correctness

$$\forall l : N_i^l \in t_i \ni \sum_{k=b_i^l}^{d_i^l} t_k^l S_k^M_i \geq c_i^N_i. \quad (5)$$  

Here, $b_i^l$ and $d_i^l$ denotes the release time and deadline of $N_i^l$, $M_i^l$ denotes the node-to-processor mapping and $S_k^M_i$ is the speed of the processor (where $N_i^l$ is allocated) at $k$th segment. Unlike to [12], at any time instant $t$, we choose either the maximum speed from all the cores running on the same cluster (Approach A) or a single speed that can guarantee the maximum execution requirement for the whole duration up to $t_i$’s deadline (Approach B). So, at any time instant, the cluster speed is larger or equals to the speed of any individual core. Considering Equations (2) and (5) we can deduce that

$$\forall l : N_i^l \in t_i \ni \sum_{k=b_i^l}^{d_i^l} t_k^l S_k^M_i \geq \sum_{k=b_i^l}^{d_i^l} t_k^l S_k^M_i \geq c_i^N_i.$$  

So, we conclude that Executing a task with speed according to the speed-profile $S_i$ guarantees real-time correctness.

An Efficient Approach for Implicit Deadline System. By adopting simple modification in Equation (4), it is possible to apply the process mentioned above for the implicit deadline tasks also. The workload $w_i$ should be divided by the period instead of the deadline. We consider the same speed through the task period, so only a single pair $(s_i, p_i)$ is required, where $s_i = w_i/T_i$ and $p_i = 1$.

**Example 6.** Now we create the speed-profile for the task described in Examples 4 and 5 considering implicit deadline. So it has $T_i = D_i = 15$ and $C_i = 6.5$. Let’s assume that it is executed at a speed of 0.6 for 5-time units, at a speed of 0.35 for 10-time units. According to Approach A, the speed-profile is

$$S_i = \left( \begin{array}{c} 0.6 \& 0.35 \\ 0.33 \& 0.67 \end{array} \right),$$  

and according to Approach B, the speed-profile is

$$S_i = \left( \begin{array}{c} 0.43 \\ 1 \end{array} \right).$$

### 4.2 Discretization of the Speed-Profile

In Section 4.1, we have described two approaches to create the speed-profile for an individual task. While creating the speed-profiles, those approaches assume a continuous frequency scheme. From a practical point of view, discrete frequency mode should be preferred over the continuous frequency mode, because a real platform supports only a set of frequencies. Now, we describe the technique to discretize all the speeds available in a speed-profile (assuming that the speed-profile is already created).

Suppose, we execute a task $t_i$ (and its speed-profile is $S_i$) in a real-platform, and this platform supports only those speeds available on a speed-set $Z$. Note that the content of $Z$ is dependent on the platform. For example, ODROID XU-3 supports a frequency range of 200-1400 MHz (LITTLE cluster) and 200-2000 MHz (big cluster) with scale steps of 100 MHz. Now, for each entry $s_{i,j} \in S_i$, we find the minimum speed $Z_k \in Z$, where $Z_k \geq s_{i,j}$. Once, we find an appropriate $Z_k$, we set the value of $s_{i,j}$ as $s_{i,j} = Z_k$.

**Example 7.** Consider a task $t_i$ with the same speed-profile from Example 4. Let us assume that we will execute $t_i$ in a platform where $Z = \{0, 0.2, 0.4, 0.55, 0.75, 1\}$, i.e., this platform supports only six discrete speeds, and all the speeds are normalized w.r.t. the maximum speed supported by this platform. Considering the speed-profile $S_i$ (from Example 4) and the speed-set $Z$, we find that:

(a) $s_{i,1} \leq \{Z_5 \text{ and } Z_6\}$  
(b) $s_{i,2} \leq \{Z_4 \text{ and } Z_6\}$  
(c) $s_{i,3} \leq \{Z_1, Z_2, Z_3, Z_4, Z_5 \text{ and } Z_6\}$.

Now, we choose the minimum $Z_k \in Z$ such that $Z_k \geq s_{i,j}$. So, we assign $Z_k$ to $s_{i,1}$, $Z_1$ to $s_{i,2}$, and $Z_1$ to $s_{i,3}$. Now, the updated (i.e., discretized) speed-profile becomes

$$S_i = \left( \begin{array}{c} 0.75 \& 0.55 \& 0 \\ 0.33 \& 0.47 \& 0.2 \end{array} \right).$$

**Theorem 1.** When a task executes with its discretized speed-profile, it guarantees that the task will not miss the deadline.
We have shown in Lemma 2 that a task \( t_i \) will not miss the deadline if executed according to its speed-profile \( S_i \). If we discretize \( t_i \)'s speed-profile and execute \( t_i \) according to this speed-profile, then the task still guarantees the real-time correctness. This is because any speed \( s_{i,j} \) of the discretized speed-profile is greater than or equal to \( s_{i,j} \) when it was continuous.

**Proof.** We have shown in Lemma 2 that a task \( t_i \) will not miss the deadline if executed according to its speed-profile \( S_i \). If we discretize \( t_i \)'s speed-profile and execute \( t_i \) according to this speed-profile, then the task still guarantees the real-time correctness. This is because any speed \( s_{i,j} \) of the discretized speed-profile is greater than or equal to \( s_{i,j} \) when it was continuous.

### 4.3 Handling Platform Heterogeneity

In this section, we discuss a specific type of multi-core platform with diverse computing abilities: heterogeneous multi-core platform. We first discuss different types of heterogeneous platforms, and then explain how our proposed techniques can be extended to handle heterogeneity. In a heterogeneous platform, different cores have different computational capabilities. In terms of speed, Funk defined a widely-accepted classification of the heterogeneous platform [26] as follows, where the speed of the processor denotes the work completed (while executing a task) in a single-time unit by this processor.

(i) *Identical multiprocessors*: On Identical multiprocessors, all tasks are executed at the same speed on any processor;

(ii) *Uniform multiprocessors*: On Uniform multiprocessors, all the tasks execute at the same speed if allocated on the same processor, but at a different speed on different processors. So, the execution speed of a task depends on the processor where the task is allocated.

(iii) *Unrelated multiprocessors*: On Unrelated multiprocessors, execution speeds of different tasks may vary on the same processor, i.e., a task’s execution speed depends on both the task itself and the processor where it is allocated.

In a heterogeneous platform, each core is designed with a different computational capability, and an efficient task-to-core mapping improves the system resource efficiency. In the context of energy efficiency, two major directions have been mentioned in [27] for any heterogeneous platform:

(i) Find an appropriate core/cluster for task mapping to reduce the overall power consumption of the whole platform.

(ii) Deploy energy-aware scheduling techniques on each core/cluster to reduce power consumption.

Our proposed approach covers both directions. First, we use speed profile to identify efficient core/cluster to task mapping and then try to reduce the overall cluster speed as much as possible. It works for an identical heterogeneous platform (a.k.a., homogeneous multiprocessor) as task-to-core mapping does not impact energy consumption much.

### TABLE 1

**Estimated Parameters for Different Cluster of an ODROID XU-3 Board**

<table>
<thead>
<tr>
<th>Cluster Type</th>
<th>( \beta(W) )</th>
<th>( \alpha(W/Hz^\gamma) )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>big</td>
<td>0.155</td>
<td>( 3.03 \times 10^{-9} )</td>
<td>2.621</td>
</tr>
<tr>
<td>LITTLE</td>
<td>0.028</td>
<td>( 2.62 \times 10^{-9} )</td>
<td>2.12</td>
</tr>
</tbody>
</table>

*This table is adopted from [15].*

### TABLE 2

**The “Uncore” Power Consumption for Different Cluster of an ODROID XU-3 Board**

<table>
<thead>
<tr>
<th>Freq (GHz)</th>
<th>2</th>
<th>1.8</th>
<th>1.6</th>
<th>1.4</th>
<th>1.2</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>big cluster(W)</td>
<td>0.8</td>
<td>0.528</td>
<td>0.39</td>
<td>0.309</td>
<td>0.244</td>
<td>0.182</td>
</tr>
<tr>
<td>Freq (GHz)</td>
<td>1.4</td>
<td>1.2</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>LITTLE cluster(W)</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

*This table is adopted from [15].*

Now, we extend our approach to apply to the uniform heterogeneous platform by modifying the parameters in the power model in Equation (1), i.e., setting different \( \alpha, \beta \), and \( \gamma \) values for the ‘big’ and ‘LITTLE’ cluster. Under such consideration, different clusters no longer share the same power model, and the same task may have different execution requirements on different clusters. We report the estimated values of \( \alpha, \beta \), and \( \gamma \) in Table 1. These parameters are adopted from [15]. The work in [15] estimated these parameters for the ODROID XU-3 board using the real power measurements along with a curve fitting method. They have also assumed that there is another contributor to the total power consumption of a cluster, i.e., the “uncore” power consumption (reported in Table 2). The “uncore” power consumption introduced in the system from some components other than a processor, e.g., a shared cache. Similar to the dynamic power consumption, the “uncore” power consumption also depends on the processor frequency. However, unlike the dynamic power consumption, there is always some “uncore” power consumption as long as the cluster remains on (even if there is no workload on a processor).

Considering all the parameters from Tables 1 and 2, we bring the following modification in Equation (1)

\[
P(s) = N_p \beta + \alpha s^\gamma + P_c(f).
\]

Here, \( N_p \) denotes the number of cores per cluster, and \( P_c(f) \) denotes the “uncore” power consumption.

We have a different power model for the “big” and the “LITTLE” cluster, but we still don’t know what the basis of assigning a task to a cluster is. Recall that, while creating the speed-profile, some vital information (e.g., the speed of a core at a specific time) were known to us (Section 4.1). If the execution speed of a task is greater than a certain threshold at any point from its release to its deadline, then we assign this task to the big cluster. Else, we assign this task to the LITTLE cluster. For the platform we consider (ODROID XU-3), we set the threshold to 0.7. It is the ratio of the maximum speed supported by the big cluster and the LITTLE cluster (see Table 2).

### 4.4 Task Partition: Greedy Merging With Speed-Profiles

We are now equipped with tools (speed-profiles) to measure the potential long-term energy saving of a cluster when partitioning any pair of DAG tasks into it. This subsection describes the scheme for selecting pair by pair so that the total number of clusters can be significantly smaller than the total number of tasks.
Let, we decide for each task whether it should be allocated on a LITTLE or a big cluster using the technique described in Section 4.3. To select a (task) pair that will share the same cluster, we greedily choose the pair that provides maximum power saving, as depicted in Algorithm 1. Note that we allow the pairing of two DAGs that are not merged previously. Also, if any task uses more cores than what is available in a cluster, that task cannot be merged with that cluster.

Algorithm 1. Greedy Merging

1: Input: Task-set \( \tau \), with speed-profile \( S_i \) (computed using approach A or approach B) for each task
2: Output: Speed-profile \( \hat{S} \) (with processor power saving).
3: \( S, \hat{S} \leftarrow \emptyset \) \( \triangleright \) All the possible/selected speed-profiles
4: for \( i = 1 \) to \( n \) do
5: \( \quad \quad \text{for } j = i+1 \text{ to } n \text{ do} \)
6: \( \quad \quad \quad S_{ij} \leftarrow S_i \cap \hat{S}_j; \quad \hat{S} \leftarrow \hat{S} \cup S_{ij}; \)
7: \( \quad \text{end for} \)
8: \( \text{end for} \)
9: while \( \exists S_{xy} \in \hat{S} \) and \( S_{xy} \) provides non-zero power saving do
10: \( \quad S_{xy} \leftarrow \text{the pair from } \hat{S} \text{ with maximum power saving} \)
11: \( \quad \hat{S} \leftarrow \hat{S} \cup S_{xy} \)
12: \( \quad \text{for } k = 1 \text{ to } n \text{ do} \)
13: \( \quad \quad \hat{S} \leftarrow \hat{S} - S_{kx} - S_{yk} - S_{ky} \)
14: \( \quad \text{end for} \)
15: \( \text{end while} \)
16: return \( \hat{S} \).

Algorithm 1 creates two empty lists \( S \) and \( \hat{S} \) that will contain all the possible and selected speed-profiles (Line 3). Lines 4–8, calculate all the possible speed-profiles and insert them into \( \hat{S} \). We greedily select a pair of DAGs that provide the maximum power saving (calculated using Equations (6) and (10) from [12]) and update the list \( \hat{S} \) by removing the pair from any further merging (Lines 9–15). The list \( \hat{S} \) is also updated by adding the selected pair (Line 11). We conclude by returning the updated list \( \hat{S} \) (Line 16).

**Theorem 2.** Executing a task with a speed according to the cluster speed-profile guarantees real-time correctness.

**Proof.** We have shown in Lemma 2 that a task \( \tau_i \) will not miss the deadline if executed according to its speed-profile \( S_i \). If \( \tau_i \) share a cluster with another task \( \tau_j \) and executes according to the merged (i.e., cluster) speed-profile \( S_{ij} \), then it still guarantees the real-time correctness, because \( S_{ij} \geq S_i \) holds at any time instant.

**Remark 2.** For \( n \) tasks, the time complexity to generate all possible speed-profiles, \( S \), is \( O(n^2Z) \), where \( Z \) is the maximum number of segments of all DAGs in the set after decomposition (related to the structure and size of the DAGs). Algorithm 1 greedily choose a speed-profile by iterating through \( S \) and then update, which takes \( O(n^2) \) time as well. Thus the total complexity of the proposed method is \( O(n^2) \).

In summary, we have proposed two methods (Section 4.1) to create the speed-profile for a constrained-deadline DAG. We also show that if a task executes according to the speed-profile, it ensures real-time correctness. According to the techniques provided in Section 3, we could evaluate and compare all potential pairs of the combination by calculating the cluster speed-profile after merging. Finally, Section 4.4 discussed how to use these speed-profiles to find suitable partners to share a cluster greedily.

## 5 System Experiments

In this section, we present experimental results conducted on an ODROID XU-3 board. The platform runs on Ubuntu 16.04 LTS with Linux kernel 3.10.105. It is fabricated with Samsung Exynos5422 Octa-core SoC, consisting of two quad-core clusters, one ‘LITTLE’ cluster with four energy-efficient ARM Cortex-A7 and one ‘big’ cluster with four performance-efficient ARM Cortex-A15. Four TI INA231 power sensors are integrated onto the board to provide real-time power monitoring for the A-7 and A-15 clusters, GPU, and DRAM. An energy monitoring script, enoxu3 [28], is used to log energy consumption of the workloads.

**DAG Generation.** In this experiment, we generate two task sets each with 300 DAGs, and use the widely used Erdos-Renyi [29] method to generate a DAG. We tune a parameter \( p \), that denotes the probability of having an edge between two nodes. In this experiment, we set \( p \) to 0.25 generate DAGs with an uncomplicated structure. If a disconnected DAG is generated, we add the fewest number of edges to make it connected. For experimentation, we have considered arbitrary task periods, and it is determined using Gamma distribution [30]. We set the periods with \( T_i = L_i + 2(C_i/m_i)(1 + \Gamma(2, 1)/4) \) [2]. Here, \( L_i \) is the critical path length of \( \tau_i \), calculated according to the definition of \( L_i \) (refer to Section 2).

After generating the topology of each DAG of a set, we partition them into two subsets according to the proposed approach, one to the “big” and the other one to the “LITTLE” cluster, and measure the energy consumption over the hyper-period of all DAGs. We use \( rt \)-app [31] to emulate the workload for each node. \( rt \)-app simulates a real-time periodic load and utilizes the POSIX threads model to call and execute threads. For each thread, an execution time needs to be assigned. In this experiment, for each node, we randomly select an execution time ranged between [300 ms, 700 ms]. \( rt \)-app itself has a latency that varies randomly between 13 – 150 ms per thread. Therefore, we add the maximum latency of \( rt \)-app, i.e., 150 ms, to the execution time of each thread from an analytical point of view.

**DAG Scheduling.** We use the Linux built-in real-time scheduler \( \text{sched} \_\text{FFFO} \) to schedule the DAGs. Compared to the other system tasks, DAGs are assigned with higher priorities so that they can execute without interference. Our approach is also applicable to other preemptive schedulers which feature the work-conserving property.

**Frequency Scaling.** According to the frequency/speed-profile (Section 4), we use \( \text{cpufreq-set} \cdot \text{program} \) (from \( \text{cpufrequtils} \) package) to change the system’s frequency online. We use the ODROID XU-3 board, where scaling-down (up) the frequency of the big cluster takes at most 60 (40) ms, respectively. On the LITTLE cluster, both the operation takes at most 15 ms. Due to this delay, the hyper-period of all DAGs becomes large (230s, in this experiment). We detail the reasons behind this delay in Appendix B.2, available in the online supplemental material.

**The Reference Approach.** Since no work has studied the same problem considered in this paper, we do not have a
direct baseline for comparison. So, we propose a reference approach based on the studies for energy-efficient scheduling of sequential tasks [32]. They assigned an operational frequency to each task, and at run-time, schedule them according to their frequency. In this reference approach, we compute an operational frequency for each DAG. This frequency stretches out execution length of these DAGs as much as possible without violating their deadlines. As stated earlier, the reference approach executes the DAGs with the same partition, but without the merging techniques proposed in Section 4.

**Results.** The experimental results are plotted in Figs. 4 and 5. In these figures, we show (i) the energy consumption over the hyper-period (230s), where the three lines show the energy consumption of the big and LITTLE cluster, and the total system; and (ii) frequency variation during the runtime, where the diamond and star marks denote the operational frequency of the big and the LITTLE cluster at a specific time instant, respectively. Note that the GPU and DRAM also contribute the energy consumption of the total system. Hence, the total energy consumption is a bit higher than the summation of the contribution of the big and the LITTLE cluster, but it is observed that there is a negligible difference for the energy consumption of GPU and DRAM between the two approaches. Besides, it is worth noticing that this energy consumption also accounts for energy consumption of the operating system.

Table 3 summarizes the comparison of the experimental results, where the energy consumption of the two clusters and the total system is presented, and the energy saving from our approach is given. As can be seen, our approach consumes 312 J and 32 J on the big and the LITTLE cluster, respectively. Comparing to the reference approach, we save energy consumption by 20 and 16 percent. In total, our approach saves energy consumption by 18 percent.

The result can be justified as the reference approach changes the frequency for each DAG, while ours have a fine-grained frequency adjustment at each segment (Section 4.1), and could scale down the frequency if required. Fig. 6 presents the frequency occurrence probability of two clusters which is recorded per second by emox3. We observe that within the same time interval the reference approach has a higher probability to execute at a higher frequency, while our approach is more likely to execute at the lower frequencies, thus reducing the energy consumption.

**Remark 3.** Each heavy DAG \((C_i > T_i)\) needs two or more cores while executing and the ODROID XU-3 board contains four cores per cluster. So, in this experimental setup, we can not execute more than four heavy DAGs at a time. Such a restriction is not applicable to the light DAGs \((C_i / C_20 < T_i)\). We also consider that a heavy DAG cannot be allocated in multiple clusters.

### 6 Simulations

For large-scale evaluation, we perform simulations and compare the results with existing baselines. We generate DAGs using the Erdos-Renyi method (Section 5). We consider two types of task periods; (a) harmonic periods, where the task period \(T_i\) is enforced to be an integral power of 2. We define \(T_i = 2^a\), where \(a\) is the minimum value such that \(2^a / L_i < T_i\), where \(L_i\) is the critical path length of \(t_i\); (b) arbitrary periods, \(T_i\) is determined using Gamma distribution (see Section 5).

We compare our approaches with some existing baselines studied in [12], [17], [33]. Total power consumption by our approach and by these baselines are calculated using...
Equation (6). As mentioned earlier, [12] considered per-core DVFS, i.e., each core individually is an island of the cluster-based platform. For a fair comparison, according to the scheduling policy of [12], when a task is allocated on some cores at any time instant $t$, we choose the maximum speed among all these cores. We consider [12] as a baseline because that work is closely related to ours. Although they have considered per-core DVFS and restrict their attention only to implicit deadline tasks, the task and the power model are same. Besides, although this work and [12] propose different approaches to power saving, the initial (preparation) steps of both approaches are based on commonly known techniques like task decomposition, task merging, etc.

The work in [17] studied a greedy slack stealing (GSS) scheduling approach considering inter-dependent sequential tasks. It considered the DAG model to represent dependencies among the tasks. In GSS, the slack (unused time in actual computation requirement of a task) is reclaimed by one task by shifting others towards the deadline. They did not consider repetitive tasks; hence it can be regarded as scheduling a single task. Besides this, the power and graph model used in [17] is different from ours. To ensure a fair comparison, we execute the GSS algorithm using the power model in Equation (1) and assume that once introduced in the system; a processor remains active. We also consider a minimum inter-arrival separation for a DAG. That work considered three different kinds of nodes: $\text{AND}$, $\text{OR}$, and $\text{Computation}$ nodes (Section 2.1 in [17]). A computation node has both the maximum and average computation requirement. To comply with our work where the focus is on energy reduction while guaranteeing worst-case temporal correctness, we execute the GSS algorithm considering only the computation nodes with their maximum computation requirement. We made all the changes in order to provide a fair comparison. Despite these differences, we chose [17] as a baseline because they studied a GSS approach for energy minimization. They considered the inter-dependent sequential tasks and their dependencies were represented by a DAG, which is similar to our task model.

We also consider [33] as a baseline because this work considered scheduling a set of independent periodic applications, where each application is modeled as a DAG. They proposed an approach for energy minimization combining the DVFS and the DPM policy. Similar to [12] and [17], the work in [33] considered per-core DVFS.

We compare power consumption by varying two parameters for each task: $\text{task periods (utilization)}$ and $\text{the number of nodes}$. We randomly generate 25 sets of DAG tasks and compare the average power consumption.

**Notations of Referenced Approaches.** For the task partitioning step, either we randomly choose any two and allocate them to the same cluster, or greedily choose the ones with lowest speed as proposed. Regarding speed-profile calculation, there are also two options (Approaches A and B in Section 4.1). Combining these options in two steps lead to four baselines: $\text{MaxSpeed_Greedy}$, $\text{SingleSpeed_Greedy}$, $\text{MaxSpeed_Random}$, $\text{SingleSpeed_Random}$. Also, three baselines mentioned above are included for comparison:

- Federated scheduling with intra-task processor merging [12], denoted by $\text{Fed_Guo}$;
- GSS algorithm [17], denoted by $\text{GSS_Zhu}$.
- $\text{DVFS}$ and $\text{DPM}$ combination [33], denoted by $\text{com_Chen}$.

### 6.1 Uniform Heterogeneous Platform With a Continuous Frequency Scheme

In this section, considering the uniform heterogeneous platform and a continuous frequency scheme, we report the power consumption comparison for different approaches mentioned earlier. Under such a platform, different clusters no longer share the same power model and we use the power model described in Equation (6). We present the power comparison results in an identical heterogeneous platform (from [34]) in Appendix A, available in the online supplemental material.

#### 6.1.1 Constrained Deadline Task

Here, we report the power consumption under the scheme for constrained deadline tasks mentioned in Section 4. We evaluate the efficiency of our proposed method by changing two parameters; task period (utilization) and the number of nodes in the task.

**Effect of Varying Task Periods (Utilization).** Here we control the average task utilization through varying the task period. In order to make the task schedulable, the critical path length $L_i$ of task $t_i$ should not exceed its deadline $D_i$. We vary the period in a range ($L_i \leq T_i \leq C_i$). The parameter $L_i$ and $C_i$ are measured once the DAG is generated according to the technique described in Section 5. We also use the following equation (according to [12]) to ensure that the value of $T_i$ satisfies the range ($L_i \leq T_i \leq C_i$).

$$T_i = L_i + (1-k)(C_i - L_i).$$

Here, $k \in [0, 1]$ is task utilization. As we are considering the constrained deadline tasks, $D_i$ is randomly picked from the range ($L_i \leq D_i \leq T_i$). The results are presented in Fig. 7a. Note that when any parameter (e.g., number of nodes in a DAG, task utilization) changes, savings in energy randomly vary within a small range and we consider the minimum value among them. The results indicate a proportional relationship between the average power consumption and average task utilization. It happens because a higher task utilization imposes tighter real-time restrictions. It restricts (refer to Fig. 2b) the space for the segment length optimization. In this experiment, the number of nodes is fixed to 30. Fig. 7a shows that $\text{SingleSpeed_Greedy}$ approach performs better for a higher utilization value. On average, the $\text{SingleSpeed_Greedy}$ approach leads to a power saving of at least 30.23 and 60.2 percent compared to $\text{Fed_Guo}$ and $\text{GSS_Zhu}$ approaches, respectively. In $\text{SingleSpeed_Greedy}$ approach, a task executes with a single speed throughout the deadline. During the task partitioning step, a suitable partner (with similar speed-profile) leads to energy efficiency. However, for the other approaches task speed may vary throughout the deadline. In that case, evil alignment and a significant variation in the speed may reduce energy efficiency (see Fig. 3 and Example 1).

**Effect of Varying the Numbers of Nodes.** Now we vary the number of nodes (10 to 55) ($T_i$ is fixed) and report the average power consumption. We report the average power consumption for harmonic deadline tasks in Fig. 7b and
We observe that the power consumption pattern does not change that much, i.e., Single-Speed_Greedy approach outperforms other approaches especially when the number of nodes (in each DAG) are high, 35 or higher. Specifically, under harmonic task periods, the Single-Speed_Greedy incurs 40.19 and 65.9 percent less power on average compared to Fed_Guo and GSS_Zhu; under arbitrary task periods, the savings potential are 33.43 and 61.96 percent, respectively.

6.1.2 Implicit Deadline Task

Effect of Varying Task Periods (Utilization). Using previous setup (Section 6.1.1), We observe that the average energy consumption is directly proportional to the average task utilization.

Fig. 8a shows that Single-Speed_Greedy approach performs better for a higher utilization value and on average, saves at least 35.21 and 62.52 percent compared to Fed_Guo and GSS_Zhu approaches, respectively.

Effect of Varying the Numbers of Nodes. Figs. 8b and 8c report the average power consumption for the harmonic and arbitrary deadline tasks, respectively. We observe that the Single-Speed_Greedy approach outperforms other approaches when the number of nodes (in each DAG) are high. Under harmonic task periods, the Single-Speed_Greedy incurs 44.84 and 67.55 percent less power on average compared to...
Fed_Guo and GSS_Zhu; under arbitrary task periods, the savings potential are 42.33 and 67.19 percent, respectively.

6.2 Uniform Heterogeneous Platform With a Discrete Frequency Scheme

In this section, we report the power consumption comparison for the (previously mentioned) approaches considering the uniform heterogeneous platform and a discrete frequency scheme. Under such a platform, we discretize the frequency using the technique described in Section 4.2.

6.2.1 Constrained Deadline Task

Here, we consider the constrained deadline tasks and report their average power consumption by changing two parameters: task period (or utilization) and the number of nodes.

Effect of Varying Task Periods (Utilization). Similar to the Figs. 7a, and 8a, we observe that the (i) average energy consumption is directly proportional to the average task utilization. (ii) SingleSpeed_Greedy approach consumes less power than other approaches (see Fig. 9a).

Effect of Varying the Numbers of Nodes. We vary the number of nodes (10 to 55) and report the average power consumption for harmonic (arbitrary) deadline tasks in Fig. 9b (Fig. 9c). Similar to the Figs. 7 and 8, we observe that the (i) Performance of SingleSpeed_Random, SingleSpeed_Greedy, MaxSpeed_Greedy, and MaxSpeed_Random does not vary that much for a small number of nodes (typically 10 to 25) per DAG. (ii) SingleSpeed_Greedy approach performs better (i.e., consume less power) than other approaches when the number of nodes per DAG is high.

6.2.2 Implicit Deadline Task

Now, we report the average power consumption using the same setup as described in Section 6.2.1, i.e., (a) for a fixed number of nodes (30) per task, change their utilization value, and (b) vary the number of nodes (10 to 55) per task, while keeping $T_i$ fixed. We report the average power consumption in Fig. 10. From this figure, we observe that the (i) Performance of SingleSpeed_Random, SingleSpeed_Greedy, MaxSpeed_Greedy, and MaxSpeed_Random does not vary that much for a smaller task utilization or when the number of nodes per DAG is small (typically 10 to 35). (ii) SingleSpeed_Greedy approach performs better (i.e., consume less power) compared to the other approaches when the number of nodes per DAG is high.

7 RELATED WORK

Much work has been done aimed at energy-efficient scheduling of sequential tasks in a homogeneous multi-core platform (see [11] for a survey). Considering the mixed-criticality task model and varying-speed processors, the works on [23], [35], [36], [37], [38] proposed an approach to handle the energy minimization problem. The work in [27], [39], [40], [41], [42], [43] presented an energy-efficient approach for the heterogeneous platform. Considering the real-time tasks in clustered heterogeneous platforms, the work in [39] studied the partitioned EDF scheduling policy, while [42] proposed an optimal task-core mapping technique that is fully-migrative. Considering the heterogeneous multi-core platform, a two-phase algorithm was proposed by [27]. In the first phase, they proposed a tasks-core allocation approach with the aim of reducing the dynamic energy consumption, while the second phase seeks for a better sleep state to reduce the leakage power consumption. A low overhead, DVFS-cum-DPM enabled energy-aware approach, HEALERS, was proposed by [43]. However, none of them considered the intra-task parallelism. Considering a clustered heterogeneous MPSoC platform, a migrative cluster scheduling approach was proposed by [15]. In this approach, run-time migration (within different cores in the same cluster) for a task is allowed to improve resource utilization. The work in [44] studied the technique to utilize the parallelism in a hard real-time streaming application (represented as a Synchronous Data Flow (SDF) graph) in a clustered heterogeneous platform.

Till date, considering both the intra-task parallelization and power minimization has received less attention. A greedy slack stealing algorithm is proposed in [17] that deals with task represented by graphs but did not consider the periodic DAGs. Assuming per-core DVFS, [33] provided the technique to combine DVFS and DPM. Considering the real-time jobs (represented as a DAG) in cloud computing systems and in a heterogeneous multi-core platform, the work in [45], [46] studied a QoS-aware and energy-efficient scheduling strategy. They proposed a scheduling policy that utilizes per-core DVFS. With the aim of improving energy-efficiency in a heterogeneous real-time platform, [47] proposed a combined
approach considering the approximate computation and bin packing strategy. [48] investigated the energy awareness for cores that are grouped into blocks, and each block shares the same power supply scaled by DVFS. Benefits of (in terms of power saving) intra-task parallelism is proven theoretically in [1]. Considering the fork-join model, [49] reported an empirical evaluation of the power savings in a real test-bed. Based on level-packing, [50] proposed an energy efficient algorithm for implicit deadline tasks with same arrival time and deadline.

None of these works allows intra-task processor sharing considering the sporadic DAG task model. The recent work in [12], [18] is most related to ours. However, these works are significantly different from ours w.r.t the task model, platform, real-time constraints (deadlines), solution techniques, and the evaluation. First, the work in [12] considered a simplified model where only one DAG task executes at a time, while the work in [18] extends this work by allowing inter-task processor sharing. However, both of these works assumed that the number of cores are unlimited. Second, Both the works in [12], [18] assumed per-core speed scaling. However, many of the existing platforms (e.g., ODROID XU-3) do not support such speed scaling—speeds of processors under the same cluster must execute at the same speed. As the number of cores fabricated on a chip increases, per-core speed scaling design is less likely to be supported due to the inefficiency on hardware levels [13]. Third, Both of these works have studied only the implicit deadline tasks and did not consider the constrained deadline tasks. Hence, the non-negligible idle gaps between the task deadline and its next release remain un-utilized. Finally, the evaluations in [12], [18] were done based on simulations without any implementation on a real platform.

8 Conclusion

In this paper, we have studied real-time scheduling of a set of implicit and constrained deadline sporadic DAG tasks. We schedule these tasks on the cluster-based multi-core platforms with the goal of minimizing the CPU power consumption. In a clustered multi-core platform, the cores within the same cluster run at the same speed at any given time. Such design better balances energy efficiency and hardware cost and appears in many systems. However, from the resource management point of view, this additional restriction leads to new challenges. By leveraging a new concept, i.e., speed-profile, which models energy consumption variations during run-time, we can conduct scheduling and task-to-cluster partitioning while minimizing the expected overall long-term CPU energy consumption. To our knowledge, this is the first work that has investigated energy-efficient scheduling of DAGs on clustered multi-core platform. Also, no work considered energy-aware real-time scheduling of constrained deadline DAG tasks.

We have implemented our result on an ODROID XU-3 board to demonstrate its feasibility and practicality. We have also complemented our system experiments on a larger scale through realistic simulations that demonstrate an energy saving of up to 57 percent through our proposed approach compared to existing methods. In this work, we have restricted our attention mainly to the CPU power consumption. In the future, we plan to consider other components that may affect the total power consumption, e.g., cache misses, context switches, I/O usage, etc. We also plan to study the effect of tasks sporadic release patterns (to the overall power consumption) and propose a task reallocation scheme.

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References

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SaberLDA: Sparsity-Aware Learning of Topic Models on GPUs

Kaiwei Li, Jianfei Chen, Wenguang Chen, and Jun Zhu

Abstract—Latent Dirichlet Allocation (LDA) is a popular tool for analyzing discrete count data such as text and images, which are required to model datasets and a large number of topics, e.g., tens of thousands of topics for industry scale applications. Although distributed CPU systems have been used to address this problem, they are slow and resource inefficient. GPU-based systems have emerged as a promising alternative because of their high computational power and memory bandwidth. However, existing GPU-based LDA systems can only learn thousands of topics, because they use dense data structures, and have linear time complexity to the number of topics. In this article, we propose SaberLDA, a GPU-based LDA system that implements a sparsity-aware algorithm to achieve sublinear time complexity to learn a large number of topics. To address the challenges introduced by sparsity, we propose a novel data layout, a warp-based sampling kernel, an efficient sparse matrix counting method, and a fine-grained load balancing strategy. SaberLDA achieves linear speedup on 4 GPUs and is 6–10 times faster than existing GPU systems in thousands of topics. It can learn 40,000 topics from a dataset of billions of tokens in two hours, which was previously only achievable using clusters of tens of CPU servers.

Index Terms—GPU acceleration, latent dirichlet allocation, topic models, machine learning

1 INTRODUCTION

Big data, such as web pages, user activities and images, are pervasive nowadays. Machine learning helps extract underlying information from the data and make predictions. Among various machine learning algorithms, Probabilistic graphical models (PGMs) is a class of popular unsupervised machine learning algorithms. PGMs provide a flexible way of defining models that incorporate human knowledge and have been used extensively in various scientific and engineering domains (see [13] for an overview).

In this paper, we focus on topic modeling, an important subclass of PGMs, and demonstrate the challenges and solutions encountered to accelerate PGMs with GPUs. Topic models provide a suite of widely adopted statistical tools for feature extraction and dimensionality reduction for bag-of-words (i.e., discrete count) data, such as text documents and images in a bag-of-words format [7]. Given an input corpus, topic models automatically extract a number of latent topics, which are in unigram distributions over the words in a given vocabulary. The high-probability words in each topic are semantically correlated. Latent Dirichlet Allocation (LDA) [4] is the most popular topic model due to its simplicity. It has been deployed as a key component in data visualization [16], text analysis [5], [42], computer vision [7], network analysis [8], [11], and recommendation systems [12].

In practice, it is not uncommon to encounter large-scale datasets, e.g., text analysis typically consists of hundreds of millions of documents [36], and recommendation systems need to tackle hundreds of millions of users [1]. Furthermore, as the scale of datasets increases, model sizes need to increase as well because we need a larger number of topics to exploit the richer semantic structure underlying the data. Having ten thousands of topics is a reasonable design goal for modern topic modeling systems, to have good coverage for both industry-scale applications [32] and researching [5], [7], [16].

However, it is highly challenging to efficiently train large LDA models. The time complexity of training LDA is high because it involves iteratively scanning the input corpus many times (e.g., 100), and the time complexity of processing each token is not constant but is related to the number of topics.

To train LDA in acceptable time, CPU clusters are often used. However, due to the limited memory bandwidth and low computational power of CPUs, large clusters are typically required to learn large topic models [1], [35], [36]. For example, a 32-machine cluster is used to learn 1,000 topics from a 1.5-billion-token corpus.

A promising alternative is to train LDA with GPUs, leveraging their high computational power and memory bandwidth. There have been several previous attempts using this approach. For example, Yan et al. [33] implemented the collapsed Gibbs sampling algorithm, BIDMach [41] implemented the variational Bayes algorithm as well as a modified Gibbs sampling algorithm, and Tristan et al. [28] proposed an expectation-maximization algorithm. These GPU-based systems are reportedly to achieved superior performance compared to CPU-based systems [28], [33], [41].

Unfortunately, current GPU-based LDA systems can only learn a few hundred topics (See Table 1), which may not be...
Compared with CPUs, GPUs have considerably:

- The size of the vocabulary; and
- The limited GPU memory capacity requires stream-

Finally, the calculation based on the skew vocabulary
requires fine-grained parallelism in multi-GPU
with hundreds of streaming multiprocessors, or it
would be imbalanced because the workload of the
largest word exceeds the average workload of all
GPU blocks.

SaberLDA addresses all these challenges by supporting
sparsity-aware algorithms on multi-GPUs. Our key technical
contributions are as follows:

- A novel hybrid data layout named partition-by-doco-
ument and order-by-word (PDOW) that simultaneously
maximizes the locality and reduces the GPU mem-
ory consumption;
- A warp-based sampling kernel that is fully vectorized,
and is equipped with a W-ary sampling tree that sup-
ports both efficient construction and sampling;
- A multi-GPU based shuffle and segmented count (SSC)
algorithm for updating sparse count matrices; and
- A split index optimization based on the Compressed
Row (CSR) format representing a sparse matrix of
vocabulary, which solves the imbalance of multiple
GPU workloads.

Our experimental results demonstrate that SaberLDA is
able to train LDA models with up to 40,000 topics, which is
more than an order of magnitude larger than previous
GPU-based systems [28], [33], [41]. SaberLDA is also highly
efficient; using a single GPU, SaberLDA converges five
times faster than previous GPU-based systems and four
times faster than CPU-based systems. With 4 GPUs, Saber-
LDA is able to learn 40,000 topics from a dataset of billions
of tokens, which was previously only achievable using clus-
ters of tens of machines [35].

The rest the paper is organized as follows. Section 2
introduces the basics of LDA. Section 3 presents the design
of SaberLDA. Section 4 contains the experiments. Section 5
discusses related work. Section 6 discusses the limitations
of this study and future work. In Section 7 we summarize
the paper.

2 LATENT DIRICHLET ALLOCATION

In this section, we introduce the Latent Dirichlet Allocation
(LDA) model, its sampling algorithm, and the sparsity-aware
optimization.

2.1 Definition

LDA is a statistical model that infers topics from a given
corpus. A corpus consists of multiple documents. Each docu-
ment is represented as a bag of words, disregarding gram-
mar and even word order, but keeping multiplicity. Each
occurrence of a word in such document is called a token. The
set of all words forms the vocabulary. The definitions of the
corpus are:

- $D$: The number of documents in a corpus;
- $V$: The size of the vocabulary; and
- $T$: The total number of tokens among all documents.
We also define the number of topics to infer, $K$, which is
given by the user.

- $K$: The number of topics to infer from the corpus.

The text corpus is represented as a token list $T : \text{List}[\{d, v, k\}]$. The terms $d$, $v$, and $k$ refer to the document-id, word-id,
and topic-id, respectively. Training LDA involves assigning a
topic assignment $k$ for each token $(d, v, k)$. Note that duplicated
words occurring in the same document are treated as multiple
tokens since they can be assigned to different topics.

According to the topics from each token in $T$, we can cal-
culate the topic count for a document or a word. Hence, we
define the document-topic count matrix $A$ and word-topic
count matrix $B$, as well as the global topic distribution
vector $C$:

- $A$: $D \times K$ sparse matrix, $A_{dk}$ is the number of occur-
rences of topic $k$ in document $d$;
- $B$: $V \times K$ sparse matrix, but usually more dense than
$A$. $B_{vk}$ is the count of those tokens with word $v$ which
has been assigned to topic $k$; and
C: a dense vector of $K$ elements, where $C_k$ is the count of all tokens with topic $k$.

Fig. 1 is an example of token list and count matrices.

### 2.2 Inference

Given the token list $T$, our goal is to infer the topic assignment $k$ for each token $t = (d, v, k)$. Informally, the LDA model is designed in a way that maximizes some objective function (the likelihood) related to the topic assignments of each token. We omit the mathematical details and refer the interested readers to standard LDA literature [4], [15].

There are many inference algorithms, such as variational inference [4], Markov chain Monte Carlo [15], and expectation maximization [9], [28]. We choose the state-of-the-art algorithm [28] to implement on GPU for its following advantages:

- It is sparsity-aware, so the time complexity is sub-linear with respect to the number of topics. This property is critical to support the efficient training of large models;
- It enjoys the best degree of parallelism because the count matrices $A$ and $B$ only need to be updated once per iteration. This matches with the massively parallel nature of GPU to achieve high performance.

The pseudo code of ESCA is shown in Algorithm 1, which is a bulk synchronous parallel (BSP) programming model. In each iteration, the topics of all tokens are sampled independently. Initially, all tokens are assigned randomly. The inference method has multiple iterations. Each iteration first calculates $A$, $B$, and $C$ by their definitions and then samples the topic $k$ of each token based on:

$$P_k = \frac{(A_{dk} + \alpha)(B_{vk} + \beta)}{C_k + V \beta}. \tag{1}$$

Here, $P_k$ is the measure of topic distribution of token $(d, v)$ where $P_k$ is proportional to the probability of choosing topic $k$. The token $(d, v)$ has larger probability in topic $k$ when its document $d$ has more tokens in topic $k$ (larger $A_{dk}$), and the word $v$ in all documents are more assigned by topic $k$ (larger $B_{vk}$). $\alpha$ and $\beta$ are two user specified parameters that control the granularity of topics. Large $\alpha$ and $\beta$ values mean that we want to discover a few general topics, while small $\alpha$ and $\beta$ values mean that we want to discover many specific topics.

### 2.3 Sampling

We now introduce the sampling function at line 12 in Algorithm 1. The input distribution array $P$ consists of $K$ floating point numbers. The output is a topic $k$, where $k$ is an integer between 1 and $K$. Algorithm 2 is a straightforward method to generate the output topic from a given array of probabilities, which takes the following steps. Compute the the prefix-sum of $P$ and set the result to the variable $S$. Random a float number $m$ from the uniform distribution of 0 to $S_K$, where $S_K$ is the summation of $P$. Use binary search to find the sampled topic $k$ where where $S_{k-1} < m \leq S_k$.

#### Algorithm 1. ESCA Algorithm for LDA

1: Input: token list $T$
2: for $(d, v, k) \in T$
3: \hspace{1cm} $k \leftarrow \text{Random}(1, K)$
4: end for
5: for $i \leftarrow 1$ to num-iteration do
6: \hspace{1cm} $A \leftarrow \text{Int}[D][K], B \leftarrow \text{Int}[V][K], C \leftarrow \text{Int}[K]$
7: \hspace{1cm} for $(d, v, k) \in T$ do
8: \hspace{2cm} $\text{Increase}(A_{dk}, B_{vk}, C_k)$
9: \hspace{1cm} end for
10: for $(d, v, k) \in T$ do
11: \hspace{2cm} $P \leftarrow (A_{dk} + \alpha) \times (B_{vk} + \beta)/(C + V \times \beta)$
12: \hspace{2cm} $k \leftarrow \text{Sample}(P)$
13: \hspace{1cm} end for
14: end for

#### Algorithm 2. Sample From Topic Distribution Array

1: Input: $K$: Int, $P$: float[] //Starting from index 1
2: $S \leftarrow \text{PREFIXSUM}(P)$ where $S_0 = 0, S_k = \sum_{i=1}^{K} P_i$
3: Random $m \in (0, S_K]$
4: Binary search $k$ where $S_{k-1} < m \leq S_k$
5: return $k$

### 2.4 Sparsity-Aware Optimization

The time complexity of constructing and sampling of $P$ is $O(K)$. However, the complexity is not sparsity-aware, which means that it is proportional to the number of topics. For example, for a given corpus, inferring 1000 topics will be 10 times slower than inferring 100 topics. The sparsity-aware optimizations [1], [28] utilize the sparsity of $A$, and improve the time complexity to $O(K_D)$, where $O(K_D) \ll O(K)$. $K_D$ is the average number of non-zero entries per row of $A$.

According to Eq. (1), for a given token with document $d$ and word $v$, its topic distribution is $P_{dv}$. The sparsity-aware optimization decomposes the original sampling problem $P_{dv}$ into two easier sampling subproblems $X_{dv}$ and $Y_{v}$. Sampling $P_{dv}$ is exactly equal to sample the subproblem $X_{dv}$ with the probability of $\frac{X_{dv}}{X_{dv} + Y_{v}}$, and sampling $Y_{v}$ with the probability of $\frac{Y_{v}}{X_{dv} + Y_{v}}$, where $|V|$ means the summation of the elements in vector $V$. The two sub-problems are derived by the following equations:

$$P_{dv} = \frac{(A_{d} + \alpha)(B_{v} + \beta)}{C + V \beta} = A_{d}Q_{v} + \alpha Q_{v} = X_{dv} + Y_{v} \tag{2a}$$

$$X_{dv} = A_{d}Q_{v}, Y_{v} = \alpha Q_{v}, Q_{v} = \frac{B_{v} + \beta}{C + V \beta} \tag{2b}$$

We now analyze the time complexity of constructing and sampling of these two subproblems $X_{dv}$ and $Y_{v}$. $X_{dv}$ has a factor $A_{d}$, which is a sparse vector most of the time, so the sparsity of $X_{dv}$ is equal to $A_{d}$. Since $A_{d}$ is the
topic count of document \( d \), the number of the non-zero element \( \text{nnz}(A_d) \) is no more than the number of tokens of document \( d \). In most corpora like NYTimes and PubMed, a document has dozens to hundreds of tokens. However, it is common to infer thousands to millions of topics, which is far more than the tokens in one document. This can be described by the following inequation:

\[
K_D \approx \text{nnz}(A_d) \leq |T_d| \approx 100 \ll K,
\]

where the function \( \text{nnz} \) is the number of non-zero elements of a vector, \( T_d \) is the group of all tokens of document \( d \), \( K_D \) is the average of \( \text{nnz}(A_d) \) among all documents. Consequently, \( \sum_{d} x_{d} \) is a sparse vector with \( O(K_D) \) elements, and the time complexity of construction and sampling of \( \sum_{d} x_{d} \) are both \( O(K_D) \), which is sparsity-aware.

\( Y_v \) is the product of \( \alpha \) and \( Q_v \), which depends on the topic count vector \( B_v \) and \( C_v \). Since many tokens belong to the same word \( v \) in the vocabulary, it is possible to reuse \( Y_v \) for those tokens. The construction of \( Y_v \) can be pre-processed before Before sampling tokens. Hence the time complexity of \( O(K) \) for constructing each \( Y_v \) and \( O(VK) \) for all unique words in the vocabulary.

At last, we look at sampling \( Y_v \). Since \( Y_v \) is a dense vector of \( K \) non-zero elements, the straightforward sampling Algorithm 2 is not sparsity-aware here. Previous works [17], [36] used AliasTable with \( O(K) \) built time for each word \( v \) and \( O(1) \) for sampling each token. Yu et al. [35] used a Fenwick tree, which was also built in \( O(K) \) and can then sample a topic in \( O(\log_2 K) \). For convenience, we respectively name the initialize and sampling functions \( \text{PreBuilt}(d) \) (line 3) and \( \text{FastSample}(d) \) (line 17).

The sparsity-aware optimization is shown in Algorithm 3. At the beginning of each iteration, the time complexity of constructing count matrices \( A, B, \) and \( C \) is \( O(T) \). Before sampling tokens, constructing \( Q_v, Y_v \) and additional data structure \( \text{Tree}_v \) have \( O(KV) \) time complexity. When sampling each token, the time complexity is \( O(K_D + 1) \) for Fenwick tree and \( O(K_D + 1) \) for AliasTable. Hence the the time complexity for sampling all tokens is \( O(KV + TK_D) \). Consequently, the overall time complexity in each iteration is \( O(KV + TK_D) \), which sparsity-aware for large topics and it is far less than the original \( O(TK) \) algorithm.

### 3 Design and Implementation

In this section, we illustrate the design and implementation of SaberLDA. We re-iterate the design goals of SaberLDA here:

- Supporting industrial-grade large models on GPUs, e.g., tens of thousands topics;
- Supporting large dataset with billions of tokens; and
- Providing significant speedup with GPUs to CPU solutions.

To achieve these goals, SaberLDA adopts the state-of-the-art embarrassingly parallel LDA algorithm with sparsity-aware optimization (Algorithm 3) in multiple GPUs. In the rest part of this section, we organize the content by demonstrating how different segments of Algorithm 3 are designed and implemented in SaberLDA.

The first challenge we faced is to design the layout of large datasets and matrices on memory-limited multi-GPU architecture. Section 3.1 discusses the computing and memory impacts of different kinds of data layouts.

We next focus on the most time-consuming part of the sparsity-aware algorithm, which is the sampling of tokens (lines 11-19). We analyze the memory access localities of different for-looping orders in Section 3.2.

Section 3.3 discusses how to assign tokens into massive GPU cores with fine-grained parallelism. It also demonstrates the vectorized optimization used in element-wise product (line 12) and sampling methods (line 15 and 17).

Since it is challenging to achieve good load balance with skew vocabulary distributions especially in multi-GPU. Section 3.4 proposes a hybrid parallelism strategy named split index to further parallelize the most frequent words.

The matrices counting methods (line 2 and line 3) take lower order of complexity than sampling, however, it is still challenging to use GPU efficiently if compute them into sparse format. We named this problem shuffle and segmented count and propose a multi-GPU solution in Section 3.5.

#### 3.1 Data Layouts

A modern GPU has more than 10 gigabytes memory on board. A typical industrial LDA model [32] learns 10k topics with 210k unique words in the vocabulary, which forms 8.4 gigabytes of a count matrix \( B \) storing 32-bit integers as its elements. Using the dense format for storing the model matrix \( B \) is suitable for most cases of LDA training.

The web-scale datasets for LDA training have no less than millions of documents, which makes the size of document-topic count matrix \( A \) tens to hundreds of times larger than word-topic count matrix \( B \). To reduce the memory requirements and enable the sparsity-aware algorithm, SaberLDA uses a sparse layout CSR to store \( A \). However, the size of \( A \) still has dozens to hundreds of gigabytes, hence it has to be partitioned before computing in GPUs.

Since the token list \( T \) and the document-topic count matrix \( A \) are too large to be kept in GPU memory, they are
divided into small partitions. A partition contains the partition of \( A \) and \( T \) that belongs to the same document group. The size of each partition is some gigabytes, so it is able to be entirely transferred to GPU memory.

During the computation, partitions of \( A \) and \( T \) are successively transferred to GPUs. When the computation of a partition finishes, the data are transferred back to the CPU. The GPU memory space is recycled for the next partition.

In multiple GPUs, the computation of tokens \( T \) can be separated evenly in multiple GPU, because they are embarrassingly parallelized. However, computing token \((d, v)\) with document-id \( d \) and word-id \( v \) requires reading the rows \( A_d \) and \( B_v \), which causes the matrices must be shared across multiple GPUs for reading. The shared data can either be replicated in all GPUs or be remote accessed through PCI-e bus or NVLink. Fortunately, if grouping the tokens of the same document (or word) into single GPU, visiting the data of count matrix \( A \) (or \( B \)) is localized.

It must be decided how to group the tokens. If grouping tokens with the same document in a GPU, we can either replicate the whole matrix \( B \) into all GPUs, or distribute \( B \) and remotely access the different parts. Unfortunately, replicating the large dense matrix \( B \) is not scalable and wasting memory, and remoting access data from other GPUs has very high latency. Grouping tokens with the same word is much more better. At this time, visiting matrix \( B \) is localized. Replicating the matrix \( A \) and the token list \( T \) into multiple GPUs take no significant overhead, because they are already spending time on CPU-GPU transferring.

Table 2 demonstrates the data layout of a typical short video dataset for recommendation. The first row is the number of elements. The second row is the estimated size. The third row is the partition strategy and the size of each partition for CPU-GPU transferring, and the last row is the grouping strategy in multi-GPU.

### 3.2 Choosing Sampling Order

In this subsection, we introduce the design and optimization of token sampling in SaberLDA. The procedure of token sampling is found in lines 11-19 in Algorithm 3. The order of sampling tokens in the GPU is the looping order of tokens \( T \). Theoretically, these tokens can be sampled in any order, but the ordering greatly impacts the locality.

As illustrated in Fig. 2a, sampling \( k \) for a token \((d, v, k)\) requires evaluating an element-wise product of two rows (Line 12 of Algorithm 3), the \( d \)th row of the document-topic count matrix \( A_d \) and the \( v \)th row of the word-topic factor matrix \( Q_v \). The element-wise product involves accessing all non-zero entries of \( A_d \) (sequential), and accessing elements of \( Q_v \) indexed by the non-zero entries of \( A_d \) (random).
partition-by-document and order-by-word (PDOW), which means to first partition the token list by document-id, and then sort the tokens within each partition by word-id. Unlike simple layouts such as sorting all tokens by document-id or by word-id in previous systems [28], [33], [35], [36], [39], [41], PDOW combines the advantages of both by-document partitioning and word-major ordering, and simultaneously maximizes cache locality with the word-major order, while keeping the GPU memory consumption small with the by-document partitioning.

### 3.3 Warp-Based Sampling

We now turn to the inner loop of sampling tokens within a word, i.e., line 7 in Algorithm 4, which is the most time consuming part of LDA. To understand the challenges and efficiency-related considerations, it is helpful to briefly review GPU architecture.

#### Algorithm 4. Sampling Tokens in Word-Major Order

1. for $i \leftarrow 1$ to num\_iteration do
2. \hspace{1em}A, B, C \leftarrow ComT
3. \hspace{1em}for $v \leftarrow 1$ to $V$ do // by block
4. \hspace{2em}shared $Q_v \leftarrow (B_v + \beta)/(C + V \beta)$
5. \hspace{2em}shared $S \leftarrow a \times |Q_v|$
6. \hspace{2em}shared $T_{vec} \leftarrow$ PreBuilt($Q_v$)
7. \hspace{2em}for $(d, k) \in T_v$ do // by warp
8. \hspace{3em}$X_{dv} \leftarrow A_d \times Q_v$
9. \hspace{3em}$R \leftarrow$ RandFloat(0, $|X_{dv}| + S$)
10. \hspace{3em}if $R \leq |X_{dv}|$ then
11. \hspace{4em}$k \leftarrow$ SparseSample($X_{dv}$)
12. \hspace{3em}else
13. \hspace{4em}$k \leftarrow$ TreeSample($T_{vec}$)
14. \hspace{2em}end if
15. \hspace{1em}end for
16. \hspace{1em}end for
17. end for

GPUs follow a single instruction multiple data (SIMD) pattern, where the basic SIMD unit is a warp, which has 32 data lanes. Each lane has its own arithmetic logic unit (ALU) and registers, and all the lanes in a warp execute the same instruction. In NVIDIA’s CUDA, each thread is executed on a lane, and every adjacent 32 threads share the same instruction. Readers can make an analogy between GPU warp instruction and CPU vector instruction.

#### 3.3.1 Thread-Based Versus Warp-Based

The most straightforward implementation of sampling on a GPU is thread-based sampling, which samples each token with a GPU thread. Therefore, 32 tokens are processed in parallel with a warp. Thread-based sampling is acceptable when $A$ is dense because the number of product operations (Line 8 of Algorithm 4) is always $K$, and there is no need to use if-branches for sparse sampling.

However, this approach has several disadvantages when using sparse sampling in word-major order. First, because each token corresponds to different rows of $A$, the numbers of product operations are different (Line 8 of Algorithm 4). In this case, all the threads need to wait for the slowest one, creating long waiting time. Second, the branch choosing the sampling function causes the thread divergence problem, i.e., if some of the threads go to one branch and other threads go to another branch, the warp needs to perform the instructions of both branches, which increases the waiting time. Finally, the memory access to $A$ is unconcealed (Line 8 of Algorithm 4) because the GPU threads are accessing different and discontinuous addresses of the global memory.

To overcome the disadvantages of thread-based sampling, SaberLDA adopts warp-based sampling, in which all the threads in a warp collaboratively sample a single token. However, there are also challenges for warp-based sampling—all the operations need to be vectorized to maximize the performance. In the next part of this section, we introduce how to individually vectorize the element-wise product, the sparse sample and the tree sampling functions.

#### 3.3.2 Element-Wise Product

The element-wise product step is vectorizable by simply letting each thread process an index, and a warp then computes the element-wise product for 32 indices at a time. All the threads are fully utilized except for the last 32 indices in the case if the number of non-zero entries of $A_d$ is not a multiple of 32. The waste is very small since the entries of $A_d$ are typically many more than 32, e.g., 100.

#### 3.3.3 Choosing the Branch

This step only consists of a random number generation and a comparison, whose costs are negligible. Note that thread divergence will not occur since the whole warp goes to one branch.

#### 3.3.4 Sparse Sampling

This step samples a topic from the sparse probability array $X_{dv}$ by vectorizing the sampling function listed in Algorithm 2.

First, we vectorize the computation of the prefix sum by using the \_shuffle intrinsic $O(\log_{2}32)$ times [20]. We refer to this routine as the warp\_prefix\_sum.

Given the prefix sum, we need to determine the index of the first element that is greater than or equal to the random value just generated. This can be achieved in two steps:

1) Use the warp-vote intrinsic \_ballot to make a 32-bit integer, whose $i$th bit is set to one if the $i$th prefix sum is greater than or equal to the random value, vice versa.

2) Use the intrinsic \_ffs to return the index of the first bit 1 of the 32-bit integer.

We call these two steps the warp\_vote, which returns an index that is greater than or equal to the given value, or is equal to -1 if there is no such index. The threads in the warp continue to fetch the next 32 values in the loop, or stop if they find such an index.

In addition to the eliminated waiting time and thread divergence, the memory access behavior of our implementation is also good. For the element-wise product, the access to $A$ is continuous. Specifically, the warp accesses two 128-byte cache lines from the global memory, and each thread consumes two 32-bit numbers (an integer index and a float32 value). The accesses to $Q$ are random, but they are still
efficient since the current row \( Q_s \) has been loaded into the shared memory or cached.

### 3.3.5 Tree Sampling

We now present the details for sampling \( Q_v \) (Lines 6 and 13 of Algorithm 4). Note that \( Tree_v \) is built from \( Y_v \), which has the same distribution as \( Q_v \) (\( Y_v = \alpha Q_v \)). Hence, \( Q_v \) can also be used to build the sampling tree. As discussed in Section 2.3, this sampling problem is essentially the same problem as sampling from \( X_{dev} \) but the basic sampling algorithm has \( O(K) \) time complexity for each token, so we want another approach. From the literature, there are two main optimizations based on the pre-processing of \( Y_v \), and we briefly review their data structures.

- An alias table [29] can then be built in \( O(K) \) time, and each sample can be obtained in \( O(1) \) time. However, building the alias table is sequential.
- The Fenwick tree [35] can be built in \( O(K) \) time, and each sample can then be obtained in \( O(\log_2 K) \) time. However, the branching factor of the Fenwick tree is only two, so the 32-thread GPU warp cannot be fully utilized.

Both approaches are designed for CPUs, and are slow to construct on a GPU because they cannot be vectorized. Vectorization is critical because using only one thread for pre-processing is much slower than using a full warp for pre-processing.

To allow vectorized pre-processing, we propose a \( W \)-ary tree (each node has no more than \( W \) children), which can be constructed in \( O(K) \) time with full utilization of GPU warp. Subsequent samples can be obtained in \( O(\log W K) \) time, where \( W \) is the number of threads in a GPU warp, i.e., 32.

We emphasize that our main focus is efficient construction of the tree instead of efficient sampling using the tree, because the cost of sampling using the tree is negligible compared with sampling from \( X_{dev} \). Moreover, the sampling using our \( W \)-ary tree is efficient because In SaberLDA, \( K \) is more than 1 million, and \( \log W K = 4 \), so \( O(\log W K) = O(4) \) is on the same level as the \( O(1) \) alias table algorithm.

Our \( W \)-ary tree is designed to efficiently find the location of a given number in the prefix-sum array. Each node of the tree stores a number, where the bottom-most level nodes stores the prefix-sums of the given array. The length of an upper level is equal to the length of the lower level divided by \( W \), and the \( i \)th node in an upper level is equal to the \((iW - 1)\)th node in the lower level. Fig. 3 illustrates the construction of the tree. This procedure is efficient because all the nodes in one layer can be constructed in parallel. Therefore, the GPU warp can be fully utilized with the aforementioned \( warp\_prefix\_sum \) function, which uses \( W \) threads to compute the prefix-sum of \( W \) numbers in parallel.

To find the position of a given value in the prefix-sum array, we recursively find the position on each level, from top to bottom (Fig. 3). Based on the particular construction of our tree, if the position on the \( l \)th level is \( i \), then the position on the \( l+1 \)th level is between \( iW \) and \( iW + W - 1 \). Therefore, only \( W \) nodes need to be checked on each level. This checking can be achieved efficiently using the aforementioned \( warp\_vote \) function. Also, the memory access is efficient because the tree can be cached in the shared memory (Section 3.2), and it only needs to read \( W \) continuous floating point numbers, i.e., a 128-byte cache line for each level.

### 3.4 Load Balancing

We now analyze the load balance of warp-based sampling in GPUs. A GPU can schedule GPU blocks overall computing units (SMs), and schedule warps of a block within one computing unit. We let a GPU block compute all tokens with the same word and each warp in the GPU block fetches one token at a time. We randomly partition the words into groups for multiple GPUs, so the computing workloads among GPUs are nearly the same. The word scheduling order is based on the descending order of word frequencies; hence, the GPU can use many blocks of small words to achieve balance among SMs.

However, using only one GPU block to compute the most frequent word may take longer than computing all other words in other blocks. For instance, one V100 GPU has 80 SMs and two V100s have 160 SMs. Each SM has to compute the 0.625 percent of the total workload for perfect balancing. By contrast, in the PubMed dataset, the computing workload of the most popular word is more than 1 percent. If we limit the computation of one word in a single GPU block, this will cause 60 percent imbalance overhead, which is unacceptable. Therefore, it is necessary to split the computation of popular words into multiple GPU blocks.

Our work proposes a split index format, which makes changes to the CSR index of the token list. In the traditional CSR format, each index represents the range of tokens belonging to a word in the vocabulary. In our split index format, the indices of those popular words are split into multiple pieces. A split parameter \( P \) is given to split the indices of words that have more than \( P \) tokens. If a word has \( t \) tokens, the index is split into \([t/P] \) pieces.

Fig. 4 shows an example. Without the split index format, block 0 computes the entire workload of word 1, which is larger than any other blocks and creates imbalance. With the help of the split index, the workload of word 1 is split into blocks 0 and 1, so each block has a similar workload and the heavy block is eliminated, avoiding the imbalance.

### 3.5 Efficient Sparse Matrix Counting

In this section, we introduce how to count the sampled topics into sparse count matrices \( \mathbf{A} \) and \( \mathbf{B} \) (line 2 in Algorithm 4). Use of a sparse matrix format can greatly reduce memory space consumption and accelerate construction time. The theoretical time complexity of counting topics is
proportional to the number of tokens, i.e., $O(T)$, which is less than the sampling tokens $(TK_D)$. However, sparse matrix counting has an undeterministic result size, which may lead to extra space exceeding the limited GPU memory. Moreover, if not well optimized in the GPU, the time consumption of counting becomes greater than the sampling step.

### 3.5.1 Count Doc-Topic Matrix A

Based on the word-major sampling order, the topics sampled from tokens are grouped by word, but are with random documents. Fortunately, since the tokens are partitioned by document, we can calculate the partition of count matrix $A$ when the sampling of the partition is finished.

A naïve approach to count the matrix is to sort all the tokens by first document-id $d$, then topic assignment $k$, and then perform a linear scan. However, the sorting is expensive since the global memory must be frequently accessed. Moreover, the efficiency of sorting decreases as the size of the token list increases.

We propose a procedure called **shuffle and segmented count** (SSC) to address this problem. We first perform a shuffle to organize the token list by the document-id’s, i.e., segment the tokens into $D$ small lists where the tokens in each list share the same document. The shuffle can be achieved by sorting all the tokens by the document-id $d$, but the sorting need not be actually performed because the target place of each token is fixed. We pre-process an index array of target places of tokens in the CPU, and transfer it to the GPU following the document partition.

Then, we parallelly count each segment (tokens of each document) in the GPU, which is a common GPU problem known as segmented count. Unlike similar problems, such as segmented sort [25], and segmented reduce [24], which have fully optimized algorithms for GPUs, an efficient GPU solution of segmented count has not yet been well studied.

We propose a solution of segmented count that is sufficiently efficient for SaberLDA. Our procedure consists of three steps, as illustrated in Fig. 5:

1. **Perform a radix sort of each segment.** Since a segment is short, it can be accelerated by caching in the shared memory;
2. **Calculate the prefix sum of the adjacent difference, to get the number of different topics and the order number of each topic; and**
3. **Assign the topic number at the corresponding order number, and increase the count of the same topic.**

In the multi-GPU system, tokens are grouped by different words and split into different GPUs. Our goal is to construct the doc-topic model $A$ and replicate it to all GPUs. Although its computing time is considerably shorter than the kernel sampling function, using only one GPU to compute still hurts the multi-GPU scalability. Our work extends the **shuffle and segmented count** method into a multi-GPU system. To achieve good scalability, we rearrange the tokens in document order and re-partition them to all GPUs. Therefore, the segmented count calculation is evenly separated in each GPU. We implement a distributed doc-topic matrix construction according to the following steps, shown in Fig. 6:

1. **For each GPU, shuffle the tokens into document order.**
2. **Use the All-to-All transfer method to send tokens to the corresponding GPU.**
3. **Shuffle the tokens in each GPU again, ordering all tokens by document id.**
4. **Call the segmented count kernel of each document partition.** Thus each document has its topic count distribution.
5. **Use the All-gather method to broadcast every document count partition to all GPUs.** Each GPU receives all document count partitions and assembles them into a complete doc-topic matrix.

Like the single GPU version, the shuffle index of each token is unchanged among all estimate iterations. All shuffle indices and all-to-all metadata can be pre-possessed in the
CPU before the first iteration. To save the transfer bandwidth between the CPU and the GPU, the doc-topic matrix is only temporarily calculated and used inside GPUs. Only indices and metadata are transferred from CPU to GPUs.

We use NVLink instead of PCI-e for communications between GPUs for its high bandwidth and extra-communication channels. For example, while learning 1000 topics from PubMed dataset in our experiment platform (in Table 3), the computation time per iteration takes about 500 ms with 4 GPUs. The communication between GPUs needs 17ms via NVLink, while it requires more than 70 ms via PCI-e.

3.5.2 Count Word-Topic Matrix B

Finally, we discuss how to efficiently update the dense count matrices B. When the sampling of all tokens with the same word is finished, the corresponding row B, of the word-topic count matrix is ready to be updated. The atomicAdd function must be used because there may be GPU blocks updating the same row, but the overhead is very low since the time complexity of updating is lower than the time complexity of sampling. The word-topic probability matrix Q can be easily generated according to Eq. (2b) from B after all the updates are finished. Maximal parallel performance can be achieved since both matrices are dense.

4 EVALUATION

In this section, we provide an extensive set of experiments to analyze various aspects of the performance of SaberLDA, and to compare SaberLDA with other cutting-edge open source implementations. We analyze the performance on a modern multi-GPU platform and process it on large datasets. The code of SaberLDA is about 3,000 lines, written in CUDA and C++.

4.1 Experiment Setup

4.1.1 Test Platforms

We have two test platforms to test our program as well as other existing solutions.

Platform A has two Intel E5-2670v3 CPUs, with 12 cores per CPU, 128 GB main memory and an NVIDIA GTX 1080 GPU. Platform B is a multi-GPU machine which consists of 4 NVIDIA V100 NVLink-connected GPUs. The bi-directional bandwidth between each pair of GPUs is 96 GB/s. A PCI-e x16 bus is in charge of transferring data between CPU and GPUs. It has 12 GB/s bandwidth for a single direction.

4.1.2 Dataset

We use the dataset from the UCI machine learning repository. NYTimes consists of news articles and PubMed has biomedical literature abstracts. We also use ClueWeb12 [6] dataset, which is a crawl of web pages, as a large dataset. We first extract text from the HTML pages, remove stop words, and tokenize them. A subset of ClueWeb12 that can be fit in main memory is randomly selected. The statistic details of the datasets we used for testing are listed in Table 4.

The hyper-parameters \( \alpha = 50/K \) and \( \beta = 0.01 \) are set according to previous works [9], [17], [34], [35].

4.1.3 Experiment Metrics

The training time of LDA depends on both the number of iterations to converge and the time for each iteration. The former depends on the algorithm, e.g., variational Bayes algorithm [4] typically requires fewer iterations than ESCA [39] to converge. The latter depends on the time complexity of sampling each token as well as the implementation, e.g., the sparsity-aware algorithm has \( O(KD) \) time complexity, and faster performance than the \( O(K) \) vanilla algorithms. We use various metrics to compare LDA implementations as follows.

We use \textit{time per iteration} or \textit{throughput} to compare different implementations of the same algorithm, e.g., SaberLDA with a CPU implementation of the ESCA algorithm, because they require the same number of iterations to converge. The throughput is defined as

\[
\text{Throughput} = \frac{\text{number of tokens } T \text{ processed}}{\text{running time}},
\]

and the unit is million tokens per second (MToken/s).

We compare different algorithms by \textit{the required time to converge to a certain model quality}, because some algorithms may require more iterations to converge, but cost less for each iteration. The model quality is assessed by \textit{holdout log-likelihood}, using the partially-observed document approach [30]. The log-likelihood of each token \( L_{d,v} \) is calculated by the following equation:

\[
L_{d,v} = \log \frac{\sum_{k} P_{k}}{|T_{d}| + K\alpha},
\]

where \( P_{k} \) and \( T_{d} \) are defined in Eqs. (1) and 3, respectively. Higher log-likelihood indicates better model quality.

4.2 Impact of Optimizations

We first investigate the impact of each optimization technique proposed in previous sections by training the LDA on the NYTimes dataset with 1,000 topics for 100 iterations, with the result shown in Fig. 7. The total elapsed time is decomposed as the \textit{Sample} function, rebuilding the document-topic matrix \( A \), constructing pre-processed data structures for sampling, and data transferring the between CPU and GPU.
G0 is the most straightforward sparsity-aware implementation on GPU which sorts all tokens by documents, performs the pre-processed sampling with the alias table, and builds count matrices by naïve sorting of all tokens. G1 adopts the word-major order proposed in Section 3.2, and the time of sampling is reduced by almost 40 percent because of the improved locality for sampling. Note that G1 takes more time than G0 to rebuild the doc-topic matrix because the tokens are ordered by word, and the sorting becomes slower.

The bottleneck of G1 is the construction of the alias table, since it is hard to vectorize. In G2, we replace the alias table with the W-ary tree in Section 3.3.5, which fully utilizes warps to greatly reduce the construction time by 98 percent.

We optimize the rebuilding of the document-topic matrix A with shuffle and segmented count (Section 3.5.1) in G3, and reduce the rebuilding time by 89 percent. Now, the time for updating A and pre-processing is negligible. Finally, in G4, we enable multiple workers running asynchronously to greatly reduce the construction time by 98 percent.

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This shows clear advantage over CPUs, given that the bandwidth between the main memory and CPU is only 40-80 GB/s. The throughputs of the L2 cache, unified L1 cache, and shared memory are 203 GB/s, 894 GB/s, and 458 GB/s respectively, while the utilization is lower than 25 percent. Therefore, these are not the bottlenecks of the overall system performance.

4.4 Comparison With Other Implementations

We compare SaberLDA with one previous GPU-based implementation, BIDMach [41], as well as three CPU-based implementations, ESCA (CPU), DMLC [38], and WarpLDA [9]. BIDMach [41] is the open-source GPU-based implementation. It is a general GPU-based machine learning tool whose algorithms are implemented in Scala, and the basic kernel functions, such as matrix manipulation are compiled to the CUDA library. BIDMach reports better performance than Yan et al.’s implementation [33], [41], and Steele and Tristan’s implementation only reports tens of topics in their paper. Therefore, we think it is reasonable to compare SaberLDA with BIDMach, and we choose the batched Variational Bayes algorithm of BIDMach because it has better convergence than other algorithms on thousands of topics. ESCA (CPU) is a carefully optimized CPU version of the ESCA algorithm that SaberLDA also adopts. DMLC has a collection of multi-thread LDA algorithms on CPU, and we choose the FTreeLDA algorithm that achieved the best performance among all algorithms provided by DMLC. WarpLDA uses a state-of-art Metropolis-Hastings sampling algorithm with O(1) time complexity per token.

We compare the time to converge of these implementations on four settings on the NYTimes and PubMed datasets, with the number of topics K = 1000. Fig. 8 shows the convergence over time. We compare the time to converge to the log-likelihood of −8.0 and −7.3, for NYTimes and PubMed, respectively. SaberLDA is 5.0 - 6.2 times faster than BIDMach. We also attempt to perform the comparison with 3,000 and 5,000 topics, and find that BIDMach is more than 10 times slower than SaberLDA with 3,000 topics, and reports an out-of-memory error with 5,000 topics. This is as expected because the time consumption of BIDMach grows linearly with respect to the number of topics, and its dense matrix format is much more memory consuming than SaberLDA. SaberLDA
is about 4 times faster than ESCA (CPU) and 5.4 times faster than DMLC on the two datasets with $K = 1, 000$. WarpLDA converges to a worse local optimum possibly because of its inexact algorithm and the different metric we used to assess model quality compared with its paper [9]. This shows that SaberLDA is more efficient than other implementations.

4.5 Multi-GPU Performance

We now analyze the performance of SaberLDA on Platform B, which has 4 V100 NVLink-connected GPUs.

4.5.1 Scalability

Fig. 9 shows the throughput and speedup in multi-GPUs. The throughput is the average among 100 iterations. We use the NYTimes dataset and split the documents into two partitions for overlapping of the peer-to-peer transferring and calculating steps. All data stays in the GPU, avoiding the PCI-e transferring bottleneck between the CPU and GPUs. The shared memory is used to fetch the current row of $Q_v$ when estimating 1000 topics while processing 3k and 10k topics would use the global memory.

Our work achieves throughput of 1.83 GToken/s with the NYTimes dataset and 1000 topics with 4 GPUs. When the number of topics increases, the throughput reduces to 1.34 GToken/s for 3k topics and 1.19 MToken/s for 10k topics. This is because of the increasing size of the vector $B$, which leads to more cache miss rate in the sampling kernel. The right subfigure of Fig. 9 shows that the speedup achieves about 1.92x with 2 GPUs and 3.61x to 3.7x with 4 GPUs.

4.5.2 Benefits of Split Index

We analyze the benefit of the Split Index proposed in Section 3.4. We test the throughput of 100 iterations with the PubMed dataset with 1000 topics. Those word indices are split if its percentage of workload is larger than the average workload per GPU SM. The results shows are presented in Fig. 10. Both versions have the same performance with a single GPU. With 4 GPUs, the original version has less than 3x speedup, whereas the split-index version has nearly 4x speedup. The split index method is effective when estimating skew vocabularies in multi-GPUs.

4.5.3 Convergence and Throughput

We then analyze the convergence and throughput in the small datasets of NYTimes and PubMed. The numbers of topics are 1k, 10k, and 40k and we use an estimated 300 iterations, which is enough for the model to attain convergence. The results are shown in Fig. 11. SaberLDA finishes in 16, 27, and 77 seconds for NYTimes with the different numbers of topics, and takes 143, 186, and 466 seconds to finish for PubMed. The throughput achieves 2.05 GToken/s for NYTimes with 1000 topics and it becomes stable at 490 MToken/s when estimating for PubMed with 40k topics.

4.5.4 Large Dataset

Finally, to demonstrate the ability of SaberLDA to process large datasets, we test the performance of SaberLDA on a large subset of the ClueWeb12 dataset. In this experiment, we used the 4-GPU platform B as our test machine. We run SaberLDA in 300 iterations with 1,000, 10,000 and 40,000 topics separately, and the result are shown in Fig. 12. The throughput of estimating 1,000 and 10,000 topics are about 500 Mtoken/s, which is limited by the PCI-e x16 link connected between the CPU and the PCI-e hub in this platform. The throughput of estimating 40,000 topics is approximately
5 Related Work

LDA Algorithms. The classic inference algorithms such as variational Bayes [4], [26], expectation propagation [21], and Collapsed Gibbs sampling (CGS) [15] have linear time complexity $O(K)$ for each token. Various algorithms have been proposed to optimize the time complexity to average topics per word $O(K/W)$ (SparseLDA [34]), average topics per documents $O(K/c^2)$ (AliasLDA [18], ESCA [39], F+LDA [35]), and constant time complexity $O(1)$ (LightLDA [36], WarpLDA [9]).

Distributed CPU LDA Systems. Distributed LDA systems were designed to accelerate training large datasets. They are based on MPI and OpenMP primitives (PLDA [31], ADLDA [23], Peacock [32], F+LDA [35], WarpLDA [9] and [27]), parameter server abstraction (LDA* [37], LightLDA [36], Yahoo!LDA [1]), and Spark (ZenLDA [40]). The state-of-the-art distributed implementation [9] can infer 1 million topics and reach good scalability at 256 servers.

GPU LDA Systems. Yan et al. [33], BIDMach [41], Steele and Tristan [28], AGA-LDA [22], and Pyro [3] implemented LDA in GPU(s), supporting at most a thousand topics because of their $O(K)$ algorithms.

LDA Model Extensions. As extensions of the classic flat topic models, Hierarchical [14] and DAG-structured [19] LDA models can learn topics of different levels of abstraction. Chen et al. [10] implemented a distributed hierarchical LDA system with 50 machines.

6 Limitations, Discussion, and Future Work

One limitation of SaberLDA is that it can only scale-up to a server with multiple NVLink connected GPUs. We plan to extend the current implementation to multiple servers to make the training process even faster.

The other limitation is that SaberLDA holds all input data in the main memory of the CPU server and streams this data to GPUs. This can be improved by using NVMe SSD disks to support larger input data size.

Although we focus on the classic LDA algorithm in this paper, we believe the techniques proposed in this paper are generally applicable to hierarchical and DAG-structured LDA models, which also have sparse models.

7 Conclusion

We presented SaberLDA, a high-performance sparsity-aware LDA system on a multi-GPU system. By adopting sparsity-aware algorithms, SaberLDA overcomes the problems of previous GPU-based systems, which support only a small number of topics. We proposed a novel data layout, a warp-based sampling kernel, and an efficient sparse count matrix updating algorithm to address the challenges induced by sparsity. We then demonstrated the power of SaberLDA with extensive experiments. It can efficiently handle large-scale datasets with up to 8.9 billion tokens and learn large LDA models with up to 40,000 topics, which is orders of magnitude larger than existing GPU-based LDA systems. With the split index structure, and workload balancing algorithm, SaberLDA has linear speedup on 4 V100 NVLink connected GPUs and is more than 20 times faster than a modern dual CPU server. We hope our work will motivate more research on supporting more sparsity machine learning algorithms with GPUs.

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CURE: A High-Performance, Low-Power, and Reliable Network-on-Chip Design Using Reinforcement Learning

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Abstract—We propose CURE, a deep reinforcement learning (DRL)-based NoC design framework that simultaneously reduces network latency, improves energy-efficiency, and tolerates transient errors and permanent faults. CURE has several architectural innovations and a DRL-based hardware controller to manage design complexity and optimize trade-offs. First, in CURE, we propose reversible multi-function adaptive channels (RMCs) to reduce NoC power consumption and network latency. Second, we implement a new fault-secure adaptive error correction hardware in each router to enhance reliability for both transient errors and permanent faults. Third, we propose a router power-gating and bypass design that powers off NoC components to reduce power and extend chip lifespan. Further, for the complex dynamic interactions of these techniques, we propose using DRL to train a proactive control policy to provide improved fault-tolerance, reduced power consumption, and improved performance. Simulation using the PARSEC benchmark shows that CURE reduces end-to-end packet latency by 39 percent, improves energy efficiency by 92 percent, and lowers static and dynamic power consumption by 24 and 38 percent, respectively, over conventional solutions. Using mean-time-to-failure, we show that CURE is 7.7× more reliable than the conventional NoC design.

Index Terms—Computer architecture, network-on-chip (NoC), reliability, deep reinforcement learning

1 INTRODUCTION

Network-on-Chips (NoCs) \cite{1}, \cite{2} have emerged as the standard interconnect solutions for connecting multiple cores, memory modules, and other hardware components. With continuous aggressive technology scaling, the reliability issue of NoCs becomes considerably more pronounced because the transistors and wires in NoCs are becoming increasingly vulnerable to faults, which are predominantly classified as permanent faults (induced by hardware aging) and transient errors (caused by runtime variations, overheated hardware, transistor delays, etc.).

A significant amount of work has been proposed to enhance the robustness of NoC \cite{3}, \cite{4}, \cite{5}, \cite{6}, \cite{7}, \cite{8}, \cite{9}, \cite{10}. Unfortunately, these existing fault-tolerant methodologies have some critical defects. First, these techniques are limited, because they only focus on either faults within routers (at the gate-level) \cite{3}, \cite{4} or faults that occur on inter-router links (at the link-level) \cite{6}, \cite{7}, \cite{8}, \cite{9}, \cite{10}. Second, these conventional error-handling techniques can be expensive: SHIELD \cite{3} and Vicis \cite{4} duplicate the logic circuitry in routing pipeline stages or use redundant input ports, which consume massive chip area, while other retransmission-based fault-handling schemes, such as \cite{5} and \cite{6}, generally incur substantial power consumption and prohibitive latency.

Deploying power-saving and performance-enhancing techniques to compensate the cost and performance degradation caused by fault-tolerant methodologies is indispensable yet extremely complex. Different optimization techniques, when being used simultaneously, can conflict and offset each other’s desired goals, which presents various design trade-offs. For example, channel buffers \cite{7}, \cite{11}, \cite{12} replace the power-consuming router buffers with link storage to save power, but result in performance loss due to the limited throughput of link storage. Power-gating techniques \cite{13}, \cite{14}, \cite{15} are proposed to take advantage of idle router periods to save power; however, they incur prohibitive wake-up latency. Dynamic voltage and frequency scaling \cite{13}, \cite{16} intends to balance power savings and network throughput, but it can lead to increased transient faults \cite{17}. Due to the explosion and complexity of the design space, we intend to use machine learning techniques to optimize the dynamic interactions of different techniques and automatically learn an optimal control policy to address the challenges of simultaneously decreasing power consumption, increasing performance, and improving reliability.

In this paper, we propose CURE, a learning-based NoC design framework, that handles permanent and transient faults at both the gate-level and link-level in a high-performance, low-power-consumption manner. CURE uniquely utilizes a per-router deep reinforcement learning (DRL)-based \cite{18}, \cite{19}, \cite{20} control policy to explore the dynamic interactions among NoC components and system-level performance metrics as well as optimizing the trade-offs.
The major contributions of this paper are as follows:

- **Improved Inter-Router Channel Design:** We improve our previous proposed channel buffers [8] by designing reversible multi-function adaptive channel (RMC) buffers. RMC buffers have three basic functions: (1) forward/backward regular repeaters for flit propagation, (2) forward/backward buffers for link storage, and (3) forward/backward re-transmission buffers for handling faults. RMC is beneficial in several ways. First, with the additional storage available on the inter-router channel, dynamic power consumption for on-chip storage is reduced at high network loads without any performance degradation. Second, RMC provides flexibility for improving reliability at the link-level via re-transmission buffers and reversing the propagation direction to avoid faulty links. Third, the reversibility of RMC enhances performance by allowing the NoC to dynamically adapt to traffic because it provides extra link bandwidth in a specific direction at high network loads. Additionally, we utilize the bypass route proposed in [8], for the purpose of enhancing reliability, to retain NoC connectivity when permanent faults occur on RMCs.

- **Robust Router Microarchitecture Design:** We significantly improve the NoC robustness for both transient errors and permanent faults. First, we propose per-router self-diagnosis adaptive error control hardware to detect/correct faults at the link-level. The proposed error correction hardware adapts to the error level of each port and dynamically deploys the most efficient error detection/correction and flit re-transmission schemes with minimized power and latency overheads. Additionally, a low-cost, fault-vulnerable detector is implemented inside the error control hardware to detect malfunctions of the error control hardware itself. Second, we modify the circuitry of the router to mitigate transient errors and permanent faults that occur in the routing pipeline stages at the gate-level. Additionally, we propose a power-gating scheme that dynamically powers off NoC components (router buffers, crossbar, error control hardware, etc.) when needed to achieve power savings and mitigate aging effects.

- **DRL-Based Control Policy Design:** We propose a set of unique operation modes for each router with a DRL-based control policy to handle the dynamic interactions and optimize the trade-offs. The goal of dynamically utilizing different operation modes is to achieve improved performance, maximized power savings, and enhanced reliability. At runtime, per-router DRL agents observe and learn from the entire NoC environment and automatically evolve optimal per-router control policies that select the optimal operation modes at any given time.

We evaluate the performance of the proposed CURE architecture using a modified Booksim2 [21] simulator with PARSEC benchmarks on an 8 × 8 2D mesh architecture. We show that the proposed CURE provides significant power savings, enhanced reliability, higher performance, and lower area overhead compared to multiple state-of-the-art NoC designs with power-saving and fault-tolerant mechanisms.

## 2 CURE Microarchitecture

In this section, we demonstrate the architectural innovations of the proposed framework. The overall microarchitecture of the proposed design is shown in Fig. 1. The proposed design consists of inter-router links based on reversible multi-function channels (RMCs), a fault-tolerant router design, dynamic fault-secure error correction hardware, and a router bypass route for power savings and stress management. Additionally, a DRL-based controller is located in each router to handle the dynamic interactions and optimize the trade-offs. The implementation of RMCs is described in Section 2.1. The fault-tolerant router design is presented in Section 2.2. The adaptive error correction (ECC) hardware is presented in Section 2.3, and the bypass route is discussed in Section 2.4.
2.1 Reversible Multi-Function Channels (RMCs)

Although previous research [7], [11], [12] has shown that the excessive power consumption of router buffers can be reduced by moving storage to inter-router links or channels, the performance loss has not been thoroughly considered. In this paper, we borrow the idea of reversible links in [7] and extend the multi-function adaptive channel (MFAC) buffers in [8] to reversible multi-function adaptive channel buffers (RMC buffers), with the objective of improving network performance. With the newly designed channel architecture, RMC storage, links, and router storage are dynamically allocated according to traffic patterns to reduce power consumption. As shown in Fig. 2, each RMC buffer consists of four physical links, with four buffer stages per link. Each physical link can be used for either storage or transmission as regular repeaters, in both directions, controlled by the additional four transmission gates. The direction and functionality of each RMC link are configured by the added RMC controller. The RMC controller first uses a single-bit reversal (rev and rev') signal to enable/disable the four transmission gates. In this way, the link can be configured in a specific direction. For example, when the rev signal is high, the RMC buffer will store/propagate flits forward. After configuring the direction, the RMC controller will send a function selection signal along with the 1-bit congestion signal from the downstream router to enable one of the functions of the RMC buffer. The RMC buffer can implement three basic functions in both directions, namely, link storage, transmission repeaters, and re-transmission buffers, which are illustrated in Fig. 3 and discussed below.

2.1.1 RMC Buffer Functionalities

Previous designs [8], [12] show that tri-state transistors can be used for propagating or storing flits. Fig. 2 shows the proposed RMC buffers based on those tri-state transistors. The proposed RMC between two adjacent routers consists of four physical links, with four buffer stages per link. Each physical link can be used for either storage or transmission as regular repeaters, in both directions, controlled by the additional four transmission gates. The direction and functionality of each RMC link are configured by the added RMC controller. The RMC controller first uses a single-bit reversal (rev and rev') signal to enable/disable the four transmission gates. In this way, the link can be configured in a specific direction. For example, when the rev signal is high, the RMC buffer will store/propagate flits forward. After configuring the direction, the RMC controller will send a function selection signal along with the 1-bit congestion signal from the downstream router to enable one of the functions of the RMC buffer. The RMC buffer can implement three basic functions in both directions, namely, link storage, transmission repeaters, and re-transmission buffers, which are illustrated in Fig. 3 and discussed below.

1) Forward/Backward Transmission Repeater (Fig. 3a): In this case, the RMC buffer links are configured as repeaters. When the RMC controller is set to forward the congestion signal and the 1-bit congestion signal is low, the transistors connected to GND and Vdd are enabled, allowing the RMC to act as a transmission channel.

Each RMC between two adjacent routers has four physical links, and the channel buffers are evenly allocated on those links.

Next, we use an RMC controller to dynamically and independently configure the transmission/buffer functions of the physical links to perform multiple RMC functions, as shown in Fig. 3. As described in detail in Section 2.1.1, RMCs can function as (1) forward/backward transmission repeaters, (2) forward/backward link storage, and (3) forward/backward re-transmission buffers. The dynamic selection of RMC functionalities is explained in Section 3. Because our design may lead to a latency penalty (due to the control overhead of the RMC buffers) and potential congestion (due to the head-of-line blocking of router buffers), we use dynamic router buffer allocation to maximize network throughput, as detailed in Section 2.1.2.
are disabled. Flits are then buffered in the configuration. If an ACK signal is received, the flit is discarded toward the flit (regular transmission). Upon receiving a "hold" signal and the other RMC buffer links for forwarding the flit, the RMC controller configures one of the RMC buffer links for storage (by applying voltage that is not powered-off for BST).

As mentioned above, determining which direction to allocate the RMC links is critical. Network traffic is measured using link utilization and buffer utilization values monitored by the corresponding router. The RMC controller analyzes the network throughput in each direction, using the number of propagated packets, and calculates the ratio of these two throughput numbers. The RMC controller will allocate the RMC links with the configuration that is nearest to the ratio. For example, if the ratio of the \(-X\) traversal to the \(+X\) traversal is 2.6, the RMC_3 configuration will be utilized.

After allocating the direction of RMC links, the RMC buffers will be allocated using a unified buffer state table (BST), as shown in Fig. 4. The proposed BST is router-associated and shared by all the input ports within the router and remains accessible even if the router is power-gated (power-gating information is recorded in the yellow and orange entries shown in Fig. 4). The proposed BST is a modified version of the conventional virtual channel (VC) state table. The conventional VC table consists of the following information, or entries: the VC identifier (VC), read pointer (RP), write pointer (WP), allocated output port (OP), output VC (OV), status (Stat), and credit count (CR). In the conventional VC state table, the header flit carries the packet information for route computation (RC) and VC allocation. The VC state table allocates a free VC slot to the header flit and records the VC information (VCID) and output information (output VC and output port). The body flits of the packet simply follow the VCID to find the correct output port from the VC state table. Thus, the packet is routed correctly.

In CURE, we modify the VC state table and implement BST to support RMC channel buffers and record routing information when the router is power-gated. Compared to input-port-associated VC state tables, the proposed BST is router-associated and shared by all input ports within the router. Other than the entries of the conventional VC state table, the proposed BST also consists of additional entries for allocating channel buffers: input port identifier (Port) that indicates the input port of the incoming flit, downstream router status (DRS) that indicates if the downstream router is power-gated, a channel buffer pointer (CBP) and channel buffer credit (CBC) that indicate the occupancy status of the associated RMC buffers. We also create two entries, OPX and OPY, to replace the conventional output port entry to support adaptive XY/YX routing. To enable BST functioning when the router is power-gated, we consider a separate supply voltage that is not powered-off for BST.

The flow control is simple. While the router is powered on, the body flits simply follow the VC and output port information carried by the header flit using the BST. Similarly, when the router is power-gated, the BST also records the VC and OP information of the header flit. Thus, the body flits can be routed to the associated output port by looking up the information. When the flit leaves the bypass switch, a credit is sent back to its upstream router for updating the credit information. This guarantees that the flow control operates normally, irrespective of whether the router is powered on or off. The BST is powered by a separate voltage supply. Additionally, the congestion control block monitors and updates the BST by recording all the available router buffer and RMC buffer slots. If all the router buffer slots and RMC buffer slots of an input direction are occupied, a congestion signal will be triggered.
To further balance traffic and utilize RMC resources, we propose an adaptive XY/YX routing algorithm to mitigate traffic in high-intensity ports/links to underutilized ports/links after RMC is configured. This can provide more routing options when flits compete for the same output port. At the RC stage, for a packet from the local injection port, the RC unit calculates the output ports out of both the XY and YX routes and stores the results in the corresponding VC state table entry. For a packet from other input ports, the routing identification bit in its head flit decides which routing computation unit is used. Because XY and YX routes can be calculated simultaneously with symmetric logic, the proposed RC unit does not introduce extra delays in the RC stage. The VC allocation stage utilizes the unified BST. All VC states are stored in the unified BST. The OP entry is extended to two parts: the output port for XY routing (OPX) and the output port for YX routing (OPY). For a packet newly injected from the injection port of the current router, the information of the output ports for XY routing (OPY) and the output port for YX routing (OPY) is calculated by the unified BST. All VC states are stored in the unified BST.

Fig. 5. Proposed fault-tolerant router.

Note that in the 4-pipeline-stage router, the arbitration algorithm has already been determined; thus, only one of two output ports is valid. The other output port will be ignored based on the additional routing identification bit in the head flit. The proposed routing scheme is deadlock-free.

2.2 Fault-Tolerant Router Design
A typical NoC router consists of input/output ports, buffers, routing logic, and a crossbar that connects the input ports to output ports for packet routing. The conventional router has four pipeline stages: routing computation stage, virtual channel allocation stage (VA), switch allocation stage (SA), and switch traversing (ST), which is also known as the crossbar stage. Permanent faults occasionally occur in the last three stages if the corresponding hardware (i.e., arbiter, switch, link, and crossbar) is faulty [3]. In this paper, several simple MUXes/DEMUXes are added to the conventional VA and SA stages to allow the router to bypass the faulty arbiter and borrow the unoccupied arbiter to perform VC and switch allocation. We modified the SHIELD [3] architecture to fit the proposed CURE using RMC buffers and unified BST. The proposed architecture is shown in Fig. 5. As shown in Fig. 5, in the VA and SA stages, arbiters from the virtual channels of the same input port can be shared.

ST (Crossbar) Stage. A crossbar is considered faulty if any of the MUXes/DEMUXes located in the crossbar are faulty and the packet fails to be forwarded to the output port. To overcome such faults, additional MUXes and DEMUXes are added to each output port to create a backup flit ejection path. The modified crossbar is shown in Fig. 5c. The crossbar is fault-free as long as either the original path or the backup path is not faulty.

2.3 Fault-Secure Adaptive ECC Hardware Design
Conventional static error correction hardware is either not power efficient or not powerful enough to handle transient faults: lightweight error control schemes (e.g., end-to-end CRC) can lead to excessive re-transmission traffic at high error rates, and powerful error correction schemes (e.g., double-error correction triple-error detection (DECTED)) are power consuming. We use three existing static error control schemes, namely end-to-end CRC, per-hop SECDED, and per-hop DECTED, to evaluate the trade-offs between NoC performance metrics of different static error mitigation techniques. The evaluation result is shown in Fig. 6. As shown in Fig. 6, for low error levels, SECDED and DECTED both negatively impact system performance due to their encoding and decoding overheads. However, as the error level increases, it is beneficial to deploy SECDED and DECTED to mitigate re-transmission packets. Therefore, there is a strong need for a dynamic error control hardware that can adapt to different error levels and apply the most efficient error mitigation technique at runtime. To this end, we propose a per-router-based adaptive error control hardware to
mitigate soft errors in RMC links for balanced NoC power, performance, and reliability. Additionally, since transistors are less reliable with aggressive technology scaling, the combinational logic of ECC hardware is also vulnerable to faults. Thus, we also enhance the router with self-diagnosis function to ensure that the error control circuitries (i.e., ECC encoder and decoder) are fault-secure.

The proposed adaptive error correction hardware is shown in Fig. 7. The proposed error control hardware can be configured as end-to-end CRC, per-hop SECDED, and per-hop DECTED. At runtime, the ECC hardware dynamically deploys the most appropriate error control scheme guided by the DRL-based control policy discussed in Section 4. We reinforce the fault-prone ECC hardware design by proposing a self-diagnosis ECC fault detector to achieve fault-tolerance in both the communication channels and the ECC circuitry. The proposed self-diagnosis detector can verify the correctness of the ECC hardware operations using low-density parity-check (LDPC) codes [22]. As shown in Figs. 7a and 7b, each ECC encoder, decoder, and corrector is assigned to an ECC fault detector. The detector applies LDPC codes to each syndrome vector (the 9-bit Hamming code) from the outputs of different ECC hardware. The circuitry details of the proposed ECC fault detector are shown in Fig. 7c. The last OR gate has 12 inputs. The first 9 inputs are the required LDPC input. Because LDPC codes are proven to detect all the error combinations in the 9-bit syndrome vector [23], the proposed detector can detect malfunctions in the ECC hardware due to transient errors. To enhance the tolerance of the ECC hardware to permanent faults, we uniquely add three more inputs to the last OR gate of the error detector. These three bits can indicate the location of the faulty bit in the 9-bit Hamming code. If an error occurs in the same bit repeatedly, the corresponding gates in the ECC hardware will be marked as faulty, and the specific faulty gate will be power-gated, while the bit-level operations will be performed by the other underutilized gates (e.g., the XOR gates for DECTED). Additionally, note that the proposed ECC fault detectors are only activated when the SECDED/DECTED scheme is enabled.

2.4 Router Bypass Route Design

In this paper, we modify the stress-relaxing bypass technique originally proposed in [8], which proactively power-gates and bypasses the NoC router to save power and prevent overheating by adding an extra escape link. All of the inter-router escape links together construct a bypass-ring network to retain connectivity when there are a significant number of failures. The bypass-ring network fully connects each router in the NoC. An example of a chip-level bypass ring in a 4 × 4 2D mesh is demonstrated with red arrows on the left-hand side in Fig. 8. When all the possible links are faulty in one direction (i.e., all the corresponding RMCs are faulty), the escape link will be activated. Each escape link has one single-flit latch as the link in the stress-relaxing bypass route. The incoming flit will be stored at that single-flit latch and propagated using a round robin scheme. In this way, packets can pass through the faulty router and proceed to the next router. In addition, the proposed design continues to utilize the BST for routing information while the router is bypassed. This retains the connectivity of that direction and eliminates resource starvation to prevent deadlock. In this way, the new bypass route with escape links can further enhance reliability in the presence of faults. Additionally, to reduce the area overhead of the cross-router bypass-ring network, the ring network does not support bypassing in all directions, which means that each network interface can accept packets from only one specific upstream router and forward packets to specific downstream router.

3 PROACTIVE OPERATION MODES

In this section, we propose ten proactive operation modes (1 power-gating mode and 9 fault-tolerant modes) for CURE routers. Each operation mode has various configurations of RMC, adaptive error control hardware, and power-gating strategies. Each CURE router occasionally and independently selects and deploys an operation mode proactively using a DRL-based control policy (described in Section 4). The operation modes are detailed below.

- **Operation Mode 0 - Power-Gating Mode:** In this mode, the router is power-gated, while the bypass route is enabled. The RMC channel is configured as 2:2 for
both directions, and the RMC buffers are used to store the incoming flits. This operation mode is activated either when the router is underutilized or when a high risk of overheating is predicted. This mode mitigates permanent faults and saves static power.

- **Operation Modes 1, 2, and 3 - CRC-Only Mode:** Operation modes 1, 2, and 3 have different RMC direction configurations: 2:2 for mode 1, 1:3 for mode 2, and 3:1 for mode 3. However, these operation modes share the same error control configurations. In these modes, the RMCs are configured as storage buffers, and the entire adaptive ECC hardware is power-gated, so that only CRC is enabled. These operation modes are beneficial to save power and eliminate ECC computational overhead when the error level is low.

- **Operation Modes 4, 5, and 6 - Per-hop SECDED Mode:** In these modes, the router’s adaptive ECC hardware is partially activated to perform per-hop SECDED. This configuration is beneficial when SECDED can handle most of the faults. Otherwise, it will either lead to unnecessary power and latency penalties (when the error level is low) or excessive re-transmissions (when errors cannot be corrected by SECDED). The RMC buffers are configured as re-transmission buffers. Similar to operation modes 1, 2, and 3, the RMC direction configurations are set to 2:2 for mode 4, 1:3 for mode 5, and 3:1 for mode 6.

- **Operation Modes 7, 8, and 9 - Per-hop DECTED Mode:** In these modes, the router activates the entire adaptive ECC hardware to enable DECTED. This is the situation where the flits are more likely to contain errors of 2 or more bits. The RMC buffers are configured as re-transmission buffers. Similarly, the RMC direction configurations are set to: 2:2 for mode 7, 1:3 for mode 8, and 3:1 for mode 9.

The dynamic selection of operation modes is performed by each router independently in a sequence of discrete time steps using the DRL-based control policy presented in Section 4. At each time step, each router decides which operation mode to apply for the following time step and passes the decision to the downstream router. In this way, the downstream router will be informed to configure the ECC decoder located in the corresponding input port to apply the correct ECC coding, such that it is synchronized with the ECC coding of the upstream router’s encoder at output port at the next time step. As demonstrated, the proposed dynamic operation modes allow individual routers to utilize the most suitable technique at runtime, resulting in greater benefits for the entire network.

### 4 Proposed Deep Reinforcement Learning (DRL)-Based Control Policy

<table>
<thead>
<tr>
<th>Category</th>
<th>Features</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Router Input</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Related Metrics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. +X link utilization</td>
<td>Input flits/cycle of +X input port</td>
<td></td>
</tr>
<tr>
<td>2. –X link utilization</td>
<td>Input flits/cycle of –X input port</td>
<td></td>
</tr>
<tr>
<td>3. +Y link utilization</td>
<td>Input flits/cycle of +Y input port</td>
<td></td>
</tr>
<tr>
<td>4. –Y link utilization</td>
<td>Input flits/cycle of –Y input port</td>
<td></td>
</tr>
<tr>
<td>5. Local port link utilization</td>
<td>Input flits/cycle of local port</td>
<td></td>
</tr>
<tr>
<td><strong>Buffer Related Metrics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. +X buffer utilization</td>
<td>the buffer utilization of +X input port</td>
<td></td>
</tr>
<tr>
<td>7. –X buffer utilization</td>
<td>the buffer utilization of –X input port</td>
<td></td>
</tr>
<tr>
<td>8. +Y buffer utilization</td>
<td>the buffer utilization of +Y input port</td>
<td></td>
</tr>
<tr>
<td>9. –Y buffer utilization</td>
<td>the buffer utilization of –Y input port</td>
<td></td>
</tr>
<tr>
<td>10. Local port buffer utilization</td>
<td>the buffer utilization of local port</td>
<td></td>
</tr>
<tr>
<td><strong>Router Output</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Related Metrics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. +X link utilization</td>
<td>Output flits/cycle of +X input port</td>
<td></td>
</tr>
<tr>
<td>12. –X link utilization</td>
<td>Output flits/cycle of –X input port</td>
<td></td>
</tr>
<tr>
<td>13. +Y link utilization</td>
<td>Output flits/cycle of +Y input port</td>
<td></td>
</tr>
<tr>
<td>14. –Y link utilization</td>
<td>Output flits/cycle of –Y input port</td>
<td></td>
</tr>
<tr>
<td>15. Local port link utilization</td>
<td>Output flits/cycle of local port</td>
<td></td>
</tr>
<tr>
<td><strong>RMC Related Metrics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16. +X RMC Previous Mode</td>
<td>Operation mode at the last timestep</td>
<td></td>
</tr>
<tr>
<td>17. –X RMC Previous Mode</td>
<td>Operation mode at the last timestep</td>
<td></td>
</tr>
<tr>
<td>18. +Y RMC Previous Mode</td>
<td>Operation mode at the last timestep</td>
<td></td>
</tr>
<tr>
<td>19. –Y RMC Previous Mode</td>
<td>Operation mode at the last timestep</td>
<td></td>
</tr>
<tr>
<td><strong>Other Metrics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20. Temperature</td>
<td>Local router temperature in °C</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9. Router attributes selected in the state vector.

The goal of the agent is to optimize its long-term return, which is represented by the discounted sum of future rewards. The return at time step \( t \) is defined as

\[
R_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \ldots
\]

The variable \( \gamma \) (where \( 0 \leq \gamma \leq 1 \)) is a discount rate, which determines the impact of future rewards on the total return: as \( \gamma \) approaches 1, the agent becomes less near-sighted by giving more weight to future rewards.

In this paper, with the goal of simultaneously improving performance, energy-efficiency, and reliability, we design the reward function for router \( i \) at time step \( t \) as
\[ r_{i,t} = - \log_\alpha (\text{Latency}_{i,t}) - \log_\beta (\text{Power}_{i,t}) - \log_\lambda (\text{Aging}_{i,t}). \] (2)

The \textit{Latency} refers to the average end-to-end latency of the specific router \( i \), \textit{Power} contains both static and dynamic power consumption. Additionally, the aging factor is calculated using the aging model, which is described in detail in Section 5.1. The \( \alpha, \beta, \) and \( \lambda \) is used to emphasize the importance of each individual desired goal. In this paper, we set all three parameters, \( \alpha, \beta, \) and \( \lambda \), to 1.

**Deep Q-Learning Approach.** In DRL, a model of the environment, specified through a probability distribution \( p(s_{t+1}, r_{t+1}|s_t, a_t) \), characterizes how the state of the environment changes as a result of an agent action, and the reward that the agent receives after each action. Correspondingly, DRL agents compute an action-value function \( Q^\pi(s, a) \) that estimates the return that they are expected to receive in this model of the environment if they start in state \( s \), take action \( a \), and follow the policy \( \pi \) for the remaining actions.

To find the optimal Q-value function \( Q^*(s, a) \) that maximizes the expected return, we use the tabular Q-learning algorithm [18]. Q-values are initialized with zeros for all possible \((s, a)\) pairs at the beginning. At each time step, the Q-learning algorithm chooses actions based on the current Q-such that, over many time steps, all actions are taken in all states. After taking an action \( a \) and observing the reward \( r \) and new state \( s' \), the action-value entry \( Q(s, a) \) is changed using the following temporal difference rule:

\[ Q(s, a) = (1 - \alpha)Q(s, a) + \alpha[r + \gamma \max_{a'}Q(s', a')]. \] (3)

The learning rate \( \alpha \) can be reduced over time and determines how well Q-learning will converge. It can be shown that for appropriate values of \( \alpha \), Q-learning converges to the optimal Q-value function \( Q^* \) and its corresponding optimal policy \( \pi^* \) [18]. To explore unvisited regions of the state-action space, an \( \epsilon \)-greedy policy is also applied to \( \pi^* \), where agents also have a probability of \( \epsilon \) to select a random action rather than always taking the action with the maximum Q-value [24].

Note that we intentionally design a reward function with a negative value to achieve a better high-level performance of RL. Because the Q-values of all the visited state-action pairs are negative and all the unvisited pairs have Q-values equal to zero, the RL agent will always select unvisited state-action pairs (zero is greater than any negative number). It allows the RL agents to explore the state-action pairs as much and as fast as possible, which leads to shorter convergence time for optimal decision making.

In conventional RL, the optimal policy \( \pi^* \) is recorded in a state-action mapping table called Q-table, where all Q-values are stored. As mentioned, each state vector consists of a number of features. When the RL agent observes any new state-action pair, it creates a new entry in the Q-table to record its actions and associated Q-values. Although the feature values are discretized into limited bins, the area consumption for the Q-table is excessive. To address this problem, we implement deep Q-learning by replacing the state-action table with an offline-trained artificial neural network (ANN) to reduce the hardware costs. The ANN calculates the state-action value rather than storing the entire state-action table in the router, thus eliminating the storage space for state-action pairs.

In CURE, each ANN consists of 20 neurons at the input layer, 30 neurons at the hidden layer, and 10 neurons at the output layer. For training, we use a cycle-accurate network simulator to simulate the dynamic interactions and the online process of reinforcement learning. At this stage, Q-values of different state-action pairs are explored using real-world applications. All the visited state vectors and their corresponding Q-values (calculated with the reward function) are recorded as training sets for the ANN. Specifically, the attribute values of the state vector are ANN inputs, and the Q-values are the desired outputs. During each time step, similarly to updating the Q-value using reward values in conventional RL, the ANN follows the Q-value update rule and record \( \Delta Q \) as the output error. Then, it uses mini-batch gradient descent to back propagate this error to the hidden layer to tune the weights. The training time is not counted to the timing overhead of the proposed NoC system. During the testing (or inference) phase, the ANN monitors the attribute values and uses the input values and weights to calculate 10 Q-values. The action with the highest Q-value will be selected.

The timing and area overheads of the proposed deep Q-learning are discussed in Section 6.4.

*The Working of the Proposed DRL-based Controller.* Fig. 10 demonstrates the working of the DRL-based control logic when running a benchmark. At each time step, the process goes through several stages. In the first stage, the router uses the feature values \( F_1, F_2, \ldots, F_{20} \) in the state vector \( s \) as inputs of the ANN. In Fig. 10, we assume current state \( s \) matches state \( s_j \) in the mapping table. In the second stage, the ANN calculates the Q-values of all possible state-action pairs in the current state entry, and the router selects an action \( a \), which has the maximum \( Q(s, a) \)-value for the next time step (we assume that \( a_0 \) in Fig. 10 has the maximum Q-value). Upon taking the action \( a \), the NoC system transits to a new state \( s' \). In the last stage of the current time step, the
NoC system provides a reward $r$ (defined in (2)) to the router. The reward will be used to update $Q(s,a)$. Each router will repeat all of these stages at each time step.

The initialization of the per-router controller and the state-action mapping table will be discussed in Section 5.2.

## 5 Evaluation Methodology

In this section, we present the fault injection model used in our experiments to inject timing errors into NoC, the simulation setup, and the benchmarks that we use. Faults that occur in the routing table and machine learning module are beyond the scope of this paper.

### 5.1 Fault Injection Models

To quantify the reliability improvement of the proposed designs for timing errors, we first create a transient error injection model to realistically produce a probability of timing errors for each link. The proposed transient fault injection model is a combination of the existing VARIUS [25] fault model and HotSpot [26] thermal model. At runtime, first, HotSpot uses the values of router supply voltage, operation frequency, and link utilization to obtain router operating temperature at runtime. The temperature values are fed into the VARIUS transient error model to generate the probability of timing errors ($R_e$) for each transmitted bit. Using $R_e$, the probability of which n-bit flit is faulty can be calculated as follows:

$$P_{fault} = 1 - (1 - R_e)^n.$$  

(4)

To assess the reliability improvement for permanent faults, we utilize the permanent fault model proposed in [8]. Specifically, we model and calculate the aging factor in (2) by correlating the shift in the threshold voltage of the transistor ($\Delta V_{th}$). The $\Delta V_{th}$ is calculated using both $\Delta V_{th_{,NBTI}}$ given by [27], [28] and $\Delta V_{th_{,HCI}}$ given by [29], [30].

We model the aging factor given in (2) as follows:

$$\Delta V_{th} = \Delta V_{th_{,NBTI}} + \Delta V_{th_{,HCI}}$$

$$Aging = 1 + \frac{\Delta V_{th}}{V_{th0}} \times 100\%.$$  

(5)

Note that the aging factor is designed to have a value greater than 1 such that it can be used in the reward function.

## 6 Evaluation Results and Analysis

### 6.1 Performance Analysis With Faulty Links

In this subsection, system-level performance metrics are evaluated when link failures are injected. Before runtime, permanent faults are randomly inserted into a percentage of links with a probability of 5 percent. At runtime, transient errors are injected into the links using the error injection model presented in Section 5. The evaluation results are presented below.

**Execution Speed-up.** The speed-up is obtained by calculating the ratio of the full application execution time of various techniques (baseline, QORE, and IntelliNoC) to the execution time using the proposed CURE technique running different benchmarks. The speed-up comparison is shown in Fig. 11. As shown in Fig. 11, CURE achieves the largest

---

**TABLE 1**

<table>
<thead>
<tr>
<th>Simulation Environment Setup</th>
</tr>
</thead>
<tbody>
<tr>
<td># of cores</td>
</tr>
<tr>
<td>Voltage and Frequency</td>
</tr>
<tr>
<td>NoC Parameters</td>
</tr>
<tr>
<td>Packet Size</td>
</tr>
<tr>
<td>Flit Size</td>
</tr>
<tr>
<td>Cycle Delay</td>
</tr>
<tr>
<td>Buffer Numbers* of Different Technologies</td>
</tr>
</tbody>
</table>

* RB: router buffer, VC: virtual channel, and CB: channel buffer.
speed-up of all evaluated techniques. QORE achieves a 14 percent speed-up over the SECDED baseline since the dual-direction links can improve the NoC throughput by dynamically allocating the limited link bandwidth for unbalanced traffic. IntelliNoC results in an average speed-up of 18 percent due to the ability to reduce ECC overhead and retransmission traffic. However, in some benchmarks (i.e., fer and vips), IntelliNoC achieves worse performance than QORE. This is because IntelliNoC’s fixed-direction MFAC can be the bottleneck when multiple link failures occur on the same channel. However, because CURE does not have such limitations, it successfully accelerates benchmark execution by 27 percent.

**Network Latency.** We define network latency as the average end-to-end latency of all transmitted packets of the full execution of each benchmark application. To measure the end-to-end latency, first, when a packet is injected from the source node, the injection time is recorded. Second, when the packet reaches the destination node and accepted by the destination node (after passing the error checking), the packet acceptance time is also recorded. The end-to-end latency for that packet is the time difference between injection time and acceptance time, which is recorded using the number of clock cycles. Fig. 12 shows the normalized network latency for different techniques. CURE achieves an average of end-to-end latency reduction of 39 percent. Note that QORE only achieves a 9 percent latency reduction over the baseline due to the timing overhead of channel buffer allocation and static error control scheme. Techniques with RL-based policies (i.e., IntelliNoC and CURE) both achieve latency reductions of over 30 percent, which implies that dynamic proactive policy such as the RL-based policy can minimize re-transmission and thus improve overall latency.

**Energy-Efficiency.** We define energy-efficiency as

\[
Energy - Efficiency = \left[ \left( P_{\text{static}} + P_{\text{dynamic}} \right) \times \frac{1}{T_{\text{exec}}} \right]^{-1}.
\]

Used to model the static power with Synopsys Library Compiler for the designed NoC. Since Synopsys cannot evaluate the dynamic power accurately for different benchmark applications, we fed the static power parameters captured by Synopsys to DSENT [35] power model. During application execution, DSENT calculates the average dynamic power by the number of buffer writes, crossbar, and VA/SA activities within full application execution time. \( T_{\text{exec}} \) is the benchmark execution time. Fig. 13 shows the energy-efficiency measurements for all techniques studied and normalized to the SECDED baseline. CURE improves energy-efficiency by 92 percent compared to the baseline, while the energy-efficiency improvements using QORE and IntelliNoC are 36 and 77 percent, respectively.

**Overall Static Power Consumption.** Fig. 14 shows the overall static power consumption for the various techniques. QORE reduces static power consumption by 11 percent due to the elimination of router buffers. IntelliNoC achieves a static power reduction of 27 percent thanks to its power-gating and bypass scheme. Similar to IntelliNoC, the use of power-gating and bypass is beneficial in CURE. However, due to the extra logic and circuitry in the modified ECC and pipeline stages, the proposed CURE only achieves a static power reduction of 24 percent.

**Overall Dynamic Power Consumption.** As discussed, dynamic power consumption can be lowered by reducing the number of router buffers and/or by mitigating faults and reducing the number of re-transmissions. CURE, using RMC and dynamic error control, is able to significantly reduce re-transmission traffic. Consequently, CURE outperforms all other techniques in reducing dynamic power consumption as shown in Fig. 15.

### 6.2 Performance Analysis With Intra-Router Failures

In this subsection, permanent faults are injected into the router circuit (including VA, SA, crossbar, and ECC hardware). Before runtime, faults are randomly inserted into a
percentage of the router components ranging from 5 to 15 percent. At runtime, transient faults are injected only to the links using the proposed error injection model. The transient fault-handling technique used by SHIELD and Vicis is static SECDED, while CURE utilizes the proposed adaptive error control hardware. The network latency and energy efficiency of different techniques are evaluated, and the results are normalized to the static SECDED baseline. Because the SECDED baseline cannot tolerate router failures, the intra-router error rate is set to 0 percent for the baseline.

**Network Latency.** Fig. 16 shows the normalized network latency for different techniques at various error rates. The proposed CURE framework achieves at least 30 percent network latency reduction under 5, 10, and 15 percent router failure rates. However, both of the other fault-tolerant techniques incur excessive network latency.

**Energy-Efficiency.** Fig. 17 shows the normalized energy-efficiency for different techniques. The energy-efficiency of all the techniques tends to decrease as the error rate increases. However, there are some exceptions to Vicis. When the error rate exceeds 10 percent, Vicis re-configures itself less often, which can significantly reduce execution time. As shown in the figure, the proposed CURE framework achieves the highest energy-efficiency among all the other techniques. This result implies that the use of channel buffers and adaptive error control hardware can substantially reduce power and execution time.

### 6.3 Reliability Improvement Analysis

We use mean-time-to-failure (MTTF) to evaluate the reliability of the proposed design for permanent faults [36], [37]. Fig. 18 shows the normalized MTTF comparison. As shown in Fig. 18, the proposed CURE framework is $7.7 \times$ more reliable than the baseline router which is unprotected from permanent faults, while the highest normalized MTTF value of other fault-tolerant designs is $4.6 \times$ (SHIELD). This improvement is achieved thanks to the reduced operation temperature by the RL-based control policy and the modified pipeline and ECC hardware in each router.

### 6.4 Overhead Analysis

We evaluated the area overhead of each technique with Synopsys and 32 nm technology library with the supply voltage set to 1.0 Volt, and clock frequency set to 2.0 GHz. The area overhead is shown in Table 2.

As shown in this table, IntelliNoC, QORE, and CURE consume less area than the baseline due to the use of link storage. Due to the additional circuitry design, Vicis has the largest area overhead. CURE and IntelliNoC incur additional area overheads thanks to the RL-based controlling modules. The area overhead of the DRL-based controller.

![Network Latency Comparison](image1)

Fig. 16. Network latency comparison using average end-to-end latency, normalized to the SECDED baseline (lower is better). Each benchmark is tested under 5, 10, and 15 percent permanent fault rates in router.

![Energy-Efficiency Comparison](image2)

Fig. 17. Energy-efficiency comparison, normalized to the SECDED baseline (higher is better). Each benchmark is tested under 5, 10, and 15 percent permanent fault rates in router.

![MTTF Comparison](image3)

Fig. 18. Mean-time-to-failure (MTTF) comparison, normalized to the SECDED baseline (higher is better).
includes the area consumption of both ALUs (adder, multiplier, and Sigmoid function) and SRAM for calculating, updating, and storing the ANN. The simulation shows that the area overhead for ALUs and SRAM are 992.2 $\mu m^2$ and 838.6 $\mu m^2$, respectively. This implies 0.8 and 0.7 percent of the area consumption of the baseline router. Therefore, the area cost is reduced as compared to the 4 percent area overhead of table-based RL in previous designs. Moreover, CURE requires additional timing overhead for calculating the Q-values using ANN. Using [38], it shows that at each time step, the timing overhead of CURE is estimated to be 160 ns. Using the similar method as [39], this latency can be overlapped by a large time step. Specifically, we use two sets of different intervals for monitoring the attributes and the controlling to minimize the negative effect of this latency. The two sets of intervals are offset by the ANN computation time, which can pipeline the overhead effectively. By doing so, the ANN control computing does not block either the monitoring process or the controlling. Therefore, the use of ANN will not negatively impact the overall performance metrics. Additionally, the timing overheads for the MFAC (IntelliNoC) and RMC (CURE) configurations are both estimated around 45 to 50 cycles. Furthermore, the control logic of the proposed DRL consumes an additional 0.26 $mw$ power, which implies 4 percent of the static power consumption of the baseline router.

### 6.5 Sensitivity Study

**Impact of Input Data Size.** In this test, we explore the impact of different input data sizes on network latency and energy-delay product (EDP), and the blackscholes application in the PARSEC benchmark is used. Lower network latency and EDP indicate better system performance. The evaluation result illustrated in Fig. 19 shows that as input data size increases, the performance metrics remain the same. This is because the traffic patterns, network intensity, and workload allocations for the same benchmark processing phase are the same, for different data input sizes. However, by reducing the NoC size to a $4 \times 4$ mesh, which incurs a higher network intensity, the proposed adaptive ECC hardware can achieve better network performance, as compared to the static SECDED baseline.

**Impact of Time Step Length.** In this test, we varied the time step $t$ starting from 200 to 10,000 clock cycles. The evaluation results using the blackscholes application are illustrated in Fig. 20a. As shown in Fig. 20a, a longer cycle time (10K cycles) has a negative impact on performance due to coarse-grain control. Meanwhile, aggressively reducing the length of time steps (200 clock cycles) also leads to a degradation in performance because the computational overhead of DRL-based control policy will dominate performance.

**Impact of Discount Rate $\gamma$.** Fig. 20b indicates the impact of the discount rate $\gamma$ on network latency and EDP. The blackscholes application is used in this test. As shown in Fig. 20b, the network latency and EDP initially improve with larger $\gamma$, yet aggressively increasing $\gamma$ can also lead to Q-learning failing to converge, which negatively affects the system performance. The best performance is achieved when $\gamma$ equals 0.9.

**Impact of Exploration Probability $\epsilon$.** Fig. 20c shows the impact of $\epsilon$ values on network latency and EDP, using the blackscholes application. When $\epsilon$ is 0, the router always selects the initial mode most of the time. As $\epsilon$ increases from 0 to 1, the router tends to explore new state-action pairs more frequently. Further, when $\epsilon$ equals 1, the router will take actions entirely at random. As shown in Fig. 20c, the best system performance is achieved when $\epsilon$ equals 0.05.

**Impact of the Hidden Layer Size of the ANN.** Because the number of neurons used in the hidden layer of the ANN can affect the accuracy of calculating $Q(s,a)$, thereby impacting the decision making of the DRL controller, we vary the size of...
the hidden layer to study its impact on calculation accuracy, as shown in Fig. 20d. Fig. 20d shows that the calculation accuracy improves as the hidden layer size increases. To save area consumption and reduce the timing overhead of the ANN, we select the hidden layer of 30 neurons.

7 RELATED WORKS

There has been considerable work in improving the energy-efficiency and reliability in NoCs. In the following, we briefly highlight some of the directly relevant works.

Power-Saving Designs for NoCs. Because static power consumption has become a substantial portion of overall network power, power-gating (PG) techniques that power off underutilized network components have been shown to be effective for static power savings [14], [15], [40]. However, conventional power-gating schemes for routers tend to substantially increase network latency due to a reduced number of active routers in the network and extra control overhead in managing power-gating. Another approach proposed for reducing network power is reducing router buffers. Michelogiannakis and Dally [11] and Kodi et al. [7], [12] have shown that eliminating router buffers is beneficial for both static and dynamic power reduction. However, simply replacing router buffers with channel buffers leads to penalties in network congestion and latency [7], [11], [12].

Fault-Tolerant Designs for NoCs. In NoC, both transient and permanent faults can manifest during transmission. CRC [6] is a basic transient fault detection technique that is often used for NoCs. Flits are encoded by a local CRC encoder in the router before transmission, and are decoded by the destination CRC decoder to perform error detection. If the destination router detects errors, a re-transmission request is sent to the source router to re-transmit the flit. To mitigate transient faults, per-hop error correction codes (ECCs) are generally deployed. SECDDED is one of the most commonly used ECC techniques in NoCs [4]. To handle permanent faults caused by transistor aging [29], a number of techniques have been proposed using load-balancing [7], circuitry redundancy [37], and adaptive routing techniques [4] among others. Note that most of the techniques are static in nature, with CRCS or SECDEDs being deployed all the time, regardless of whether faults are present. Reliability-enhancement mechanisms based on static techniques have been shown to require excessive power consumption, and longer delays, thereby significantly degrading NoC performance [17], [37], [41].

Learning-Enabled NoC Designs. Multiple machine learning techniques have been introduced to balance design trade-offs or predict traffic in NoCs. These works target achieving high power efficiency [8], [16], [24], [39], [42], [43], [44], [45], and enable fault-tolerant design [5], [8], [45]. For example, [44] discovers that the wake-up latency of PG and performance degradation of using DVFS are the bottlenecks of implementing PG and DVFS simultaneously. Hao et al. [44] have shown that applying machine learning to handle the dynamic trade-offs of DVFS and PG can achieve optimal power savings. [45] introduces a proactive fault-tolerant mechanism to optimize energy efficiency and performance with reinforcement learning (RL). Further, [8] proposes to use channel buffers to achieve higher power savings in addition to [45]. We extend these existing works by proposing link reversibility, adding hard error tolerance, and using DRL to reduce control overhead.

8 CONCLUSION

In this paper, we propose CURE, a learning-based NoC design that can simultaneously improve performance, energy-efficiency, and reliability. CURE consists of reversible multi-function adaptive channel, enhanced fault-tolerant router circuitry, ten unique operation modes, and a DRL-based dynamic control policy. With DRL, each router learns from the NoC behavior and updates a control policy to select an optimal operating mode at any given time. The experimental results illustrate that CURE decreases network latency by 39 percent, improves energy efficiency by 92 percent over the static SECDED baseline. Using the mean-time-to-failure metric, we show that the proposed framework is 7.7× more reliable than the baseline NoC architecture.

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Heterogeneous Edge Offloading With Incomplete Information: A Minority Game Approach

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Abstract—Task offloading is one of key operations in edge computing, which is essential for reducing the latency of task processing and boosting the capacity of end devices. However, the heterogeneity among tasks generated by various users makes it challenging to design efficient task offloading algorithms. In addition, the assumption of complete information for offloading decision-making does not always hold in a distributed edge computing environment. In this article, we formulate the problem of heterogeneous task offloading in a distributed environment as a minority game (MG), in which each player must make decisions independently in each turn and the players who end up on the minority side win. The multi-player MG incentivizes players to cooperate with each other in the scenarios with incomplete information, where players don't have full information about other players (e.g., the number of tasks, the required resources). To address the challenges incurred by task heterogeneity and the divergence of naive MG approaches, we propose an MG based scheme, in which tasks are divided into subtasks and instructed to form into a set of groups as possible, and the left ones are scheduled to perform decision adjustment in a probabilistic manner. We prove that our proposed algorithm can converge to a near-optimal point, and also investigate its stability and price of anarchy in terms of task processing time. Finally, we conduct a series of simulations to evaluate the effectiveness of our proposed scheme and the results indicate that our scheme can achieve around 30% reduction of task processing time compared with other approaches. Moreover, our proposed scheme can converge to a near-optimal point, which cannot be guaranteed by naive MG approaches.

Index Terms—Heterogeneous edge offloading, incomplete information, minority game

1 INTRODUCTION

Edge computing is a new computing paradigm that allows data produced by end devices (e.g., vehicles, cameras, IoT devices) to be processed at the edge of the network where the data is being generated, instead of sending it back to the cloud (or data center) along long routes. By exploiting computation resources of edge nodes to process offloaded tasks, edge computing can provide real-time local data analytics capability, which cannot be offered by the cloud. As an example, self-driving cars are commonly equipped with a multitude of sensory devices, including cameras, LIDAR, sonar devices, and so on. For these cars to operate safely, it is essential to process the huge amount of collected data (e.g., over 1.38 GB per second for a self-driving car [1]) in a timely manner. The latency in processing speed is critical for vehicle safety, but transmitting such a large volume of data back-and-forth over a network is very time-consuming. One feasible solution is to process the sensory data at the edge, so that the cars can react to their surroundings right away and take driving actions accordingly [2]. Although at a nascent stage, edge computing has proven its superiority in quite a few areas. According to the report by CB Insights[1], the global market of edge computing is estimated to reach $6.72 billion by 2022.

It should be pointed out that the edge and the cloud are complementary components of a whole computing system. Compared to high-end cloud servers, edge servers (or edge nodes) in the proximity commonly have much lower capacity. The edge needs to work with the cloud in tandem to process tasks offloaded by end users more efficiently [3], [4], [5]. However, it is challenging to determine how to conduct efficient task offloading in a distributed environment, where cloud (or edge) servers are physically distributed in different locations. The reasons are multi-fold: first, edge servers have limited processing capacity and transmission bandwidth, and an improper offloading decision may overload edge servers and cause wastage of CPU cycles; second, given that complete information is not always available in a distributed environment, it is difficult to find the optimal offloading scheme. We need to design offloading algorithms that only require local and incomplete information, especially for users with their own interests.

From the perspective of any end user, its objective is to minimize the amount of processing time for its tasks. The user needs to determine whether to conduct task offloading, where to offload the task (e.g., to the edge or the cloud), the
amount of offloaded tasks, and so on. However, as complete
information is not available, each user has no idea about
decisions made by others when making its own decision.
Therefore, they have to make offloading decisions indepen-
dently, which may cause unexpected results. It is possible
that too many tasks are assigned to the same edge server
and the processing time is significantly prolonged due to
server overloading. In another extreme, if too few tasks are
assigned to an edge server, it will lead to low execution ef-
ciciency of edge resources.

Due to the importance of task offloading in edge comput-
ing, researchers have made significant efforts in this field. In
most of previous studies, the scheduler of task offloading is
very aggressive in collecting task-related information. It is
based on the assumption that a better offloading decision
can be made with more information. Centralized schedulers
proposed by [6], [7], [8], [9], [10] extensively collected informa-
tion from end users before making offloading decisions.
Due to the high volume of message traffic, the above cen-
tralized approaches cannot work well in a large-scale dis-
tributed environment. Some work [11], [12] started to
investigate the design of distributed schedulers for task off-
loading in edge computing systems. It normally requires
pair-wise information exchange among all users to obtain a
complete view of the system. Inevitably, the communication
overhead will increase considerably with the increase of
user population in the system. The problem will be further
complicated by the loss of direct communication between
two users. In the real-world environment, it is more com-
mon that each user needs to make decisions with partial
information.

In this paper, we formulate the problem of heteroge-
neous task offloading in a distributed environment as a
multi-player game, in which complete information is not
required by the scheduler to make offloading decisions. We
apply the theory of minority game (MG) to design the
scheduling algorithm for task offloading. MG is efficient in
modeling collective behaviors of users where they have to
compete for limited resources without complete informa-
tion [13]. However, a naive MG solution cannot guarantee
to converge to the optimal point and also is not applicable in
the case with heterogeneous task loads. To overcome
the above obstacles, we propose an MG based scheme for
the heterogeneous edge computing scenario, in which tasks
generated by the same user are divided into subtasks and
instructed to form into groups, and compete for computa-
tion resources in a batch in order to minimize the task pro-
cessing time. The left subtasks are scheduled to make their
offloading decisions independently with incomplete
information.

- We propose an MG based scheme for efficient task
  offloading in heterogeneous edge computing, in
  which groups are formed among subtasks to com-
  pete resources with others. For subtasks not included
  in any group, they are scheduled by a probabilistic
  decision adjustment policy. We further prove that
  the proposed scheme can converge to a near-optimal
  point, and an expression of the gap is derived follow-
  ing derivations on attendance and price of anarchy.
- We conduct a series of simulations to evaluate the
effectiveness of our proposed scheme and the results
indicate that our scheme can achieve around 30%
reduction of task processing time compared with
other approaches. Moreover, our scheme is verified
that can converge to a near-optimal point.

The remainder of this paper is organized as follows. Section 2 introduces the system model and formulates a
task offloading game with incomplete information. The MG
based offloading framework and properties are introduced
in Section 3. An extension to a fully distributed edge com-
puting architecture is presented in Section 4. Experimental
results are shown in Section 5, and some related studies are
summarized in Section 6. Finally, Section 7 concludes our
work and points out possible future work.

2 System Model and Game Formulation

In this section, we introduce the system model and formulate
the problem of heterogeneous task offloading in a distrib-
uted environment as a multi-player game with incomplete
information.

2.1 System Model

Fig. 1 shows a typical heterogeneous edge computing sys-
tem for processing video streams captured by cameras,
where the video analytics tasks (e.g., object detection, track-
ning, recognition) have different resource requirements [14].
In such a system, there exist multiple edge servers with dif-
cerent capacity (e.g., CPU, memory), which work together
with remote clouds for task execution. We first consider a
partially distributed architecture, in which all edge servers

![Fig. 1. A partially distributed architecture of a heterogeneous edge computing system.](image-url)
can be abstracted as a single cluster and the master edge server is the entry point of task offloading, as illustrated in Fig. 1. Such a partially distributed architecture has been proven to be effective in managing large-scale networked systems (e.g., SDN controllers, Kubernetes). Anyway, the whole edge computing system can also be organized in a fully distributed way (as illustrated in Fig. 2).

In Fig. 1, end users can choose to offload their tasks to either cloud servers or edge servers. The internal scheduling of task offloading within the edge server cluster is transparent to end users. Our main contribution in this architecture is to determine which side (i.e., edge server side, cloud server side) should be chosen when only incomplete information is available. The extension to a fully distributed architecture will be discussed in Section 4.

Suppose that there are $K$ users (e.g., cameras) in the system, each of which generates tasks with different amounts of computation load. Each task is further divided into a set of subtasks with equal size in terms of computation load. For a parallelizable task focused in this paper, it can be divided into a set of subtasks that can be run in parallel. A typical example is video analytics task, in which a video can be divided into a series of frames and the video analytics task (e.g., object detection) can be conducted on each frame separately [15], [16]. The size of a subtask is denoted by $C_0$ in the unit of CPU cycles. Let $R_k$ be the computation requirement of a user $k$, and $N_k$ be the number of subtasks generated by the user $k$. Thus, we have $R_k = N_k C_0$. In one decision period, all users generate $N = \sum_k N_k$ subtasks in total. Similar to many previous studies (e.g., [11] and [12]), to enable tractable analysis and get useful insights, we consider a quasi-static scenario where the set of end users remains unchanged during a computation offloading period (e.g., several hundred milliseconds). For a better understanding of the work, we list the key defined symbols in Table 1.

As the key performance metric, the task processing time consists of task delivery time and task execution time. The task delivery time from the user to the server is influenced by the communication model. While task execution time refers to the time to process a task on the server, which is mainly affected by the computation model. In the following, we will introduce the communication model and the computation model, respectively.

(1) Communication Model: To analyze our scheme in the wireless environment, we introduce a wireless communication model [17] into our system model, based on which we derive the user uplink rate and transmission delay. The wireless communication model has been widely adopted in the previous literature (e.g., [12], [17], [18]). According to the model, with the channel state information at the user side, we have the uplink capacity from user $k$ to server $m$ as:

$$C_{UL,k,m} = B \log_2 \left(1 + \frac{SNR_k}{\sigma^2} \right),$$

where $B$ represents the receiver bandwidth, and $SNR_k = |h_{m,k}|^2 P_k / \sigma^2$ denotes the signal to noise ratio (SNR) for user $k$, where $|h_{m,k}|^2$ is the gain of the channel between the transmitter $k$ and the base station for server $m$ and $P_k$ represents the transmission power of user $k$. The uplink capacity $C_{UL,k,m}$ can be achieved by certain physical layer technologies, which has been commonly assumed in many previous studies, e.g., [19], [20], [21]. For the partially distributed case, the uplink rate is denoted as $C_{UL,k}$ and $C_{UL,k,m}$ for the edge server cluster and cloud server, respectively.

Let $\tau_{t,CE}$ and $\tau_{t,CL}$ denote the task transmission time from user $k$ to the edge server cluster and the cloud server, respectively. Then we have

$$\tau_{CE,k} = \frac{D_{CE,k}}{C_{UL,k}} \quad \text{and} \quad \tau_{CL,k} = \frac{D_{CL,k}}{C_{UL,k}} + \tau_{RTT},$$

where $D$ is the data size of one subtask in terms of bits, and $\tau_{RTT}$ denotes the expected transmission delay between the base station and the cloud server. The reference value of $\tau_{RTT}$ can be obtained from statistics in field experiments [22].

(2) Computation Model: In our system, servers support the processing of multiple tasks by using the technique of virtual parallel processing [23]. Let $\tau_{C,k}$ and $\tau_{C,s}$ denote the task execution time on the edge server and the cloud server, respectively. Then we have

$$\tau_{C,k} = \frac{D_{CE,k}}{C_{c,k}} \quad \text{and} \quad \tau_{C,s} = \frac{D_{CL,k}}{C_{c,s}} + \tau_{RTT},$$
\[ \xi_{c,k} = \frac{n_{c,k}C_0}{C_{c,k}} \quad \text{and} \quad \xi_{s,k} = \frac{n_{c,k}C_0}{C_{s,k}}, \] 

where \( n_{c,k} \) is the number of subtasks from user \( k \) processed by the edge, and \( n_{s,k} \) represents the number of subtasks from user \( k \) processed by the cloud. The parameters \( C_{c,k} \) and \( C_{s,k} \) are defined as the computation capacity of the edge server and cloud server respectively. In terms of the unit of measures for computation capacity, similar to the definitions in the previous work (e.g., [9], [20], [21]), we focus more on CPU speed, which can be measured by the number of CPU cycles per unit time (e.g., second). Specifically, the amount of allocated computation resource on the edge (or cloud) (namely, \( C_{c,k} \) or \( C_{s,k} \)) is determined by the results of competition among multiple users. Our goal in this paper is to minimize the overall task processing time, and thereby it is important to avoid the congestion of any particular edge server, unless the whole system is overloaded with too many tasks for processing.

### 2.2 Game Formulation

We model the decision-making problem of heterogeneous task offloading as a multi-player game, in which users are modeled as players. Within a decision-making period \( t \), each user can choose its own task offloading strategy independently. A strategy is a vector, in which each element indicates the number of subtasks that should be offloaded to each resource provider.

Let \( \mathbf{v} = [v_1, v_2, \ldots, v_k]^T \) denote a vector of user offloading strategies, where \( v_k \) is the number of subtasks generated by user \( k \) that are offloaded to the edge server. Furthermore, we let \( \mathbf{s} = [s_1, s_2, \ldots, s_N]^T \) denote the subtask scheduling profile, where \( s_n \) denotes the decision on subtask \( n \) and \( s_n \in \{0, 1\} \). If \( s_n = 1 \), subtask \( n \) is offloaded to an edge server; otherwise, subtask \( n \) is offloaded to the cloud. Thus, we have \( v_k = \sum_{n \in N_k} s_n \), where \( N_k \) denotes the subtask set of user \( k \). Apparently, the payoff of a user depends on the actions conducted by the other users. Let \( \mathbf{v}_{-k} = [v_1, v_2, \ldots, v_{k-1}, v_{k+1}, \ldots, v_K]^T \) denote the offloading decisions made by all other users except user \( k \).

Following the classical definition [13], the game can be formally defined as below.

**Definition 1.** A minority game model for task offloading with incomplete information. We formulate a minority game \( \Gamma = (\mathbb{K}, \mathbb{V}_k, U(v_k, \mathbf{v}_{-k})) \), where the set of users \( \mathbb{K} \) is regarded as the set of players, \( \mathbb{V}_k \) is the set of strategies for user \( k \), and the payoff for user \( k \) can be defined as the utility function \( U(v_k, \mathbf{v}_{-k}) \).

The set of strategies \( \mathbb{V}_k \) are only known by the user instead of any opponent, however, the payoff function \( U(v_k, \mathbf{v}_{-k}) \) depends not only on your own strategy but on strategies chosen by your opponents.

Our formulated minority game belongs to the game with incomplete information due to the following two reasons:

1. Each player in our game does not know its opponents’ possible action spaces. In our paper, an action strategy is defined by a profile indicating the number of subtasks that are offloaded to each server. Formally, let \( \mathbf{s} = [s_1, s_2, \ldots, s_N]^T \) denote the action profile, where \( s_n \) denotes the decision on subtask \( n \) and \( s_n \in \{0, 1\} \). If \( s_n = 1 \), subtask \( n \) will be offloaded to an edge server; otherwise, subtask \( n \) will be offloaded to the cloud. The action spaces are related to the value of \( N_k \), i.e., the number of generated tasks. As \( N_k \) varies with \( k \) and a player may not know the properties of tasks generated by its opponents, the action spaces of players can be treated as incomplete information.

2) Each player in our game does not know its opponents’ payoffs. Recall that the payoff function is defined as the utility \( U(v_k, \mathbf{v}_{-k}) \), \( \mathbf{v} = [v_1, v_2, \ldots, v_K]^T \). This indicates that the payoff is determined by not only the individual action \( v_k \), but also actions of the other players \( \mathbf{v}_{-k} \). Diverse action spaces induce different payoff functions for our game.

In the formulated MG model, user \( k \) aims to make a decision \( v_k \) to maximize the utility for its own tasks, i.e.,

\[
\max_{v_k \in \{0, 1, \ldots, N_k\}} U(v_k, \mathbf{v}_{-k}).
\]

For an edge (or cloud) server, the processing time is a function of the amount of assigned task load. Let \( T_c(n_c) \) denote the task processing time on an edge server, and \( T_c(n_c) \) denote the task processing time on a cloud server. We consider the total latency determined by the slowest subtask, then the payoff function can be represented as:

\[
U(v_k, \mathbf{v}_{-k}) = \begin{cases} 
1/T_c(n_{c,-k} + v_k), & \text{if } v_k = N_k \\
1/T_c(n_{c,-k} + N_k - v_k), & \text{if } v_k = 0 \\
1/\max\{T_c(n_{c,-k} + v_k), T_c(n_{c,-k} + N_k - v_k)\}, & \text{otherwise}
\end{cases}
\]

where \( n_{c,-k} \) denotes the number of subtasks being offloaded to the edge server from the other users except user \( k \), and \( n_{c,-k} \) denotes the number of subtasks being offloaded to the cloud server from other users except user \( k \).

Combining Eqs. (2) and (3), we have the time experienced by tasks that are offloaded to the edge server as:

\[
T_c(n_c) = \tau_e + \frac{n_c C_0}{\tau_e},
\]

where \( \tau_e = E_k\{\tau_{c,k}\} \) refers to the expected value and \( n_c = \sum_{n \in N} s_n \). Similarly, we have the task processing time on the cloud server as:

\[
T_c(n_c) = \tau_c + \frac{n_c C_0}{\tau_c},
\]

where \( \tau_c = \tau_e + \tau_{R_{TT}} \) and \( n_c = N - n_c \).

For each player, minimizing its task processing time is the strategy to win the game. However, without the knowledge of actions chosen by the other users, it is not easy to make the right choice. A solution to the MG based task offloading design with incomplete information will be presented in the following sections.

### 2.3 Preliminary of Minority Game

Minority game is efficient in modeling collective behaviors of users when they have to compete for limited resources with incomplete information. In its original form, namely, the El Farol Bar problem, players make their decisions on whether to attend a bar each night. Going to a bar is only enjoyable only if it is not too crowded, otherwise people would rather
stay at home. Intuitively, players adjust their behavior based on their expectations on what other players are going to do next, and these expectations are generated by the attendance decision of the other players. At each round of MG, each player determines his/her own action based on historical and preference factors. After all decisions are made, the action associated with less players is declared as minority side strategy and those players get the chance to win certain payoffs. The game result is broadcast back to all players such that they can update their information and make necessary adjustments in future expectations. As each player makes his/her own decision independently, the game is carried out in a decentralized manner [23], [24], [25], [26].

In a basic MG, the players select between two alternatives and the players belonging to the minority group win. The minority is typically defined by using a cut-off value. The collective sum of the selected actions by all players is referred to as the attendance. These two key terminologies, the cut-off value and attendance, are defined as below.

**Definition 2.** Cut-off value $\psi$ is defined as a threshold value for the number of subtasks offloaded to the edge server such that the task processing time is minimized, i.e.,

$$\min_{\psi \in \mathbb{R}} l(\psi) = \min_{\psi \in \mathbb{R}} \max\{T_e(\psi), T_c(N - \psi)\}.$$  

From Eqs. (5) and (6), $T_e(n_e)$ and $T_c(n_c)$ are monotonically increasing functions with the number of subtasks. Then, $T_e(\psi)$ monotonically increases with $\psi$, while $T_c(N - \psi)$ monotonically decreases with $\psi$. To obtain an optimal cut-off value $\psi^*$, it should be guaranteed that $T_e(N - \psi^*) = T_c(\psi^*)$, and we have

$$\psi^* = \frac{NC_0C_e - (\tau_e - \tau_c)C_eC_c}{C_0(C_c + C_e)}. \quad (7)$$

In the practical system, the cut-off value $\psi$ can be approximated by an rounding integer, and we have

$$\psi = \begin{cases} \lfloor \psi^* \rfloor, & \text{if } l(\lfloor \psi^* \rfloor) \leq l(\lfloor \psi^* \rfloor), \\ \lceil \psi^* \rceil, & \text{otherwise}. \end{cases} \quad (8)$$

The cut-off value $\psi$, as the key factor of minority game, explains how users play the game by making offloading decisions. If $n_e \geq \psi$ (equivalent to $T_e \geq T_c$), subtasks off-loaded to the edge (i.e., majority) lose and subtasks off-loaded to the cloud (i.e., minority) win. In contrast, if $n_e \leq \psi$ (equals to $T_e \leq T_c$), the subtasks offloaded to the edge (i.e., minority) win and the others lose.

In the standard MG, attendance is defined as the collective sum of the selected actions by all players, originally for decision scenarios with homogeneous tasks [13]. For example, in the El Farol Bar problem mentioned in Section 2.3, players make their decisions on whether to attend a bar every night. Let “to attend the bar” as strategy 1 and “not to attend the bar” as strategy 2. It is introduced in MG by labelling strategy 1 as 1 and strategy 2 as -1, which is called a spin. Then the attendance $A$ can be represented as $A = n_1 - n_2$, where $n_i$ denotes the number of users choosing strategy $i$. To accommodate heterogeneous computing capacity in the practical system, we redefine it as below.

**Definition 3 (Attendance).** Let $\rho$ denote a ratio on the capacities between edge servers and cloud servers, where $0 < \rho \leq 1$. The derivation of $\rho$ will be introduced later. We have redefined “attendance” as:

$$A = n_e - \rho n_c. \quad (9)$$

Specifically, this definition considers the asymmetric processing capacity between edge servers and cloud servers [23], [24], [25], [26]. As our objective is to minimize the task processing time, we derive a lemma to establish the connection between attendance and the payoff function.

**Lemma 1.** The system performance (e.g., task processing time) is optimized when the absolute value of the attendance is minimized.

**Proof.** Recall Eq. (9), since $n_e = N - n_c$, we have

$$A = n_e - \rho(N - n_c) = (1 + \rho)n_e - \rho N. \quad (10)$$

With Eq. (5), we have that $T_e$ increases as $n_e$ increases, which is linearly proportional to a positive $A$ from Eq. (10). Similarly, $T_c$ increases with the decrease of $A$. Overall, the task processing time increases with $|A|$. Therefore, we can conclude that the task processing time can be optimized when the absolute value of $A$ is minimized.

Recall the optimal cutoff value defined in Eq. (7), we define the ratio $\rho$ such that $A = \psi^* - \rho(N - \psi^*) = 0$. By solving this equation, we have

$$\rho = \frac{NC_e + (\tau_e - \tau_c)C_cC_e}{NC_e - (\tau_e - \tau_c)C_cC_e}, \quad (11)$$

and $0 < \rho \leq 1$. Especially, in the general condition $C_e(\tau_e - \tau_c) < C_c(\tau_c - \tau_e) < < N$, we have $\rho \approx C_c/C_e$.

### 3 Task Offloading for Partially Distributed Architecture

In this section, we propose an MG based task offloading algorithm for the partially distributed architecture of a heterogeneous edge computing scenario with incomplete information, and prove that the proposed algorithm can approach the optimality.

#### 3.1 MG-Based Task Offloading Schedule

In the standard MG based offloading schemes, tasks are assumed to be homogeneous in terms of computation requirements, based on which the offloading decisions are made. However, tasks generated by users in real environments are commonly heterogeneous. Moreover, a naive MG solution cannot guarantee to converge to the optimal point. To address these challenges, we propose a novel MG based offloading algorithm, which not only makes decision with incomplete information but handles heterogeneous tasks generated by different users and servers with heterogeneous processing capacities. Besides, our algorithm is proven to be able to converge to a near-optimal point.

To handle the general case that tasks are commonly heterogeneous in the computation requirements, we introduce the concept of group, based on which our MG based task offloading algorithm is proposed.

**Definition 4 (Group).** A “group” is the smallest unit when conducting the offloading procedure in our algorithm. Namely,
all the subtasks are offloaded in groups and one group of subtasks is mapped to only one server.

By introducing groups, we can achieve high flexibility when offloading heterogeneous tasks and reduce the complexity of offloading decision-making. In our algorithm, the problem of task offloading is decomposed into two phases:

1) Server selection, i.e., mapping each group to a server side, either edge server side or cloud server side.
2) Task scheduling, i.e., matching each subtask with a specific server.

These two phases will be introduced as follows.

3.1.1 MG Based Server Selection

In Algorithm 1, we propose an MG based server selection algorithm, which contains three major steps:

**Algorithm 1. MG Based Server Side Selection**

1: Given: $N, K, N_k, \rho, s_n = \{0, 1\}, t = 1$
2: % initial allocation
3: for $k = 1 : K$ do
4: Calculate $n_{e,k}, n_{c,k}$ and $n_{r,k}$ following Eq. (12);
5: Allocate $n_{e,k}$ subtasks to side $1$, $n_{c,k}$ subtasks to side $0$;
6: Allocate $n_{r,k}$ subtasks following Eq. (14);
7: end for
8: repeat
9: Calculate the attendance $A$ following Eq. (9);
10: if $A > 0$ then
11: $b(t) = 0$
12: else
13: $b(t) = 1$
14: end if
15: % scheduler broadcasts the winner information $b(t)$
16: % user adjusts decision with probability $p$ in Eq. (17)
17: for $n = 1 : N$ do
18: if $s_n \neq b(t)$ then
19: if $(\text{rand}() \% N) < \text{abs}(A) - 1$ then
20: $s_n = (s_n + 1) \% 2$
21: end if
22: end if
23: end for
24: $t = t + 1$
25: until $p = 0$

1) Subtask grouping and initial allocation. First, subtasks are grouped to match servers with different processing capacities. Given the capacity balancing ratio $\rho$, we initially allocate the number of subtasks generated by user $k$ to the edge server cluster as:

$$n_{e,k} = \left\lfloor \frac{\rho N_k}{1 + \rho} \right\rfloor.$$

Our allocation follows the same procedure as that in the classical papers (e.g., [12], [27], [28]). The crux of the allocation refers from the fact that the transmission time increases with the workload offloaded to the server. Similarly, the number of subtasks offloaded to the cloud server is initially set as

$$n_{c,k} = n_{e,k}/\rho.$$

After initial subtask grouping, there may exist residual subtasks without being scheduled, i.e., $n_{r,k} = N_k - n_{e,k} - n_{c,k}$, where $n_{r,k}$ denotes the number of subtasks that are not in an initial group division. The offloading decisions for these subtasks follow probability

$$p_e = \frac{\rho}{1 + \rho} \quad \text{and} \quad p_c = \frac{1}{1 + \rho}. \quad (14)$$

This principle is proposed to minimize the absolute value of the attendance.

2) Information collection and winner broadcast. Following the offloading principle in Eqs. (12)-(14), the scheduler can calculate the values of $n_e$ and $n_c$ as

$$n_e = \sum_{k=1}^{K} n_{e,k} \quad \text{and} \quad n_c = \sum_{k=1}^{K} n_{c,k}. \quad (15)$$

Then, the scheduler broadcasts the winning group with one-bit control information $b(t)$, defined as

$$b(t) = \begin{cases} 1, & \text{if } n_e < \psi \\ 0, & \text{otherwise}. \end{cases} \quad (16)$$

When $b(t) = 1$, the subtasks offloaded to the edge server win; otherwise, the subtasks offloaded to the cloud server win. Given the control information, users could change their strategies to improve decision-making for the next round offloading.

3) Decision adjustment and iterative scheduling. It is essential for the subtasks in the majority to adjust their decisions to achieve a better performance. For example, suppose that the subtasks offloaded to the “cloud server” win, then, in the next round, we keep decision for subtasks that choose the “cloud server” action, however, subtasks in the “edge server” may set to change their decisions by the user.

We propose a decision adjustment policy. The subtasks in the majority change their offloading decisions according to the probability

$$p = \begin{cases} \frac{|A| - 1}{N}, & \text{if } |A| > 1 \\ 0, & \text{otherwise}. \end{cases} \quad (17)$$

The task offload decision adjustment and scheduling iteratively continue until $p = 0$.

For clarity, we use an example with two users to describe our algorithm (as shown in Fig. 3). User 1 generates seven subtasks in one time slot, while user 2 generates five subtasks in one slot. We set the edge/cloud server capacity ratio as $\rho = 1/2$. In Step 1, initial groups are formed among users. Following the principles defined in Eqs. (12) and (13), we have $n_{r,1} = 2$ and $n_{r,2} = 4$ for user 1. While for user 2, we have $n_{r,2} = 1$ and $n_{r,2} = 2$ in the initial two groups. With these operations, we have the number of residual subtasks that are not included in any initial group, i.e., $n_{r,1} = 1$ and $n_{r,2} = 2$. In total, we have $N = n_{r,1} + n_{r,2} = 3$ subtasks for random offloading schedule. In this example, we assume the three residual subtasks selecting “edge server”, i.e., the edge offloading group is in “majority” while the cloud offloading group is in “minority”. In Step 2, we have the attendance $A = n_e - \rho n_r = 3$, which is broadcast to all the users in the network. Since the attendance value is greater than 1, we need to adjust the decision profile in Step 3. As defined
in Eq. (17), we have the decision adjustment probability as 
\[ p = \left(\frac{|A| - 1}{|A|} \right) \frac{N}{N - 1} = \frac{3 - 1}{3} = 1/6. \]
That is, the subtasks in the majority change their decisions with probability \( p \), while the subtasks in the minority stay unchanged. With such a policy, the decision profile iteratively changes until achieving the balanced state, when no subtask can change decision to achieve a smaller attendance value. Finally, we have four subtasks offloaded to the edge server and eight subtasks to the cloud server.

### 3.1.2 Intra-Cluster Task Scheduling

Here we propose an intra-cluster task scheduling algorithm. Assume that the edge cluster is composed of \( J \) edge servers. Let \( n_{e,j} \) denote the number of subtasks allocated to the edge server \( j \), and \( \sum_j n_{e,j} = n_e \). Recall that edge servers support computation on multiple tasks with virtual parallel processing. The number of subtasks allocated to each edge server should be proportional to its capacity. This model has been widely used for designing allocation policies (e.g., [12], [27], [28]) for computation resources. For example, in [27], the CPU cycles at the edge server are proportionally allocated to each device such that they experience the same execution latency. Then we have the optimal number of subtasks scheduled to the edge server \( j \) as
\[ n_{e,j} = \frac{n_e C_j}{\sum_j C_j}. \]

Note that the number of subtasks in Eq. (18) might not be an integer, and we refine the allocation as
\[ n'_{e,j} = \left\lfloor \frac{n_e C_j}{\sum_j C_j} \right\rfloor. \]

and we have \( \sum_j n'_{e,j} \leq n_e \). Let \( \Delta_j \) denote the gap between \( n_{e,j} \) defined in Eq. (18) and \( n'_{e,j} \) defined in Eq. (19), and we have \( \Delta_j = n_{e,j} - n'_{e,j} \). Let \( J_\Delta \) denote the set containing the first \( n_e - \sum_j n'_{e,j} \) values on \( \Delta_j \). Then we have the revised subtask allocation among edge servers as
\[ n^*_{e,j} = \begin{cases} 
\frac{n_e C_j}{\sum_j C_j}, & \text{for } j \in J_\Delta \\
\frac{n_e C_j}{\sum_j C_j}, & \text{otherwise.}
\end{cases} \]

The intra-cluster task allocation, conducted by the master node of the edge cluster, is illustrated in Algorithm 2.

### Algorithm 2. Intra-Cluster Task Scheduling

1. **Given:** the number of subtasks offloaded to edge server cluster \( n_e \), the capacity \( C_j \) for edge server \( j \).
2. **Step 1: Subtasks Allocation Policy Calculation**
   - for \( j = 1 : J \) do
   - Calculate \( n^*_{e,j} \) following Eqs. (18)-(20).
   - end for
3. **Step 2: Subtasks Allocation**
   - for \( j = 1 : J \) do
   - Allocate \( n^*_{e,j} \) subtasks to edge node \( j \).
   - end for

To theoretically verify the performance of our intra-cluster scheduling, we derive the gap of task processing time between the practical case with a single edge server cluster and the optimal case with resource pool. In the optimal case, edge servers are perceived as a resource pool by consolidating the computation resources of all edge servers.

In the optimal case, we have the task processing time as
\[ T_e^* = \tau_e + \frac{n_e C_0}{\sum_j C_j}. \]

With the intra-cluster scheduling, we have the practical task processing time as
\[ T'_e = \tau_e + \max \left\{ \frac{n_{e,1} C_0}{C_1}, \frac{n_{e,2} C_0}{C_2}, \ldots, \frac{n_{e,J} C_0}{C_J} \right\} \]

which might be greater than \( T_e^* \) due to the heterogeneity of servers. For the performance gap between \( T'_e \) and \( T_e^* \), we have the following proposition.

### Proposition 2. The performance gap on task processing time between the practical case with a single edge server cluster and the optimal case is not greater than \( C_0/\min_j(C_j) \), where \( C_0 \) is the computation requirement for one unit subtask and \( C_j \) is the computing capacity of edge server \( j \).

### Proof.
Comparing Eqs. (21) and (22), we can conclude that the practical task processing performance can achieve the optimal value if and only if Eq. (18) holds. As we assume that servers support computation on multiple tasks with virtual parallel processing, subtasks allocated to edge servers are proportional to the capacity. It should be noted that the difference between \( n^*_{e,m} \) and \( n_{e,m} \) is not greater than one. As the system performance is mainly constrained by the slowest edge server, we have the
difference on task processing time between the practical system and optimal value as $T_r - T_r^c \leq C_0 / \min_j \{ C_j \}$. \hfill \qed

In practical, the computing capacity of an edge server is generally greater than 10 Gigacycles per second. Meanwhile, with efficient task partition, the computation requirement for each subtask can be lower than 100 CPU Megacycles. Thus, the performance gap will be lower than 10 milliseconds, showing the effectiveness of our design. With Proposition 2, a way to reduce the gap is to set a small size of partitioned subtask $C_0$.

3.2 Performance Analysis

In this section, we analyze the properties of our MG based scheduling algorithm, and check its equilibrium, convergence, and performance bound.

3.2.1 Nash Equilibrium

Nash equilibrium (NE) is used to discuss the equilibrium state in general game models. An Nash equilibrium is defined as a strategy profile that maximizes the expected payoff for each player given their beliefs and the strategies played by the other players. That is, a strategy profile $V_k$ is an NE if and only if for every player keeping the strategies of every other player fixed, strategy $v_k$ maximizes the expected payoff of player $k$ according to his/her belief. In the following, we prove the existence of at least one NE point for the proposed game.

Based on the utility function defined in Eq. (4), the NE in a pure strategy for the MG is given as below.

**Definition 5.** For player $k$, the strategy profile $\{v^*_k, N_k - v^*_k\}$ is a NE of the MG if at the equilibrium, the utility cannot be improved by unilaterally changing the strategy, i.e.,

\[
U(v^*_k, V_{-k}) \geq U(v^*_k + 1, V_{-k}),
\]

and

\[
U(v^*_k - 1, V_{-k}) \leq U(v^*_k, V_{-k}).
\]

Thus, the individual player’s utility cannot be improved by unilaterally deviating from the equilibrium.

**Proposition 3.** The NE state can be reached if and only if $v^*_k$ subtasks are offloaded to the edge server from player $k$, given that $l(\lfloor \psi^* \rfloor) \neq l(\lfloor \psi^* \rfloor)$.

**Proof.** Please see Appendix A, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TPDS.2020.2988161, for details. \hfill \qed

**Proposition 4.** If $l(\lfloor \psi^* \rfloor) = l(\lfloor \psi^* \rfloor)$, NE can be achieved by assigning either $\lfloor \psi^* \rfloor$ or $\lceil \psi^* \rceil$ subtasks to the edge server.

**Proof.** Similar to the proof for Proposition 3, we can obtain two NE points $\lfloor \psi^* \rfloor - n_{e-k}$ and $\lceil \psi^* \rceil - n_{e-k}$, since both of these points satisfy Eqs. (23) and (24) simultaneously. That is, if $l(\lfloor \psi^* \rfloor) = l(\lfloor \psi^* \rfloor)$, NE can be achieved by assigning either $\lfloor \psi^* \rfloor - n_{e-k}$ or $\lceil \psi^* \rceil - n_{e-k}$ subtasks from user $k$ to the edge server. This completes the proof. \hfill \qed

3.2.2 Convergence

For the convergence of our MG based task offloading algorithm, we have the following proposition.

**Theorem 5.** With the MG based algorithm, the attendance $|A|$ can converge to a value not greater than 1. That is, $\exists \epsilon \leq 1$ and $\exists T > 0$, such that $|A(t)| = \epsilon$ for $t \geq T$.

**Proof.** Please see Appendix B, available in the online supplemental material, for details. \hfill \qed

With Proposition 5, we find that the attendance $|A|$ can converge to a value no more than one with our proposed MG based offloading scheme. Recall that attendance measures how tasks are allocated to servers with different capacities. Although we cannot directly acquire task processing performance with attendance, the conclusion is still essential as the state with large attendance will result in bad performance in terms of task processing time.

3.2.3 Performance Bound

We then study the performance bound of the task processing time by utilizing the price of anarchy (PoA).

Let $Z$ denote the set of Nash equilibria of the multi-player offloading game and $V^* = \{v_1, v_2, \ldots, v_K\}$ denote the optimal solution that maximizes the number of users that can maximize their payoffs. Then the PoA is defined as

\[
PoA = \frac{\max_{v \in Z} \sum_{k \in K} t_k(v)}{\min_{v \in \nu} \sum_{k \in K} t_k(v)}.
\]

Note that, for the metric of the task processing time, a smaller PoA is better.

For our proposed MG based offloading scheme, we have the following theorem on PoA.

**Theorem 6.** For the multi-player MG offloading game, the PoA of the task processing time satisfies that

\[
1 \leq PoA \leq \frac{\min \left\{ t_r + \frac{n_e^0 + 1}{C_0}, t_r + \frac{n_e^0 C_0}{C_0} \right\}}{\min \left\{ t_r + \frac{n_e^0 C_0}{C_0}, t_r + \frac{n_e^0 - 1}{C_0} \right\}},
\]

where $n_e^0 = \lfloor \psi^* \rfloor$.

**Proof.** Please see Appendix C, available in the online supplemental material, for details. \hfill \qed

Theorem 6 indicates that when the computation of one subtask decreases (i.e., $C_0$ is small), the worst-case performance of Nash equilibrium can be improved. Moreover, when servers have plenty of computing resources (i.e., $C_0$ and $C_s$ are large), the worst-case Nash equilibrium is closer to the optimum and hence the PoA is lower.

4 TASK OFFLOADING FOR FULLY DISTRIBUTED ARCHITECTURE

To further prove the feasibility of our approach, we extend our design to a fully distributed scenario with multiple distributed edge servers. This is a general case since edge servers deployed by different operators might be restricted in mutually resource sharing. Although the standard MG theory provides a scalable extension (e.g., simplex game [29]) to the cases where each decision maker owns multiple choices, the task offloading scheduling for the heterogeneous edge computing scenario is still challenging.

Let $M$ denote the number of edge servers and $M$ the server set. The cut-off value can be generalized as below.
**Definition 6.** General cut-off value. The general cut-off value $\psi_m$ denotes a threshold for the number of subtasks offloaded to edge server $m$, such that the task processing time can be minimized. Let $C_m$ denote the processing capacity of server $m$, then we have

$$\psi_m = \frac{NC_m}{\sum_{m=1}^{M} C_m}.$$  

(27)

With the cut-off value set $\Psi = \{\psi_1, \psi_2, \ldots, \psi_M\}$ in conjunction with multiple distributed servers as offloading choices, there will be more than one winner (i.e., minority). The tasks offloaded to server $m$ is in minority if $n_m \leq \psi_m$. Otherwise, the tasks in server $m$ are in majority.

With the general cut-off values, we can extend the MG based algorithm to the case with multiple choices, with three key procedures presented as follows.

1) **Subtask grouping and initial scheduling.** For the $N_k$ subtasks generated from the user $k$, the number of subtasks offloaded to edge server $m$ is initially set as

$$n_{m,k} = \left[ \frac{C_m}{\sum_{m=1}^{M} C_m} N_k \right].$$  

(28)

The key idea of the initial subtask scheduling is to make the most efficient decision from the user’s perspective. After initial scheduling, some residual subtasks might still be unscheduled, i.e.,

$$n_{r,k} = N_k - \sum_m n_{m,k},$$  

(29)

where $n_{r,k}$ denotes the number of subtasks that have not been scheduled initially. These subtasks are initially scheduled to edge server $m$ with probability

$$p_m = \frac{C_m}{\sum_{m=1}^{M} C_m}.$$  

(30)

2) **Information collection and winner broadcast.** After the offloading decision is made, the scheduler can collect the information on the number of subtasks offloaded to edge server $m$ as

$$n_m = \sum_{k=1}^{K} n_{m,k}.$$  

(31)

With network information for the $t$th iteration, the edge server $m$ is in minority, if $n_m$ is not greater than $\psi_m$, then, the subtasks offloaded to edge server $m$ win; otherwise, the subtasks offloaded to edge server $m$ lose. After information collection, the scheduler broadcasts the winning side by sending $M$-bit control information $\{b_m(t)\}_{m \in M}$ defined as

$$b_m(t) = \begin{cases} 1, & \text{if } n_m < \psi_m, \\ 0, & \text{otherwise}. \end{cases}$$  

(32)

After receiving the control information, users could evaluate their strategies to improve decision-making for the $(t + 1)$-th round offloading.

3) **Decision adjustment and iterative scheduling.** Following the insights from the MG theory, subtasks in the majority side should change their decisions to achieve better performance. Let $M^+$ denote the server set in majority, i.e., $m \in M^+$ if $n_m > \psi_m$. Otherwise, let $M^-$ denote the server set in minority, i.e., $m \in M^-$ if $n_m \leq \psi_m$.

Suppose that edge server $m$ wins, then we keep decision unchanged for subtasks that choose edge server $m$ in the next round. Otherwise, subtasks in edge server $m$ are set to adjust their decisions following probability defined as follows. For $m \in M^+$, $m' \in M^-$,

$$q_{mm'} = \frac{n_m - \psi_m}{n_m} \cdot \frac{\psi_{m'} - n_m}{\psi_{m'} - n_m} \cdot \frac{1}{1 + e^{-\gamma n_m / \tau_{m'}}}.$$  

(33)

where Part I represents a high decision adjustment probability with a large gap between the actual offloading subtasks $n_m$ and the optimal $\psi_m$ for servers in the majority, Part II denotes a high decision adjustment probability for servers in the minority with a large gap between the optimal $\psi_m$ and the actual offloading subtasks $n_m$, Part III illustrates the probability that the offloading decision changes from a server in the majority with transmission delay $\tau_m$ to a server in the minority with transmission delay $\tau_{m'}$.

Note that some subtasks in the majority should keep the current offloading decision with the probability of

$$q_{mm} = \frac{n_m - \psi_m}{n_m},$$  

(34)

where Part II and Part III in Eq. (33) are not included as the offloading decision is not changed.

As a normalized form of $q_{mm'}$, we define the practical decision change probability $p_{mm'}$ as

$$p_{mm'} = \frac{q_{mm'}}{\sum_{m' \in M^-} q_{mm'} + q_{mm}}.$$  

(35)

The task offloading determination is conducted until $p_{mm'} = 0, \forall m, m' \in M$.

Note that the cut-off value defined in Eq. (27) cannot be guaranteed to be an integer, which might cause the algorithm not to be convergent since $p_{mm'} \neq 0$. Alternatively, we refine the definition of the general cut-off value as

$$\psi'_m = \left[ \frac{NC_m}{\sum_{m=1}^{M} C_m} \right].$$  

(36)

However, this might cause $\sum_{m=1}^{M} \psi'_m \neq N$, which might also introduce a divergent case. Nevertheless, let $\Delta_m$ denote the gap between $\psi'_m$ defined in Eq. (27) and $\psi'_m$ defined in Eq. (36), and we have

$$\Delta_m = \frac{NC_m}{\sum_{m=1}^{M} C_m} - \left[ \frac{NC_m}{\sum_{m=1}^{M} C_m} \right].$$  

(37)
Let $\mathcal{M}_\Delta$ denote the set containing the first $N - \sum_{m=1}^{M} \psi^*_m$ values of $\Delta_m$. Then we have the revised cut-off generalization value as

$$\psi^*_m = \begin{cases} \lceil NC_m/\sum_{m=1}^{M} C_m \rceil, & \text{for } m \in \mathcal{M}_\Delta \\ \lceil NC_m/\sum_{m=1}^{M} C_m \rceil, & \text{otherwise.} \end{cases} \quad (38)$$

Following the idea in Proposition 4, we can also prove that our MG based offloading algorithm with multiple choices can converge to the revised cut-off generalization value defined in Eq. (38). The detailed proof is omitted here due to the page limit.

The MG based task offloading algorithm for the fully distributed scenario is illustrated in Algorithm 3.

**Algorithm 3. MG Based Task Offloading for Fully Distributed Architecture**

1: Given: $C_m, N, N_k, s_n = \{1, 2, \ldots, M\}$
2: % Step 1: Subtask grouping and initial allocation
3: Calculate general cut-off value $\psi^*_m$ following Eq. (38);
4: for $k = 1 : K$ do
5: Calculate $n_m,k$ and $n_r,k$ following Eqs. (28) and (29);
6: Allocate $n_m,k$ subtasks to edge server $m$;
7: end for
8: % Step 2: Information collection and winner broadcast
9: for $t = 1 : T$ do
10: Collect information and calculate $n_m$ following Eq. (31);
11: if $n_m < \psi^*_m$ then
12: $b_m(t) = 1$
13: else
14: $b_m(t) = 0$
15: end if
16: % Step 3: Decision adjustment and iterative scheduling
17: Calculate normalized prob. $p_{wn}$ following Eq. (35)
18: end for

With multiple edge servers, the competition between distributed servers and the diverse communication delays to different servers cause a gap of task processing time compared to the scenario where edge servers can form a single cluster. Thus, we introduce an upper bound on the gap of the task processing time between the case with fully distributed edge servers and the optimal solution with an edge resource pool. The result is shown as follows.

**Proposition 7.** The performance gap between the practical case with fully distributed servers and the optimal case with a resource pool is not greater than $C_0/\min_m \{C_m\} + \max\{\tau_m - \tau_m'\}$, where $C_0$ is the computation requirement of one unit subtask, $C_m$ is the computing capacity of edge server $m$, and $\tau_m$ is the transmission delay to edge server $m$.

**Proof.** As the processing capacities of servers are identical for the scenario with distributed multiple servers, and the scenario where edge servers form a cluster. They all support computations on multiple tasks with virtual parallel processing, and the number of subtasks allocated to edge servers is proportional to their capacity. Recall the conclusion in Proposition 2, the task execution time difference is less than $C_0/\min_m \{C_m\}$.

Different from Proposition 2, we should take the transmission delay into account as the task delivery time to different servers is usually different. We have the gap shown as the maximum difference between delivery time of all tasks, i.e., $\max\{\tau_m - \tau_m'\}$.

In total, the performance gap between $T_c$ and $T_g$ is not greater than $C_0/\min_m \{C_m\} + \max\{\tau_m - \tau_m'\}$. □

5 PERFORMANCE EVALUATION

In this section, we conduct simulations with various settings to verify our analysis and demonstrate the performance gain achieved by the proposed algorithm.

5.1 Simulation Settings

In our experiments, we simulate an edge computing system that contains both cloud and edge servers. In default, there are 20 users in the system and each user can generate tasks independently. The default number of CPU cycles to complete the basic unit computation load of one subtask $C_0$ equals to 50 Megacycles. Based on the empirical results in [28], [30], we set the range of the workload as $[2.5 \cdot 10^8, 5 \cdot 10^9]$ CPU cycles. For each user, the number of generated task loads obeys a uniform distribution as referred in [27], which is set to be in the range of (0, 100) in our simulations.

Considering the difference in terms of distance to the cloud (or edge) servers, the round-trip-time from a user to cloud (or edge) servers is set as $\tau_c = 100 \text{ ms}$ (or $\tau_e = 10 \text{ ms}$) respectively according to [22]. In addition, the computing capacity of the edge server and the cloud server is set as $C_e = 2.5 \text{ GHz}$ and $C_c = 10 \text{ GHz}$ in default, respectively. In reality, the cloud capacity is configurable and can be expanded by installing more CPU cores if necessary [27]. In each experiment, algorithms are executed 32 times to obtain the average results. In each simulation run, decisions on 1000 periods are evaluated and statistically collected [26]. Table 2 lists the key parameters with corresponding values adopted in our simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>the number of users</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>the number of subtasks</td>
<td>$125 - 2000$</td>
<td></td>
</tr>
<tr>
<td>$C_e$</td>
<td>edge server capacity</td>
<td>2.5</td>
<td>GHz</td>
</tr>
<tr>
<td>$C_c$</td>
<td>cloud server capacity</td>
<td>$2.5 - 12.5$</td>
<td>GHz</td>
</tr>
<tr>
<td>$N_{e_r}$</td>
<td>iterations per run</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>$N_{pl}$</td>
<td>the number of periods</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>cloud communication delay</td>
<td>100</td>
<td>ms</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>edge communication delay</td>
<td>10-50</td>
<td>ms</td>
</tr>
</tbody>
</table>

For performance evaluation, the scheduling results are compared among the following schemes:

- **Random scheme**, in which each task randomly determines whether offloading its computation to the edge server or the cloud server [31].
- **Standard MG scheme**, where each subtask is scheduled based on strategy iteration techniques [23], and subtasks respond to the aggregate action of all subtasks in the previous round.
- **Bonding MG scheme**, where users make offloading decisions according to the standard MG principle, where tasks generated by one user are scheduled in a bonding manner [26]. The bonding MG scheme is
an extension of the standard MG scheme under scenarios for heterogeneous task load.

- **Heterogeneous MG scheme** ("hetero MG" for short), which is proposed in Algorithm 1 and applicable for the heterogeneous edge computing scenarios.
- **Optimal scheme**, where each players know the actions taken by the rivals, i.e., this is the solution for game with complete information.

### 5.2 Results for Partially Distributed Architecture

In this section, we will report and analyze our simulation results in terms of major metrics, including task processing delay and convergence time.

#### 5.2.1 Task Processing Time

Fig. 4a compares the task processing time $E(T)$ versus the number of subtasks in the system $N$ and the cloud processing capacity $C_c$, where "-1c" denotes the case with a single cluster (partially distributed architecture).

![Fig. 4a](attachment:fig4a.png)  
**Fig. 4a.** The task processing time $E(T)$ versus the number of subtasks in the system $N$ and the cloud processing capacity $C_c$, where "-1c" denotes the case with a single cluster (partially distributed architecture).

Fig. 4b presents the task processing time versus the cloud processing capacity $C_c$, given a static edge capacity $C_e$. Again, the "hetero MG" scheme is the best one consuming the least amount of time to process tasks. As the cloud capacity increases, we can observe that the task processing time decreases for all schemes except the random scheme. The "hetero MG" scheme is always better than baselines, while approaches the optimal solution with complete information. Again, the random scheme, which does not take the beliefs of the other players into the offloading design, is the worst one in most cases.

![Fig. 4b](attachment:fig4b.png)  
**Fig. 4b.** The task processing time $E(T)$ versus the cloud processing capacity $C_c$ with different basic task loads.

Fig. 5 compares the task processing time versus the edge communication delay $t_e$.

![Fig. 5](attachment:fig5.png)  
**Fig. 5.** The task processing time $E(T)$ versus the edge server $t_e$ with different basic subtask loads.

pretty good performance which is close to the optimal solution for the scenario with complete information.

The figure, we can observe that our "hetero MG" scheme is always the best one achieving the lowest task processing time. The "hetero MG" scheme will be prolonged significantly for all schemes because of the increase of the basic subtask load from $C_0 = 10$ to $C_0 = 50$ Megacycles. By comparing the two cases with different basic subtask loads, we find that the task processing time $T$ will go upwards slightly as we increase $t_e$ with $C_0 = 10$ Megacycles. However, such phenomena cannot be observed for the case with $C_0 = 50$ Megacycles, in which the task processing time is very stable with $t_e$. This
difference can be explained from the influence of the task delivery time with different $C_0$. The larger the value $C_0$ is, the smaller influence the delivery time will play on the task processing time.

### 5.2.2 Algorithm Convergence

Fig. 6 evaluates the algorithm convergence by comparing the number of iterations taken to reach the equilibrium state versus the number of subtasks in the system. For the proposed “hetero MG” scheme, the criteria of the equilibrium state is the attendance less than 1 (i.e. $|A| < 1$). Since the other two schemes might not converge to the equilibrium, we set the identical convergence principle for a convenient comparison. The simulation results show that all schemes except our “hetero MG” scheme need more iterations to converge to the optimal state with the increase of $C_c$. In Fig. 6a, we can see that there is a clear upward trend of the number of iterations for both “standard MG” and “bonding MG” with the increase of the number of subtasks in the system. We can also observe that $C_c$ increases the number of iterations for “standard MG” and “bonding MG” schemes significantly, especially when the number of subtasks is not very large by comparing Figs. 6a and 6b. The large number of iterations consumed by “standard MG” and “bonding MG” schemes imply that it is difficult for them to achieve the equilibrium state in reality. In contrast, the number of iterations of our scheme is almost unchanged for all cases. In fact, our “hetero MG” scheme will organize subtasks (generated by the same user) to form groups, which can substantially reduce convergence iterations.

Fig. 7 illustrates the converging tendency on the attendance $A$ versus the decision epoch. Importantly, neither the “standard MG” nor the “bonding MG” could guarantee the convergence to a small value on the attendance, which is an essential factor representing the server utilization efficiency. The task processing performance cannot be guaranteed because of a large difference between the number of subtasks offloaded to the two kinds of servers. While for our “hetero MG” scheme, it can fast converge to a small attendance value, which verifies the effectiveness of the group formation and the decision adjustment policy.

### 5.3 Results for Fully Distributed Architecture

The experiment results on the extended case for the fully distributed architecture in Section 4 are shown as follows. In our experiments, the computing capacity of the edge servers follows the uniform distribution with the expectation $\mu$ and the variance $\sigma^2$, i.e., $C_m \sim U(\mu, \sigma^2)$. The variance $\sigma^2$ is an essential factor indicating the heterogeneity of the processing capacity of the servers. With a higher value of $\sigma^2$, it achieves a larger gap between the server with maximum capacity and the server with minimum capacity. In the following experiments, the expected server capacity is set as a constant (e.g., 5 GHz), while we evaluate the offloading performance versus different levels of server heterogeneity (i.e., variance of server capacity).

Fig. 8a illustrates the task processing time versus the variance of server capacity for the scenarios where edge servers form a cluster. We find that our “hetero MG” scheme outperforms the other two schemes, including the “standard MG” and the random schemes. The performance improvement mainly comes from taking the heterogeneity issues of the edge computing environment into account.

For the fully distributed scenarios, Fig. 8b illustrates the task processing time versus the variance of server capacity for the scenarios where edge servers form a cluster. We find that our “hetero MG” scheme outperforms the other two schemes, including the “standard MG” and the random schemes. With the increase of the capacity variance, the results from the other two schemes show an obvious increasing trend. This might be due to the mismatch between the scheduled tasks and their computation requirements without considering the influence of the heterogeneity of the edge computing scenario.
Similarly, Fig. 8b illustrates the performance gap between the results from “hetero MG” scheme and the optimal solution. Again, the gap on the task processing time increases with the diversity among edge capacities. Compared to the gap in Fig. 8a, the gap of the scenario with multiple edge servers is a bit larger, which might be caused by various transmission delays to different edge servers.

Fig. 9 compare the task processing time versus the number of split tasks. Different from Fig. 4a with constant subtask load, the task load of the network is set as a constant in Fig. 9. Again, for both partially and fully distributed architectures, the proposed “hetero MG” scheme outperforms the other two compared schemes, while approaches the optimal solution with complete information. This verifies the performance of the MG based algorithm design for heterogeneous edge offloading.

For the performance gap, in Fig. 9a with a single edge cluster, we find a decreasing trend on the practical gap between “hetero MG-1c” solution and the optimal solution with the increase of the number of split tasks. This might be due to that, the finer the granularity of task split is, the better the match between subtasks and heterogeneous computing resources will be. In Fig. 9b, we can find a similar trend on the gap between the “hetero MG-mc” solution and the optimal solution. Differently, the gap for the “hetero MG-mc” solution is relatively larger than that for the “hetero MG-1c” solution. This is reasonable since the computing resources are more heterogeneous for the fully distributed architecture compared to the partially distributed architecture.

6 RELATED WORK
The edge computing framework has been designed to meet the time-critical real-world use cases, as referred in many studies [32], [33], [34]. In the environment of edge computing, a task can be offloaded to a nearby edge server or a remote cloud server to reduce latency and increase throughput. Many previous studies formulated the two-way offload decision as a knapsack problem and solved it using methods, such as dynamic programming and Markov decision process (MDP). For example, Wu et al. [6] modeled the task scheduling problem as a knapsack problem, and proposed both offline and online algorithms to maximize the user’s quality of experience satisfaction degree cost effectively. Goudarzi et al. [8] proposed a genetic algorithm to timely achieve the best offloading outcome in a heterogeneous multisite context. Sun et al. [9] developed a user-centric task offloading scheme to optimize task processing delay, based on Lyapunov optimization and multi-armed bandit theories. Kochovski et al. [35] proposed a new MDP based management method for service level agreements to facilitate an automated decision-making process. However, these schemes require a scheduler to collect information and make decisions, which is time consuming and not suitable for decentralized edge computing scenarios.
For distributed scheduling, game theory-driven schemes have attracted much attention. Chen et al. [11], [12] formulated the computation offloading decision among devices as a multi-player strategic game. Some work (e.g., [36]) established a non-cooperative game framework to systematically study the stabilization of a competitive mobile edge computing environment, and proposed an iterative algorithm to find the Nash equilibrium of the games. He et al. [37] proposed a game-theoretic approach that formulates the cost-effective edge user allocation problem as a potential game, and designed a novel decentralized algorithm for finding a Nash equilibrium. To consider the computation requirement difference between users, these game-based schemes always require state information exchange between task initiators, which might introduce problems in scalability and internode interaction overhead.

The above-mentioned games assume the players have complete information about other players’ strategies, including the knowledge of other players’ choices (e.g., Stackelberg game) in some cases. But in some real scenarios, there is a lack of information about the environment. In the cases with incomplete information, minority game shows its efficiency on players’ distributed cooperation decision [13]. Ranadheera et al. [23] studied the applicability of MG to solve the distributed decision-making problems in wireless networks. The authors in [26] developed an MG based distributed server activation mechanism for computation offloading in order to guarantee energy-efficient activation of servers. Furthermore, the asymmetric processing capacity between edge servers and clouds has been investigated by a revision of MG with arbitrary cut-offs [23], [26]. However, these works assumed that users are homogeneous with respect to tasks potentially to be offloaded, which cannot be directly adapted to heterogeneous task scheduling scenario.

7 Conclusion

In this paper, to address the challenges incurred by incomplete information and task heterogeneity, we proposed a minority game (MG) based offloading algorithm to perform task offloading in a heterogeneous environment with incomplete information. In our scheme, tasks are divided into subtasks and instructed to form into a set of groups as possible, and the left ones are scheduled to perform decision adjustment in a probabilistic manner. We investigate the properties of our proposed algorithm theoretically and prove that our algorithm can approach a near optimal point. Finally, we conducted extensive simulations to validate our heterogeneous MG scheme and compared its performance with other alternatives. In the future, we plan to explore task offloading in more realistic scenarios with user dynamics and time-varying network conditions.

Acknowledgments

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Minority Disk Failure Prediction Based on Transfer Learning in Large Data Centers of Heterogeneous Disk Systems

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Abstract—The storage system in large scale data centers is typically built upon thousands or even millions of disks, where disk failures constantly happen. A disk failure could lead to serious data loss and thus system unavailability or even catastrophic consequences if the lost data cannot be recovered. While replication and erasure coding techniques have been widely deployed to guarantee storage availability and reliability, disk failure prediction is gaining popularity as it has the potential to prevent disk failures from occurring in the first place. Recent trends have turned toward applying machine learning approaches based on disk SMART attributes for disk failure predictions. However, traditional machine learning (ML) approaches require a large set of training data in order to deliver good predictive performance. In large-scale storage systems, new disks enter gradually to augment the storage capacity or to replace failed disks, leading storage systems to consist of small amounts of new disks from different vendors and/or different models from the same vendor as time goes on. We refer to this relatively small amount of disks as minority disks. Due to the lack of sufficient training data, traditional ML approaches fail to deliver satisfactory predictive performance in evolving storage systems which consist of heterogeneous minority disks. To address this challenge and improve the predictive performance for minority disks in large data centers, we propose a minority disk failure prediction model named TLDFP based on a transfer learning approach. Our evaluation results in two realistic datasets have demonstrated that TLDFP can deliver much more precise results and lower additional maintenance cost, compared to four popular prediction models based on traditional ML algorithms and two state-of-the-art transfer learning methods.

Index Terms—Disk failure, machine learning, transfer learning, cloud computing, data center

1 INTRODUCTION

Hard disks are widely used as the common and primary storage devices for large-scale storage systems in modern data centers. In such data centers, it has been an extremely challenging undertaking to ensure high availability and reliability for IT management, as various disk failures constantly occur in the field, whether being hard disks [2], [3], [4], flash-based SSDs [5], [6] or NVMe. Disk failures can lead to temporary data loss and thus system unavailability or even permanent data loss if the lost data cannot be recovered by existing data protection schemes, e.g., replication and erasure codes [7], [8] due to disk failures exceeding the designed correction capability. A disk is a rather complex system consisting of a variety of magnetic, mechanical, and electronic components, each of which could fail. As a result, disk failures show different manifestations and extents of severeness [9] for numerous reasons, which has been observed in data centers from major IT companies [10], [11]. Compared with the traditional passive fault tolerance techniques like Erasure Code (EC) and Redundant Arrays of Independent Disks (RAID) [12], proactive disk failure prediction tends to ensure the reliability and availability of large-scale storage systems in advance. Therefore, successful disk failure prediction not only reduces the risk of losing data but also reduces the data recovery cost (i.e., network bandwidth) associated with recovering the data residing on failed disks.

Disk manufacturers implement the self-monitoring, analysis and reporting technology (SMART) technology [13] in the disk firmware. Most of the SMART attributes contain information about gradual degradations and possible defects of disks. Internally, a disk uses the so-called “threshold method” [14] based on SMART values to claim its failure status, which means the hard disk would raise an alarm if the value of an SMART attribute crosses the corresponding predefined threshold. However, this “threshold method” only achieves a failure detection rate (FDR) of 3-10 percent with 0.1 percent false alarm rate (FAR) [14] (FDR and FAR see Section 5.1.2 in detail). In other words, these numbers highlight the...
conservative nature of this method, i.e., it would rather miss chances to detect more disk failures than report false alarms at a higher rate.

To improve the predictive performance, several machine learning (ML) algorithms based disk failure prediction models [15], [16], [17], [18] have been proposed, which leverage training SMART data to predict disk failures. Unfortunately, these works focused on a large number of homogeneous disks which have sufficient training data. In large-scale storage system scenarios, bunches of new disks enter gradually to replace failed disks, resulting in storage systems consisting of heterogeneous disks from different vendors and different models from the same vendor as time goes on. Heterogeneous disks with numerous disk models are common in data centers [19], [20], [21]. Moreover, in evolving storage systems, some disk models are dramatically fewer than others and we call this relatively small amount of disks minority disks (conversely the large amount of disks as majority disks) in large data centers of heterogeneous disk systems. We found that about 25 percent of disks with numerous models (more than 50) are minority disks in two real-world data centers as detailed in Section 3.1. Due to the small sample and insufficient training data of minority disks, traditional ML algorithms using the training data of minority disks would dramatically increase the risk of overfitting (Section 3.1) or poor generalization [22] which will weaken the performance of predictive models and seriously affect the reliability of the storage system. Therefore, we are poised to develop a disk failure prediction model TLDFP to predict failures for minority disks under the condition of having rich heterogeneous disk datasets. Our basic idea is to predict minority disk failures from the available majority disk datasets, which is an application of transfer learning.

In this paper, we aim to seek answers to the following problems: (1) What is the definition of a minority disk dataset as far as failure predication is concerned? (2) Why should we use transfer learning for minority disks failure prediction? (3) How to use transfer learning methods to predict minority disks failure? (4) When to use transfer learning for minority disks failure prediction? Besides, when applied to three real-world datasets from the public Backblaze and Tencent which is one of the largest social network companies in the world, our method TLDFP achieves on average 96 percent failure detection rate with 0.5 percent false alarm rate based on 5 disk manufacturers (Hitachi, Seagate, Hitachi Global Storage Technologies and Western Digital) in 3 different storage media (HDD, SSD and NVMe) when making cross-disk models failure prediction in addressing realistic system challenges.

2 BACKGROUND AND RELATED WORK

2.1 SMART Technology

Almost all hard disk drives, flash-based SSDs and NVMe come with built-in Self-Monitoring, Analysis and Reporting Technology, which are indicators of disk health status. The specification of SMART technology contains up to 30 attributes, reporting various disk operating conditions. SMART data directly or indirectly reflects the health condition of disks and even contains some statistical information. SMART data can be obtained through specified disk protocols upon which the disk manufacturer reached agreement. The disk would raise an alarm if the value of an SMART attribute crosses the corresponding predefined threshold. Each SMART attribute entry consists of five elements described as a tuple \((\text{ID}, \text{Normalized}, \text{Raw}, \text{Threshold}, \text{Worst})\).

- \text{ID}: The designated sequence number of the SMART attribute.
- \text{Normalized}: Current or last normalized value (most are normalized to a value between the best value 253 and the worst value 1 calculated by manufacturer-specific algorithms using its raw value).
- \text{Raw}: The original value corresponding to counts or physical states provided by a sensor and vendor-specific.
- \text{Threshold}: The threshold value beyond which a disk alarms a failure.
- \text{Worst}: The lowest or worst value for a given attribute.

Not all five elements in a tuple are used. In our paper, we focus on the first three elements \((\text{ID}, \text{Normalized}, \text{and Raw})\) in our collected datasets. For convenience, we use “\text{smart_ID_Raw: } V” to denote the raw value of a SMART attribute whose \text{ID} is \(V\). For example, \text{smart_1_RAW: 10} means that the raw value of the read error rate attribute \((\text{ID}: 1)\) is 10 and \text{smart_5_Normalized: 56} means that the normalized value of the reallocated sectors count attribute \((\text{ID}: 5)\) is 56. More specific information about the SMART attributes we use in our evaluation is given in Table 6.

2.2 Large Scale SMART Data Collection

We not only used the publicly available SMART datasets from Backblaze,1 but also collected the real-world datasets from Tencent Inc., Tencent Inc., founded in November 1998, is currently one of the world’s biggest internet companies. In China, Tencent has four major data centers located in Shenzhen, Tianjin, Shanghai and Chongqing, respectively. Among them, Tianjin data center is the largest in Asia. According to Tencent’s statistics, as of 2019, Tencent hosts a total of more than 700 thousand servers and 8 million disks. In the Tencent Cloud Foundation Department where we obtained our SMART dataset in this paper, there are about 400 thousand servers (5 million disks) supporting a variety of business applications, such as WeChat, QQcloud (Cloud of Tencent), TencentVideo, and the QQ photo store QQphoto.

We configure the collection interval as one hour. In practice, the size of one sampled SMART data per disk is about 2 KB, which results in 201.4 GB data per day. Moreover, suppose the disk life is about 4-5 years, then we need to store 350.1 TB SMART data for a period of five years. In each hour, nearly 400 thousand servers send their SMART data to one server, which parses, processes and stores massive amounts of data at the same time. It imposes a great challenge to collect and store such a large-scale set of SMART data. To well handle this situation, we have proposed a scalable framework for collecting large-scale SMART data and it has been deployed in the Tencent’s data centers in practice.

As it is shown in Fig. 1, the scalable SMART data collection framework mainly contains five components:

- **Agent**: An agent runs on the same server with multiple disks, typically 11 disks in Tencent’s servers. It periodically issues protocol-compliant requests to disks to obtain SMART data and sends SMART data files to two dedicated servers via Tencent Gate Way (TGW).

- **TGW**: Due to all servers in Tencent data centers not locating in the same network and high concurrency easily happening, we leverage the open source technology Tencent Gate Way to forward data file send requests. TGW enables multi-network unified accesses and also supports automatic load balancing. Upon receiving SMART data files from agents, TGW forwards them to one of two dedicated servers in a random manner (For illustration purpose, we draw two servers in the figure. However, TGW can easily scale up to connect more servers as needed).

- **TDAgent**: A dedicated server which receives raw binary SMART data files from TGW and converts them to regular text files for data transformation (these files will remain for 3 days temporarily). After this processing, it then periodically forwards the resultant text files to TDW for further storage and analysis.

- **Alarm Log**: The extracted disk alarm events and the feedback information from disk manufacturers, which are used for disk labeling. This alarm log is important for us to redeploy the collected data in new settings and to label disks in supervised learning.

- **TDW**: Tencent Distributed Data Warehouse (TDW), is an open-source system of distributed data processing based on Hadoop, Hive and PostgreSQL which provides massive data storage and analysis functionality.

The framework of large-scale SMART data collection we proposed can effectively solve the problem of mass storage and difficult retrieval. Besides, these key technologies of our framework are all open source, thus more conducive to achieve and apply. Last but not the least, the high-quality SMART data is essential for a firm foundation for modeling of disk failure prediction.

### 2.3 Related Work

There have proposed a host of ML algorithms for disk failure prediction models based on SMART data. Hamerly and Elkan [23] employ two Bayesian methods to model disk failure based on SMART data from Quantum Inc., which consists of 1,927 good drives and 9 failed drives. They categorize the problem as anomaly detection and establish a mixture model called NBEM, short for Naive Bayes clusters trained using expectation-maximization and another method called naive Bayes classifier. They achieve failure detection rates of 75-35 percent for NBEM and 75 percent for naive Bayes classifier at about 40 percent FAR. Hughes et al. [24] explore two statistical methods to improve predictive performance. They explore the capabilities of statistical tests like the rank sum test and OP-ed single variate test, and test both methods with 7,744 drives data (out of which 36 are failures) from two different disk models spanning across a period of 3 months. They achieved a failure detection rate (FDR) of 60 and 0.5 percent false alarm rate (FAR). Murray et al. [25] compare the predictive performance of Support Vector Machine (SVM), rank-sum test, unsupervised clustering and reverse arrangements test.

Zhu et al. [15] explore the capability of a Backpropagation (BP) neural network and an improved SVM model to establish the prediction model based on SMART data. Many researchers use a SVM [26] because they claim SVM can efficiently perform a non-linear classification using the kernel trick, implicitly mapping their inputs into high-dimensional feature spaces [27], [28]. In order to improve the stability and interpretability of the disk failure prediction model, Li et al. [29] propose a new hard drive failure prediction model based on Classification And Regression Trees (CART). The Regression Tree can give the disk a health assessment rather than a simple classification result. Gradient Boosted Regression Tree (GBRT [30]) has been proposed to model disk failure [31], [32], where GBRT is a gradient descent boosting technique based on tree averaging, and is an accurate and effective ML method that can used for both regression and classification problems. To avoid over-fitting, the GBRT algorithm trains many tree stumps as week learners, rather than full, high variance trees. Moreover, the Regularized Greedy Forests (RBF [33]) approach is a powerful, non-linear classification method. It is a variation of GBRT in which the structure search and optimization are decoupled and it utilizes the concept of structured sparsity to perform greedy search directly over the forest nodes based on the forest structure. Mirela Madalina Botezatu et al. employ this method to model disk failure and achieve good results [34]. Xu et al. [35] present a Recurrent Neural Networks (RNN [36], [37]) method to leverage sequential information for predicting hard disk failure. They use a dataset collected from a real-word data center containing 3 different disk models represented as $W$, $S$ and $M$ and establish the prediction model for those disk models, respectively. They model the long-term dependent sequential SMART data and demonstrate the capability of their predictive model. More recently, Mahdisoltani et al. [38] propose to use traditional ML algorithms to predict disk sector errors using SMART.

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3. [https://github.com/tonycody/tencent-tdw](https://github.com/tonycody/tencent-tdw)
datasets. Our goals in this paper are to make whole disk failure predictions which require much higher accuracy due to cost consideration. Note that all these studies focus only on HDDs and none of them has investigated NVMe SSDs.

As previously mentioned, the need for transfer learning occurs when there is a limited availability of training data from a new disk model, which regularly happens to evolving storage systems. With big data repositories becoming more prevalent, using existing datasets that are related to, but not exactly the same as, a target domain of focus point or interest makes transfer learning solutions an attractive approach. There are various applications in which transfer learning has been successfully applied to, including multi-language text classification [39], [40], image classification [41], human activity classification [42], text sentiment classification [43], Web document classification [44] and so on. Unsurprisingly, in recent years, researchers have started to use transfer learning method to solve minority disks failure prediction problems [20], [34]. Mirela Madalina Botezatu et al. proposed the sample selection de-biasing method [34], which we denoted as SSDB in our paper. Its main idea is to train a classifier that can rank the observations linked to a specific disk model based on their similarity to samples pertaining to the target disk model. This method is also a single-source domain transfer learning method in spirit similar to TLDFP algorithm. FLF Pereira et al. proposed the multi-source domain transfer learning for Bayesian network [20], which we denoted as TLBN in our paper. It proposes a new source building method called clustering-based information source and groups several HDDs according to their similarity to build a novel information source for transfer learning. Although these methods also provide a solution to minority disks failure prediction, we have compared TLDFP with them and show our approach delivers better predictive performance. Besides, note that our work is the first to systematically (What, Why, How and When) propose using the transfer learning method to solve minority disks failure prediction based on SMART attributes for large-scale, active, evolving storage systems.

3 Preliminary Study and Motivation

In this section, we define minority disk datasets via experimental examination, investigate the distributions of SMART data, and justify why we use transfer learning for minority disk failure prediction.

3.1 Minority Disk Datasets

As mentioned previously, we aim to improve disk failure predictive performance for a disk dataset which has insufficient training data and where traditional ML algorithms deliver suboptimal performance. In this section, we give the definition of a Minority Disk Dataset and quantitatively evaluate them via experimentally showing the training loss and testing loss [45] of four popular ML algorithms (GBRT, RGF, SVM, and RNN) in disk failure prediction [28], [31], [34], [35]. A loss is a number indicating how bad a model’s prediction. Note that we have added a regularization term to construct the loss function in all four methods which can effectively prevent over-fitting caused by the model having a very large number of parameters. Fig. 2 illustrates the results. As can be seen from the figure, with the dataset increasing, the loss of training set increases to a certain extent while the loss of testing set decreases because the increase in dataset leads to more complex situations where the training model needs to be fitted. More specifically, when the amount of disks is less than 1,500, the gap between the loss of training and testing decreases as the dataset enlarges, which is called over-fitting caused by minority disks. When the amount of disks goes over 1,500, the gap becomes smaller and stabilizes. Therefore, we can draw the following conclusions: (1) A disk dataset containing fewer than 1500 disks could lead to over-fitting which we name it as Minority Disk Datasets; (2) The four popular traditional ML algorithms cannot deliver satisfactory performance when the dataset contains fewer than 1,500 disks. As far as we know, we are the first to define minority disk datasets and quantitatively evaluate them through extensive data analysis and experiments. We studied two real data centers and categorized the disk quantities by the threshold of 1500. As shown in Table 1, in data center BackBlaze, 91 different disk models only account for less than 24 percent of all disks while 12 models account for more than 76 percent. We call these 91 models minority disks. A similar observation has been found in data center of Tencent.

The above description and analysis implies that making disk failure prediction for minority disks is a realistic problem that needs to be resolved.

3.2 The Baseline Results of Using Traditional ML Only Trained on Minority Disk Datasets

In order to investigate the predictive results of using traditional ML methods only trained on minority disk datasets, we use 7 minority disk models from 5 disk manufacturers

<table>
<thead>
<tr>
<th>Data Center</th>
<th>Disk Number</th>
<th>Disk Models</th>
<th>Total Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backblaze</td>
<td>≥ 1500</td>
<td>12</td>
<td>114,570</td>
<td>76.61%</td>
</tr>
<tr>
<td></td>
<td>&lt; 1500</td>
<td>91</td>
<td>34,978</td>
<td>23.39%</td>
</tr>
<tr>
<td>Tencent</td>
<td>≥ 1500</td>
<td>8</td>
<td>52,235</td>
<td>73.32%</td>
</tr>
<tr>
<td></td>
<td>&lt; 1500</td>
<td>52</td>
<td>18,996</td>
<td>26.67%</td>
</tr>
</tbody>
</table>
TABLE 2

<table>
<thead>
<tr>
<th>Type</th>
<th>Methodology</th>
<th>Manufacturer</th>
<th>FDR</th>
<th>FAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDD</td>
<td>GBRT</td>
<td>HDS/STX/HGST/WDC</td>
<td>27.3%/37.5%/31.6%/38.5%</td>
<td>29.0%/19.4%/17.6%/21.8%</td>
</tr>
<tr>
<td></td>
<td>RGF</td>
<td>HDS/STX/HGST/WDC</td>
<td>36.4%/50.0%/47.4%/53.8%</td>
<td>44.3%/22.4%/53.4%/36.6%</td>
</tr>
<tr>
<td></td>
<td>SVM</td>
<td>HDS/STX/HGST/WDC</td>
<td>50.0%/47.9%/57.9%/30.8%</td>
<td>20.7%/47.8%/24.0%/43.7%</td>
</tr>
<tr>
<td></td>
<td>RNN</td>
<td>HDS/STX/HGST/WDC</td>
<td>40.9%/33.3%/36.8%/30.8%</td>
<td>28.3%/31.3%/39.7%/38.7%</td>
</tr>
<tr>
<td>SSD</td>
<td>GBRT</td>
<td>STX/WDC/SAMSUNG</td>
<td>23.7%/38.2%/52.8%</td>
<td>25.5%/27.2%/40.1%</td>
</tr>
<tr>
<td></td>
<td>RGF</td>
<td>STX/WDC/SAMSUNG</td>
<td>35.6%/46.0%/60.4%</td>
<td>20.5%/32.7%/30.1%</td>
</tr>
<tr>
<td></td>
<td>SVM</td>
<td>STX/WDC/SAMSUNG</td>
<td>15.6%/39.5%/47.2%</td>
<td>16.3%/41.4%/22.4%</td>
</tr>
<tr>
<td></td>
<td>RNN</td>
<td>STX/WDC/SAMSUNG</td>
<td>40.7%/52.6%/62.3%</td>
<td>14.3%/22.5%/30.9%</td>
</tr>
</tbody>
</table>

to conduct this experiment based on four popular traditional ML methods: GBRT, RGF, SVM and RNN which include the popular tree structure algorithms and deep learning algorithms and have been commonly used in disk failure prediction. Note that we only use 70 percent of the minority disk datasets as training sets and the remaining 30 percent as testing sets without any other datasets. Besides, we consistently use the following acronyms for these five vendors throughout the paper which are also used in their disk models: Hitachi (HDS) and Seagate (STX) from Backblaze, Hitachi Global Storage Technologies (HGST), Western Digital (WDC) and SAMSUNG come from Tencent. As can be seen from the Table 2, none of the four traditional ML methods can deliver a high FDR and low FAR. We know the poor predictive performance due to over-fitting caused by using small homogeneous datasets based on traditional ML.

We have also conducted experiments to directly use large datasets of the available majority disks to predict minority disk failures based on the four popular traditional ML techniques, the performance is not satisfactory either (details are shown in Fig. 6 in Section 5). To understand the reason, we analyze the SMART data distributions as below.

3.3 SMART Data Distributions

It is interesting to observe that the values of SMART attributes indicating disk health conditions of different disk models from the same manufacturer exhibit similar distribution patterns. We have analyzed both the publicly available SMART dataset from Backblaze and a dataset from the data center of Tencent. Fig. 3 shows the revealed SMART data distribution patterns. Each subfigure shows a pair of SMART attribute values’ distribution pattern (we also investigate many other different SMART attributes, which lead to similar results) of two different disk models from the same manufacturer and each circle is used to highlight different disk models. As it is evidently shown, the relationship between Abnormal and Normal states indicated by the two SMART attributes of two disk models shows a similar pattern, with only the difference of SMART values being in different ranges. Figs. 3a, 3b, 3c and 3d respectively show that the Abnormal state is above, below, and to the left of the Normal state for the two disk models from the same manufacturer. Furthermore, the SMART values are distributed in different spectrums. Take Fig. 3b Seagate as an example, the distribution region of model STX-B is right-lower than that of model STX-A. Note that even if we ignore the y-axis (smart_241_Raw, LBAs written), there are still differences in the distribution of SMART data from these two different disk models (the distribution region of model STX-B is right-to that of model STX-A). Traditional ML algorithms deliver good predictive performance only when both training and testing data are drawn from the same distribution. Therefore, they fail to perform satisfactorily when it comes to cross-disk models failure prediction due to different distribution spectrums as revealed in Fig. 3.

In order to take an in-depth look at the distributions of SMART values of different disk models from the same manufacturer and further motivate the transfer learning from one disk model to a different model and explain why we use transfer learning for minority disks failure prediction, we investigate the differences in the distributions of SMART data and present a intuitive analysis comparing different disk models from the same manufacturer. Probability Density

![Fig. 3. The distributions of two SMART attributes of two disk models from four manufacturers, i.e., Hitachi, Seagate, HGST, and WDC. Each subfigure shows the Abnormal and Normal states indicated by a randomly chosen pair of SMART attributes of two disk models. These four subfigures indicate that two disk models of each manufacturer exhibit similar failure patterns represented by the two SMART attributes distributions and the SMART data are distributed in different value ranges, which motivates us to apply transfer learning to make cross-model disk failure predictions.](image-url)
Function statistic (PDFs) is frequently used to describe the intensity of continuous random variable. For easy observations, we use the Gaussian Kernel Density Estimation (GKDE) as the kernel function, which results in smooth curves. Fig. 4 shows the PDFs of the value of a SMART attribute of two models from four manufacturers. The distributions of the SMART data of two disk models are different but similar in that they show similar spikes though at different points and magnitudes. We refer to this phenomenon as covariate shift [47] among relevant predictors between different models from the same manufacturer. Therefore, we conclude that different disk models from the same manufacturer exhibit varying SMART value distributions. For the problem of minority disks failure prediction, the implication is that a prediction model built upon traditional ML algorithms using training data from one disk model is not applicable to other different models even from the same manufacturer. Therefore, to leverage a prediction model for a disk model built on adequately sufficient SMART training data to build a predictive model for a different model for which there is only limited training data, we could employ transfer learning algorithm which is inherently suitable for transferring health state information from one disk model to another disk model from the same manufacturer.
Considering the above regularity of relationship between Abnormal and Normal disk states and varying SMART data distribution spectrums across different disk models, we are motivated to apply transfer learning to predict disk failures for minority disks using the knowledge from the majority disks, which we name as TLDFP.

4 MINORITY DISK FAILURE PREDICTION

In this section, we answer the questions of how and when to use transfer learning for minority disk failure prediction. Specifically we detail our transfer learning method for minority disks failure prediction TLDFP, followed by our method of selecting source domain based on KLD.

4.1 TLDFP: Transfer Learning for Minority Disk Failure Prediction

We elaborate on how to use transfer learning to predict minority disk failure in this section. Fig. 5 illustrates the overall structure of our proposed predictive method TLDFP. It main consists of two components: a transfer learning algorithm TrAdaBoost [48] and an instance map to disk algorithm IMDA. Note that we randomly divide the SMART data of minority disk model B into two parts. The first part includes a small portion (e.g., 10 percent) of the labeled target domain data, which is then put together with the data of majority disk model A as a combined source domain to establish the relationship between the two different disk models so as to reduce variation between their distributions. The other part contains the remaining unlabeled data as testing data. Then we use our TLDFP method to establish a predictive model and make failure prediction for the minority disk model B training data. With the above description, the problem we aim to solve in this paper can then be formally defined as: given enough labeled training data S_a, a small amount of labeled disk model B data S_b, the target domain containing all unlabeled disk model B data T_b, and the maximum iteration T.

Note: n is the number of disk model A data and m is the number of disk model B data in the source domain.

Algorithm 1. TrAdaBoost Algorithm

Require:
The source domain containing labeled disk model A data S_a = \{a_1, a_2, \ldots, a_n\}, a small amount of labeled disk model B data S_b = \{b_{n+1}, b_{n+2}, \ldots, b_{n+m}\}, the target domain containing all unlabeled disk model B data T_b, and the maximum iteration T.

1: Begin;
2: Initialize weights distribution W_j, where \( t \) is the sequence of iteration, i.e., W = \{w_1, w_2, \ldots, w_{n+m}\}:

\[
\begin{align*}
w_j &= \frac{1}{n}, & j &= 1, 2, \ldots, n \\
&= \frac{1}{m}, & j &= n, n+1, \ldots, n+m
\end{align*}
\]

3: Set \( \varphi = 1/\sqrt{2 \ln n/T + 1} \);
4: for \( t = [1, T] \) do
5: Set the weights of instances as:

\[
I_t = \frac{W_j}{\sum_{j=1}^{n+m} w_j}
\]

6: Apply the basic learner to \( S_a \) and \( S_b \) with weights of instances \( I_t \), and also to the unlabeled target domain disk model B dataset \( T_b \). We can achieve a classifier \( h_t \);
7: Calculate the error rate of \( h_t \) on the labeled source domain disk model B datasets \( S_b \), note that \( e(x_j) \) is the true label of the \( j \)th SMART sample \( x_j \):

\[
e_t = \sum_{j=n+1}^{n+m} \frac{|e(x_j) - h_t(x_j)| w_j}{\sum_{j=n+1}^{n+m} w_j}
\]

8: Set \( \varphi_t = \varphi (1 - \epsilon_t) \). Note that if \( \epsilon_t \) is greater than 1/2, it needs to be reset to 1/2;
9: Update the weight distributions as below:

\[
w_j^{t+1} = \begin{cases} 
w_{j} \varphi_t & j = 1, \ldots, n \\
w_{j} \varphi_t \frac{|e(x_j) - h_t(x_j)|}{\epsilon_t} & j = n, n+1, \ldots, n+m
\end{cases}
\]

10: end for
11: Map the instance to disk results using IMDA;

Ensure: Classify the disk as good(-1) or failed(1);

The input of the TrAdaBoost algorithm includes two disk models’ training and testing data, and the maximum number of iterations \( T \). It initializes the weights of training data and performs the iteration process. In each cycle, we use the basic learner, such as GBR, RGF, SVM, RNN, and the weight distribution \( I_t \) to build a classifier \( h_t \) on testing data and calculate the error rate on the labeled source domain disk model B dataset \( S_b \). Lastly, we set the new weights based on the previous iteration results and the error rate. Note that if majority disk model A instances of the source domain are misclassified, they are considered to be different from the minority disk model B. As a result, we reduce the weights of these instances in order to reduce their influences.
on the predictive model in the next iteration. Specifically, we multiply the instances by $w^j_\varphi(x_j^{(t)}) - h(x_j^{(t)})$, where $\varphi$ ranges from 0 to 1 and $c(\cdot)$ is the true label of a SMART attribute. On the other hand, if the disk model $B$ instances in the combined source domain are misclassified, we increase the weights of these instances to gain more attention in the next iteration through multiplying these instances by $w^j_\varphi(x_j^{(t)}) - h(x_j^{(t)})$, where $\varphi_i$ is greater than 1. After several iterations (we will investigate the impact of this value on the predictive performance in Section 6.4), the instances of majority disk model $A$ in the source domain that fit minority disk model $B$ will gain greater weights and those different from disk $B$ will have smaller weights.

Since the inputs of the failure prediction model include many SMART instances from a lot of disks at different moments, each output result indicates the prediction result for a particular instance rather than the disk health state. Therefore, we need to map the results of multiple SMART instances to the final disk state. To achieve that, we propose an Instances Map to Disk Algorithm (IMDA) by extensive experiments and analysis using the majority disk dataset in the training progress. IMDA determines the final health state of a minority disk in testing progress (practical use). Specifically, it performs a prediction for each minority disk once every day, if any instance of one disk is predicted as a failure, the corresponding disk will be considered as failure. Besides, we discuss other alternative options are discussed in Section 6.5 and our method outperforms all others in terms of predictive result.

### 4.2 Source Domain Selection Based on KLD

*When* can we use TLDFP for minority disks failure prediction? To answer this question, we use Kullback Leibler Divergence (KLD), which is a metric measuring the divergence degree of one probability distribution from another expected probability distribution [49]. KLD values indicate the disparities between two random variable distributions. A zero KLD value means that the two random distributions are the same, while the KLD value increases as the differences between two random distributions widen. In general, the bigger a KLD value is, the greater the differences between two distributions will be and the more difficult the knowledge transfer between two distributions will be. Table 3 gives KLD values corresponding the PDFs showed in Fig. 4. Table 3 shows that all KLD values are not equal to zeros, confirming that the respective SMART data distributions are indeed not the same. Note that as indicated in Table 3, the KLD value trend, which is consistent with the PDF differences increase from HDS, to STX, to HGST, to WDC shown in Fig. 4. Considering that TLDFP is a method to decrease the data distribution differences between source domain and target domain, so we infer that the bigger KLD value between one disk model and another is, the harder TLDFP can transfer experience. The predictive results in Section 5.2 prove our conjecture and we also conduct detailed experiments and discussions in Section 6.1. Therefore, the value of KLD can guide us to select appropriate majority disk dataset (source domain) for training the minority disk failure predictive model. As far as we know, we are the first attempt to present a novel method based on KLD values as an effective indicator to select proper majority disk models and improve disk failure prediction. Note that we also use the Jensen-Shannon Divergence (JSD) and Wasserstein Distance (WD) but find KLD is the best metric to guide us to select an appropriate majority disk dataset and results in better prediction performance. More specifically, JSD (WD) only achieves on average 79.6 percent (83.1 percent) $FDR$ and 3.2 percent (2.1 percent) $FAR$. Our evaluation results in Section 6.1 demonstrate that our approach of using KLD is very effective and practical.

### 5 Experimental Evaluation

In this section, we evaluate the predictive performance of TLDFP. We first describe the methodology, followed by the experimental results of comparing TLDFP against four ML algorithms and two state-of-the-art transfer learning methods according to the evaluation metrics.

#### 5.1 Methodology

We describe the characteristics of three real-world SMART datasets in our experiments and SMART attributes selection. Then we introduce four evaluation metrics commonly used in ML and some testing methods we use to conduct all our experiments.

##### 5.1.1 Datasets and Attribute Selections

**Datasets.** Defining a failed disk is a difficult task in varying deployment scenarios. Based on experience and post-mortem analysis in Tencent Inc., we define a disk that cannot function properly as failed if it was replaced as part for repair. Specifically, it includes three cases: the disk has a write operation error, the system loses connection to the disk and an operation (i.e., disk scrubbing, read and write calls) exceeds the timeout threshold. Note that not all samples of failed disks need to be used in the training set; otherwise, those good samples of failed disks which are far from the actual failure would disturb the training of the detection model. Therefore, only the last 14 continuous samples (our goal is to predict disk failure 14 days in advance) before the moment of failure of the training disks can be regarded as failed samples. We use three SMART datasets from real-world data centers for evaluations. Table 4 gives the overall characteristics of the two datasets. Every disk is classified either as “Good” or “Failed”. “Sample” indicates SMART records. Each good disk or failed disk has many SMART records. As the original dataset has more samples of good disks than failure disks, we use majority class undersampling to improve training in the case of imbalanced classes to create training datasets. We have chosen a 1:3 ratio of failure disk to good disk [38]. When performing the training in traditional ML method, we divide the data into 70 percent
training and 30 percent testing data for our experiments which is in line with existing work [20]. Note that all the results of our experiments are obtained by cross-validation [50] in order to avoid fortuitous accident which is common used in ML. Table 5 lists our chosen disks for evaluations.

SMART Attribute Selections. Each SMART observation can contain up to 30 meaningful SMART attributes. However, some attributes are irrelevant to our disk failure predictive model because they are immutable or have not experienced noticeable abnormal changes. Therefore, we selectively keep those attributes that are relevant to the disk health state according to a feature selection process based on Principle Component Analysis (PCA) while ignoring other irrelevant attributes. The selected SMART attributes of HDDs, SATA SSDs and NVMe SSDs are listed in Table 6. For each SMART sample, we use the Normalized Value and Raw Value. The normalized value typically represents the current value of an attribute. However, certain normalized values lose accuracy when transformed from the raw value and some raw values are more sensitive to the predictive model. We also use other methods of feature selection like [20], [34], [38]. However, the impact on the experimental results is not significant, so we do not discuss further due to limited space.

In addition, different SMART attributes have different output ranges, which will lead to different impacts on the predictive model. In order to make a fair comparison among different SMART attributes in our disk failure predictive model, we normalize the range of all selected SMART attributes using the min-max scaling which is in line with existing work [38]

\[ x_{\text{norm}} = \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \]

where \( x \) is the original value of a SMART attribute, \( x_{\text{max}} \) and \( x_{\text{min}} \) are the maximum and minimum value of the attribute in the training set, respectively.

5.1.2 Evaluation Metrics

Confusion matrix [51] is a tool of visualization for interpreting the performance of machine learning algorithms. Table 7 shows the confusion matrix used in disk failure prediction. We refer to a failed disk sample as a Positive instance (denoted as “P”), and a good disk sample as Negative instance (denoted as “N”). The prediction result takes on only two values: True (denoted as “T”) or False (denoted as “F”). Therefore, “TP” standing for “True Positive”, means that a failed disk is correctly predicted and “FP” standing for “False Positive”, means that a good disk is falsely predicted as a failed disk. By a similar reasoning, we can get the meanings of both “FN” and “TN”. Using the statistics information about the four metrics, we can construct many evaluation criteria which are often utilized in judging machine learning algorithms. In our evaluations, we use the following four metrics to report the results in our experiments which are commonly used for evaluating the capability of a classification model in ML [52].

### Table 4

<table>
<thead>
<tr>
<th>Data center</th>
<th>Duration</th>
<th>Good</th>
<th>Good Sample</th>
<th>Failed</th>
<th>Failure Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>BackBlaze</td>
<td>50 months</td>
<td>141,891</td>
<td>106,867,099</td>
<td>7,657</td>
<td>7,689</td>
</tr>
<tr>
<td>Tencent</td>
<td>26 months</td>
<td>68,436</td>
<td>774,994,430</td>
<td>2795</td>
<td>31,574,341</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Type</th>
<th>#ID</th>
<th>SMART Attribute Name</th>
<th>Attribute type</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDD</td>
<td>001</td>
<td>Raw Read Error Rate</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>003</td>
<td>Spin-Up Time</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>005</td>
<td>Reallocated Sectors Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>007</td>
<td>Seek Error Rate</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>009</td>
<td>Power-On Hours</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>184</td>
<td>I/O Error Detection and Correction</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>187</td>
<td>Reported Uncorrectable Errors</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>188</td>
<td>Command Timeout</td>
<td>Raw</td>
</tr>
<tr>
<td></td>
<td>189</td>
<td>High Fly Writes</td>
<td>Normalized&amp;Raw</td>
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<td>190</td>
<td>Airflow Temperature</td>
<td>Normalized&amp;Raw</td>
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<td></td>
<td>193</td>
<td>Load/Unload Cycle Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>194</td>
<td>Temperature</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>197</td>
<td>Current Pending Sector Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>240</td>
<td>Head Flying Hours</td>
<td>Raw</td>
</tr>
<tr>
<td></td>
<td>198</td>
<td>Offline Uncorrectable Sector Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>241</td>
<td>Total LBAs Written</td>
<td>Raw</td>
</tr>
<tr>
<td></td>
<td>242</td>
<td>Total LBAs Read</td>
<td>Raw</td>
</tr>
<tr>
<td>SATA SSD</td>
<td>001</td>
<td>Raw Read Error Rate</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>005</td>
<td>Retired Block Count</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>012</td>
<td>Power On Hours Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>017</td>
<td>Program Fail Count</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>172</td>
<td>Erase Fail Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>174</td>
<td>Unexpected Power Loss Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>177</td>
<td>Wear-Range Data</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>187</td>
<td>Reported Uncorrectable Errors</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>188</td>
<td>Command Timeout</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>195</td>
<td>On the Fly Reported Uncorrectable Error Count</td>
<td>Raw</td>
</tr>
<tr>
<td></td>
<td>196</td>
<td>Reallocated Event Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>197</td>
<td>Read Failure Block Count</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>206</td>
<td>Write Error Rate</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>208</td>
<td>Erase Count Average</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td>NVMe SSD</td>
<td>001</td>
<td>Temperature</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>002</td>
<td>Available Spare</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>003</td>
<td>Available Spare Threshold</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>004</td>
<td>Percentage Used</td>
<td>Normalized</td>
</tr>
<tr>
<td></td>
<td>005</td>
<td>Controller Busy Time</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>006</td>
<td>Power Cycles</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>007</td>
<td>Power On Hours</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>008</td>
<td>Unsafe Shutdowns</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>009</td>
<td>Media and Data Integrity Errors</td>
<td>Normalized&amp;Raw</td>
</tr>
<tr>
<td></td>
<td>010</td>
<td>Error Information Log Entries</td>
<td>Normalized&amp;Raw</td>
</tr>
</tbody>
</table>
The goal of our approach is to balance between high precision and high recall. Note that the precision (Precision = TP / (TP + FP)) also called PP, is a probability curve and represents the proportion of true failed disks that are correctly predicted as failed. The higher the PP is, the better the model is.

FAR. False Alarm Rate (FAR = FN / (FN + TN)), the proportion of good disks that are falsely predicted as failed. The lower the FAR is, the better the model is.

F-Score. F-Score is a balance between FDR and Prediction Precision (PP = TP / (TP + FP)). PP is the proportion of predictive failed disks that are correctly predicted as failed. Therefore, the specific calculation formula of F-Score is 2 * FDR * PP / (FDR + PP). The higher the F-Score is, the better the model is.

AUC-ROC Curve. The Area Under the Curve-Receiver Operating Characteristic (AUC-ROC) curve is a performance measurement for classification problem at various threshold settings. ROC is a probability curve and AUC represents degree or measure of separability. It is plotted with FDR against the FAR where FDR is on y-axis and FAR is on the x-axis. In disk failure prediction, a higher the AUC means the model is better at distinguishing failed and good disks.

5.1.3 Testing Methods and Configurations
To verify the effectiveness of our proposed TLDFP, we conduct experiments in three scenarios: 1) to use traditional ML methods only trained on the minority disk datasets, 2) to compare TLDFP with traditional ML techniques (baseline), and 3) to compare TLDFP with other transfer learning approaches. The settings are described below.

1) Traditional ML methods only trained on minority disk:
The detailed description see Section 3.2.
2) TLDFP with traditional ML techniques:
In this scenario, we conduct experiments to investigate the performance of minority disks failure prediction based on four traditional ML algorithms using large heterogeneous datasets for training from both different disk models and the same disk manufacturer. Table 8 shows the training and testing datasets. Note that we randomly choose 10 percent testing datasets for training with all training datasets.

3) TLDFP with other transfer learning approaches:
In addition to traditional ML techniques, we also compare our TLDFP with two state-of-the-art transfer learning methods (SSDB and TLBN) of predicting minority disks failure. Note that we use the same datasets for the two methods in [34] and [20] for fair comparison in all our experiments.

5.2 Experimental Results
In this section, we show the HDD, SATA SSD and NVMe SSD results of the TLDFP compared to traditional ML methods and other transfer learning methods with four evaluation metrics mentioned in Section 5.1.2 respectively. Note that we had showed the poor baseline results of using traditional ML methods only trained on the minority disk datasets in Section 3.2.

5.2.1 Evaluations Compared to Traditional ML Approaches
- FDR/Recall Rate: We conduct experiments to investigate the FDR of TLDFP and four popular traditional ML methods using four HDD models from two real data centers. As can be seen from the Fig. 6a, none of the four traditional ML methods can deliver a high FDR using large heterogeneous datasets. However, the TLDFP use the above GBRT, RFG, SVM, RNN algorithms respectively as the basic learners all achieved higher FDR.

- FAR: Note that the goal of our TLDFP is not only to achieve high FDR but also low FAR for minority disk failure prediction. The results of FAR are showed in Fig. 6b. All other methodologies show higher FAR which is unacceptable in realistic data centers. Further, none of the four traditional ML methods can deliver both a high FDR and a low FAR on minority disks except for TLDFP. Based on the analysis in Section 3.3, we know the poor predictive performance caused by the traditional ML methods do not have the ability to reduce the distribution difference between the minority disk datasets in target domain and majority disk datasets in source domain.

### Table 7: Confusion Matrix Used in Disk Failure Prediction

<table>
<thead>
<tr>
<th>Predictive failed disk</th>
<th>True failed disk</th>
<th>True good disk</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TP</strong></td>
<td><strong>FP</strong></td>
<td></td>
</tr>
<tr>
<td><strong>FN</strong></td>
<td><strong>TN</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Table 8: Datasets of Minority Disk Failure Prediction Using Large Heterogeneous Dataset Based on Traditional ML

<table>
<thead>
<tr>
<th>Data Center</th>
<th>Manufacturer</th>
<th>Training Disk Model</th>
<th>Testing Disk Model</th>
<th>Training set</th>
<th>Testing set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backblaze</td>
<td>Hitachi</td>
<td>HDS-A</td>
<td>HDS-B</td>
<td>4774 good HDS-A and 225 failed HDS-A 105</td>
<td>943 good HDS-B and 65 failed HDS-B</td>
</tr>
<tr>
<td>Tencent</td>
<td>HGST</td>
<td>HGST-A</td>
<td>HGST-B</td>
<td>13367 good HGST-A and 451 failed HGST-A 68</td>
<td>611 good HGST-B and 57 failed HGST-B</td>
</tr>
<tr>
<td>WDC</td>
<td>WDC-A</td>
<td>WDC-B</td>
<td></td>
<td>6847 good WDC-A and 259 failed WDC-A 47</td>
<td>425 good WDC-B and 38 failed WDC-B</td>
</tr>
<tr>
<td>STX</td>
<td>SSD-S-A</td>
<td>SSD-S-B</td>
<td></td>
<td>8672 good SSD-S-A and 397 failed SSD-S-A 92</td>
<td>830 good SSD-S-B and 121 failed SSD-S-B</td>
</tr>
<tr>
<td>WDC</td>
<td>SSD-W-A</td>
<td>SSD-W-B</td>
<td></td>
<td>13972 good SSD-W-A and 489 failed SSD-W-A</td>
<td>611 good SSD-W-B and 68 failed SSD-W-B</td>
</tr>
<tr>
<td>SAMSUNG</td>
<td>NVMe-A</td>
<td>NVMe-B</td>
<td></td>
<td>2653 good NVMe-A and 136 failed NVMe-A 39</td>
<td>353 good NVMe-B and 48 failed NVMe-B</td>
</tr>
</tbody>
</table>
- **F-Score**: Fig. 6c compares the F-Score of the different prediction models on the two datasets using four disk models. As can be seen, TLDFP has much higher F-Score than other different traditional ML methods. As an example, the F-Score of TLDFP with RBF as its basic learner TLDFP(RBF) is almost 9 times as high as the algorithm RBF for WDC disks from data center of Tencent. As we recall in Section 4.2, the bigger KLD value of HGST dataset will lead to more difficult knowledge transfer. This conclusion is also confirmed by the F-Score observations given that the F-Score of TLDFP for HGST are generally lower that other cases. We will further discuss this issue in detail in Section 6.1.

- **AUC-ROC Curve**: We plot the AUC-ROC curve in Fig. 6d using WDC disk model in Tencent. As it is shown, the AUC-ROC curve of TLDFP are all close to the top left corner and TLDFP(RGF) achieving the higher AUC value compared to other two transfer learning methods. The four traditional ML methods achieved lower AUC values, reflecting their poor classification ability in performing cross-disk model failure predictions.

### 5.2.2 Evaluations Compared to Other Transfer Learning Approaches

As it is shown in Table 9, TLDFP shows higher FDR/F-Score and lower FAR than SSDB and TLBN. The reason is that although SSDB matches the distribution of the source domain with the target domain, it only ranks the observations in source domain, while TLDFP makes more effective weight adjustments to every observation. Considering the TLBN is a multi-source domain transfer learning method and TLDFP is a single-source domain transfer learning method, we analyze the KLD values between each disk model in the source domain and the minority disk model in the target domain. The results are showed in Table 10. We find the data in the disk model with a large KLD value in the source domain (such as ST320005XXXX and ST1500DL003). However, TLDFP only uses the disk model which has the smallest KLD value as source domain. A large KLD value leads to difficulties for TLBN in mitigating the distribution differences between the source and target domains. This result also shows that single-source domain transfer learning performs better than multi-source domain transfer learning and there is a good metric (e.g., KLD) for evaluating differences between different domains.

### 5.2.3 Evaluations on SATA SSD and NVMe SSD

In addition to the experimental results of HDDs failure prediction, we also conduct experiments on SATA SSD and NVMe SSD in the three scenarios mentioned in Section 5.1.3 and verify the good performance of our TLDFP. Table 2 illustrates the poor results of first scenarios (see Section 3.2). As can be seen from the Table 11, whether with traditional ML methods or other transfer learning approaches, our TLDFP can achieve higher FDR, F-Score, AUC and lower FAR at same time. That shows the effectiveness of our TLDFP on different storage media.

In summary, the results have demonstrated that TLDFP can effectively solve the problem of minority disk failure prediction with much better predictive performance than traditional ML methods and two other transfer learning methods for the same datasets. More specifically, TLDFP not only delivers high FDR, F-Score and AUC, but also shows a rather low FAR at the same time. The main reason for the comparison results is that our TLDFP algorithm is able to utilize the small number of labeled target disk data to establish the relationship between source and target disk models, which helps the large heterogeneous disk model to be well trained toward the characteristics of the minority target disk model. In other words, TLDFP reduces the difference in data distribution between the source and target domain, which we will further discuss in Section 6.2. Note that we don’t include the results of all methods or disk models due to space limit. From all our tests, TLDFP demonstrates the best performance.

### 6 Observations and Sensitivity Study

In this section, we provide several additional sensitivity studies from six aspects.

#### 6.1 The Impact of KLD in Source Domain Selection

In order to verify our conjecture in Section 4.2 and further explain the results in Section 5.2.1 and Section 5.2.2, we analyze the relationship between KLD and F-Score in TLDFP.
using several minority disk models as testing disk model and large heterogeneous datasets of one disk model for training from the same manufacturer in two real data centers. The results are shown in Fig. 7a and Table 12. We observed that the value of F-Score keeps rising as the value of KLD decreases. In other words, it shows that the smaller the difference in SMART data distribution between the source and target domain, the easier knowledge transfer in TLDFP can be. Therefore, we use KLD as an effective indicator for source domain selection of large heterogeneous dataset and usually select the one which has the smallest KLD value.

6.2 The Change of KLD Value in TLDFP

In order to more intuitively observe how the TLDFP method reduces the difference in data distribution between the source domain and the target domain, we record the value of KLD between the training set and the dataset after each iteration of the model during the training process of TLDFP(GBRT), as illustrated in Fig. 7b. We can see that as the number of iterations of the model increases, the KLD value between the source domain and the target domain decreases continuously, and stabilizes at a smaller value when the number of iterations is about 22. This shows that our TLDFP model continuously reduces the difference of the SMART data distribution between the two domains in the training process, enabling us to use the large heterogeneous disk data to predict the minority disk data and realize knowledge transfer.

6.3 Varying Samples from Target Domain

As discussed previously our prediction model TLDFP uses a portion of labeled dataset from target domain as part of its source domain dataset. In this section, we investigate how the percentage number affects predictive performance of TLDFP. We report the results of TLDFP with RGF as its basic learner and using the disk data of WDC model from Tencent data center, as the other three basic learners show similar results. Fig. 7c shows the relationship between the FDR and the percentage of target domain data put in the source domain. As it clearly shows, the FDR increases dramatically when the percentage increases from 2 to 10 percent. When the percentage goes beyond 10 percent, the FDR does not continue to increase but remains a relatively high level, meaning putting more target domain data to the source domain does not help further improving predictive performance. Consequently, we randomly choose 10 percent target domain data in our previous experiments.

6.4 The Impact of Iterations

The TrAdaBoost algorithm takes an input parameter to cap the iterations performed by the algorithm. In this section, we study how the iteration parameter affects predictive performance in terms of F-Score. We report the results of TLDFP with RGF as its basic learner. Fig. 7d shows how F-Score changes as the number of iterations varies for different datasets. From this figure, we can make similar observations as in the preceding subsection. As the number of iterations increases, the F-Score increases quickly and reaches a stable level at 22 iterations (fast convergence). It also shows that as the algorithm adjusts instance weights in each iteration, TLDFP gradually adjusts to converge toward the testing data. Since the number of 22 represents a inflection point, we adopt this iteration number in our previous experiments.

6.5 The Sensitivity Study of IMDA

In Section 4.1, we introduced the algorithm KLD. Specifically, if any instance is classified as failure, the corresponding disk is considered as failure. Here we conduct experiments to evaluate the impact of different instances, specifically, 1 instance, 1/3 of all instances, 1/2 of all instances, 2/3 of all instances, and all instances being classified as failure. The result of this experiment using TLDFP(RNN) based on the WDC disk model data in Tencent is showed in Table 13. It is clear that the option we use (one instance failure indicates the corresponding disk failure) achieves the

---

**TABLE 12**
The F-Score Varies With the Value of KLD

<table>
<thead>
<tr>
<th>Data Center</th>
<th>Method</th>
<th>Training Model</th>
<th>Testing Model</th>
<th>KLD</th>
<th>F-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>BackBlaze</td>
<td>TLDFP</td>
<td>HGST-K</td>
<td>HGST-L</td>
<td>2.67</td>
<td>76.8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HGST-M</td>
<td>1.76</td>
<td>85.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HGST-N</td>
<td>0.91</td>
<td>93.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HGST-O</td>
<td>0.69</td>
<td>95.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HGST-P</td>
<td>0.47</td>
<td>97.6%</td>
</tr>
<tr>
<td>Tencent</td>
<td>TLDFP</td>
<td>STX-K</td>
<td>STX-L</td>
<td>3.57</td>
<td>71.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>STX-M</td>
<td>2.58</td>
<td>78.2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>STX-N</td>
<td>2.26</td>
<td>83.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>STX-O</td>
<td>1.35</td>
<td>93.1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>STX-P</td>
<td>1.17</td>
<td>92.2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>STX-Q</td>
<td>0.71</td>
<td>95.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>STX-R</td>
<td>0.66</td>
<td>96.6%</td>
</tr>
</tbody>
</table>
best performance. In practice, we perform a prediction for each minority disk once a day. If any instance of the disk is predicted as a failure, the corresponding disk will be considered as failure.

### 6.6 Cost Benefits

As we have demonstrated so far that TLDFP achieves both high FDR and low FAR. It is easily understandable that improving FDR, i.e., disk failures are correctly predicted, can reduce the probability of data loss occurring. In this section, we discuss the benefits brought by low FAR in ML methods. A FAR will trigger responsive procedures to be launched, e.g., data migration, disk replacement, etc., which incurs extra cost to the IT management. Assume that disk failures are independent events with a probability of $\epsilon_d$ and the number of disks in a data center is $N$. A disk will fail after a certain period of time, which is called disk lifespan. For instance, the Tencent’s data centers assume the lifespan $T$ to be 4-5 years. The disk failure rate over a period of time is dependent on the time $T$ and $\epsilon_d$. Therefore, we use $f(T, \epsilon_d)$ to denote the failure rate of disks in a data center that are failed over a period of $T$. The total number of failed disks in the period $T$ can be expressed as $N \times f(T, \epsilon_d)$ and the total number of failed disks per unit time can be described as:

$$\frac{N \times f(T, \epsilon_d)}{T}.$$  

The total number of good disks is thus given by

$$N \times (1 - \frac{f(T, \epsilon_d)}{T}).$$

Given a FAR, the number of all non-faulty disks which are wrongly predicted to be failed can be described as

$$\text{FAR} \times N \times (1 - \frac{f(T, \epsilon_d)}{T}).$$

Assume $c(\$)$ is the statistic average cost of replacing one disk (including many disks out of the warranty period), which contains the cost of manual replacement $c_1$ and the cost of a new disk $c_2$ as well as the cost $c_3$ caused by copying the data from the failed disk to a new disk. The total additional cost is

$$\text{Cost} = c \times \text{FAR} \times N \times (1 - \frac{f(T, \epsilon_d)}{T}).$$

Therefore, if FAR = 0, the cost will be 0. On the other hand, if FAR = 1, meaning all good disks are wrongly predicted to be failed and replaced, the cost would be maximum. Last but not the least, the total additional cost caused to the data center is proportional to the FAR of the prediction model. The larger FAR is, the more additional cost will be. The traditional machine learning methods and other transfer learning methods, lead to an average of 60 and 4 percent FAR, when they are used to predict disk failures for minority disks, while our TLDFP achieves an average of 0.5 percent FAR, which can be translated to 120X and 8X reduction in additional cost. According to an average cost was estimated as $c = \$426$ in Tencent data centers in 2018, the Tencent company can save about $105 to $1,800 millions per year. In addition, we get a higher FDR with lower FAR which means TLDFP predicts true failed disks as a good disk rarely, reducing the inestimable cost due to disk failures.

### 7 Conclusion

In this paper, we develop a model called TLDFP to effectively predict minority disk failure leveraging the transfer learning based on different storage media, where traditional ML approaches perform poorly. Our main contributions include: (1) we are the first to define minority disk datasets and quantitatively evaluate them through extensive data analysis and experiments in data centers of heterogeneous disk models, (2) we are the first to present a novel method based on KLD values to select proper majority disk models, and (3) we develop a method of making crossing-disk model (HDD, SATA SSD and NVMe SSD) failure prediction, which has important practical applicability as different disk models are gradually placed into the realistic storage systems to replace failed disks. Our experiments with three datasets from real-world data centers have shown that TLDFP well outperforms representative traditional ML methods and existing transfer learning approaches in terms of failure detection rate and false alarm rate. TLDFP achieves on average 96 percent failure detection rate with only 0.5 percent false alarm rate in performing crossing-disk models failure prediction and reduces inestimable cost to data centers.

### Acknowledgments

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### References


---

**TABLE 13**

<table>
<thead>
<tr>
<th>Methods</th>
<th>Metrics</th>
<th>1</th>
<th>1/3</th>
<th>1/2</th>
<th>2/3</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>TLDFP</td>
<td>FDR</td>
<td>95%</td>
<td>32%</td>
<td>24%</td>
<td>8%</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td>FAR</td>
<td>0.7%</td>
<td>0.7%</td>
<td>0.5%</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

\[ \text{FDR} = \frac{\text{TP}}{\text{TP} + \text{FP}} \]
\[ \text{FAR} = \frac{\text{FP}}{\text{TP} + \text{FP}} \]


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Abstract—Realistic, relevant, and reproducible experiments often need input traces collected from real-world environments. In this work, we focus on traces of workflows—common in datacenters, clouds, and HPC infrastructures. We show that the state-of-the-art in using workflow-traces raises important issues: (1) the use of realistic traces is infrequent and (2) the use of realistic, open-access traces even more so. Alleviating these issues, we introduce the Workflow Trace Archive (WTA), an open-access archive of workflow traces from diverse computing infrastructures and tooling to parse, validate, and analyze traces. The WTA includes >48 million workflows captured from >10 computing infrastructures, representing a broad diversity of trace domains and characteristics. To emphasize the importance of trace diversity, we characterize the WTA contents and analyze in simulation the impact of trace diversity on experiment results. Our results indicate significant differences in characteristics, properties, and workflow structures between workload sources, domains, and fields.

Index Terms—Workflow, open-source, open-access, traces, characterization, archive, survey, simulation

1 INTRODUCTION

Workflows are already a significant part of private datacenter and public cloud infrastructures [1], [2]. This trend is likely to intensify [3], [4], as organizations and companies transition from basic to increasingly more sophisticated cloud-based services. For example, 96 percent of companies responding to RightScale’s 2018 survey are using the cloud [5], up from 86 percent in 2012 [6]; the average organization combines services across five public and private clouds. To maintain, tune, and develop the computing infrastructures for running workflows at the massive scale and with the diversity suggested by these trends, the systems community requires adequate capabilities for testing and experimentation. Although the community is aware that workflow traces enable a broad class of realistic, relevant, and reproducible experiments, currently such traces are infrequently used, as we summarize in Fig. 1 (left) and quantify in Section 2. Toward addressing this problem, we focus on improving trace availability and understanding by proposing a new, free and open-access Workflow Trace Archive (WTA), as detailed in Fig. 1 (right) and in the remainder of this work.

The need for workflow traces is stringent [4], [7]. In this work, we adopt the workflow model of Coffman and Graham [8]. In this model, a workflow is considered a directed acyclic graph (DAG) where each vertex represents a task and an edge a computation/data constraint. As such, we do not consider workflow formalisms with iteration (loops) and human interaction, such as BPMN/BPEL [9] and Petri nets [10]. We consider as tasks a broad range of activities, that is, black boxes ranging from simple compute and data operations to entire workflows, recursively.

A workflow trace is a recording of useful, relevant information during the processing of the workflow. Traces can be used to create models with, or used in emulations and simulations to replay the execution of a workflow in a controlled environment, etc. Not only the sheer volume of workloads has increased significantly over time [2], but also the users of datacenters and cloud operations are expecting increasingly better Quality of Service (QoS) from the workflow-management systems, including elasticity, reliability, and low-cost, under strong assumptions of validation [4], [7] and reproducibility [3], [11]. Developing workflow management systems to meet these requirements requires considerable scientific and technical advances and, correspondingly, comprehensive trace-based experimentation and testing. This can be conducted (i) in vivo, i.e., experimenting in live/production settings, (ii) in vitro, i.e., experimenting using emulation, and (iii) in silico, i.e., experimenting in simulation [12].

Testing such systems, especially at cluster and datacenter scale, often cannot be done in vivo, due to
Fig. 1. A visual map to this work: (left) The problem: infrequent use of realistic (≈ 40%) and open-source (≈ 15%) workflow traces in representative articles (see Section 2), which can affect the relevance and reproducibility of experiments for the entire community. (right) Toward an answer: the WTA stakeholders, process, and tools provide the community with open-source traces of relevant workflows running in public and private computing infrastructures.

The current scarcity of traces forces researchers to either use synthetically generated workloads or to use one of the few available traces. Synthetic traces may reduce the representatives and quality of experiments, if they do not match relevant real-world settings. Using realistic traces that correspond to a narrow application-domain may result in overfitting; Amvrosiadis et al. [15] demonstrate this for cluster-based infrastructures. Additionally, a lack of realistic traces may lead to limited or even wrong understanding of workflow characteristics, their performance, and their usage, which hampers the reuse of the systems tested with such workloads [16]. This gives rise to the research question RQ-1: How diverse are the workflow traces currently used by the systems community?

We identify the need to share workflow traces collected from relevant environments running relevant workloads under relevant constraints. Effective sharing requires unified trace formats, and also support for emerging and new features. For example, since the introduction of commercial clouds, clients have increasingly started to ask for better QoS, and in particular have started to increasingly express non-functional requirements (NFRs) such as availability, privacy, and security demands in traces [4], [17]. This leads us to research question RQ-2: How to support sharing workflow traces in a common, unified format? How to support in arbitrary NFRs?

Persuading both academia and industry to release data is vital to address the problems stated prior. We tackle this issue with two main approaches. First, by offering tools to obscure sensitive information, while still retaining significant detail in shared traces. Second, by encouraging the same organization to share the data across its possibly multiple workflow management systems (sources), and by explicitly aiming to collect data across diverse application domains and fields. The availability of diverse data and tools stimulates the benefits of making available such traces, while simultaneously reducing the concerns of competitive disadvantage or of an (accidental) disclosure of sensitive information.

The community is already helping with both approaches, by increasingly focusing on the problem of reproducibility. For example, ACM introduced artifact review and badges to stimulate the release of both software and data artifacts for reproducibility and verification purposes [18]. We add to this community-effort ours, which is scientific in nature: RQ-3: What is the impact of the source and domain of a trace on the characteristics of workflows?

Addressing research questions 1–3, our contribution is four-fold:

1) To answer RQ-1, we conduct the first comprehensive survey of how the systems community uses workflow traces (Section 2). We collect, select, and label articles from top conferences and journals covering workflow management. We analyze the types of traces used in the community, and the domains and fields covered in published studies. To improve reproducibility and promote extensions, we make public all (raw) data used for this survey.

2) To answer RQ-2, we design the WTA for open-access to traces of workloads of workflows (Section 3). We identify a comprehensive set of requirements for a workflow trace archive. A key conceptual contribution of the WTA is the design of a unified trace format for sharing workflows, the first to generalize NFRs support at both workflow- and task-levels. The WTA currently archives a diverse set of (1) real workflow traces collected from real-world environments, (2) realistic workflow traces used in peer-reviewed publications, and (3) workflow traces collected from simulated and emulated environments commonly used by the systems community. WTA also introduces tools to detail and compare its traces.

3) To address RQ-3, we compare key workload characteristics across traces, domains, and sources (Section 4). Our effort is the first to characterize the new trace from Alibaba, and the first to investigate the critical path task length, level of parallelism, and burstiness using the Hurst exponent on workloads of workflows. Overall, the archive comprises 96 traces, featuring more than 48 million workloads containing over 2 billion CPU core hours.

4) To validate our answers to RQs 1–3, we analyze various threats (Section 5). We conduct a trace-based simulation study and qualitative analysis. Our results for the former indicate systems should be tested with different traces to validate claims about the generality of the proposed approach.
2 A SURVEY OF WORKFLOW TRACE USAGE

To assess the current usage of workflow traces in the systems community and the need for a workflow archive, we systematically survey a large body of work published in top conferences and journals, and investigate articles that perform experiments using workflow traces, either through simulation or using a real-world setup. The process and outcome of this survey answer RQ-1.

2.1 Article Selection and Labeling

Selection. Fig. 2 displays our systematic approach to select articles relevant to this survey, based on [19]. First, we collect data from DBLP [20] and Semantic Scholar [21]. We filter them by venue, retaining only articles from the 10 key conferences and journals in distributed systems listed in the caption of Table 1, including TPDS. While not an exhaustive list, this covers a significant part of the systems community. This yields 18,412 articles. Next, we automatically select all articles from the last decade (2009–2018) containing the word “workflow” in either title or abstract, yielding 397 articles. This step provides articles that focus on all aspects of workflows, e.g., scheduling, analysis, and design. Finally, to obtain insights into workflow traces usage, we manually check the 397 articles. Overall, this systematic process yields 104 articles using workflow traces. To highlight the relevance of papers, we use Google Scholar to obtain citation counts. In total the 104 papers have been cited 3,965 times.

Labeling. We label for each of the 104 articles the type of trace usage. For articles explicitly describing their use, we use the label realistic for traces collected from real-world workflow executions. For all others, including workflows extrapolated from real-world data or generated from known statistical distributions, we use the label synthetic.

We further label traces as open-access (or open-source) if they are available online and to a broad audience, and closed-access (or closed-sources) otherwise. In our analysis, we include among the open-access traces only those that are also realistic.

We also label traces by domain and field. Domains are corresponding to the area of study of which the trace originates from. We label sub-domains within these domains as fields. We adopt the domains and fields reported by the respective authors, where mentioned. If the domain or field are not mentioned, yet the application appears in another article by name and with labels, we remain consistent in our labeling by adopting the domain/field from this prior article. We have not encountered cases where an application is labelled as belonging to multiple domains or fields. We identify in articles explicit use of traces from the domains “scientific”, “engineering”, “multimedia”, “governmental”, and “industry”, and from fields such as “bioinformatics”, “astronomy”, “physics”, etc. We further label a trace with uncategorized when its origin remains unexplained.

All data used in this survey is available as open-access data¹ and can be used to verify and extend this survey.

2.2 Types of Traces Used in the Community

We analyze here the types of traces used by the community, with the following Observations (Os):

O-1: Less than 40 percent of articles use realistic traces.
O-2: Only one-seventh of all articles use open-access traces.

Table 1 presents the types of traces used in the community, focusing on realistic (R) and open-access (O) traces. The community uses traces for experiments across both conference and journal articles, across various levels of (high) quality. In contrast to this positive finding, the results indicate that, from the total number of articles using traces at all, the fraction of articles using realistic and even open-access traces is relatively small. Across all venues, only 38 percent of the articles use at least one realistic trace, and only 13 percent of the articles use at least one open-access trace.

These findings match the perceived difficulty in reproducing studies in the field [11], [12], and may hint why so few of these seemingly successful designs are adopted for use in practice [22].

2.3 Workflow Domains and Fields

We analyze the domains and fields from which the community sources workflows, with as main observations:

O-3: The community sources workflows from 5+ domains and 25+ fields.
O-4: Traces containing scientific workflows are used significantly more (20x) than workflows from other domains, e.g., industry and engineering, in the surveyed articles.
O-5: Bioinformatics workflows are the most commonly used, but three other fields exhibit usage within a factor of 5.
O-6: Many traces have uncategorized domain (14 percent) and/or field (31 percent).

Overall, we find that the community uses diverse workflows, sourced from 5+ domains and 25+ fields. We further investigate the distribution of use, per domain and per field. Fig. 3 (top) shows that the scientific domain is over-represented in the literature in the top-five trace domains encountered, due to the large number of available open-access traces and from their conventional use in prior work. In particular, a large portion of the articles use workflow traces from the Pegasus project, which covers the scientific domain. The number of traces in this domain exceeds 200, which is larger than the number of

¹ https://github.com/atlarge-research/wta-analysis
articles in the study as each article uses multiple traces. In contrast, the next-largest domains are industry and engineering, each with less than 10 traces representing less than one-twentieth of the scientific domain.

We remark the positive diversity of the workflow domains, considering that the entire community is tempered by the extreme focus on scientific workflows. This confirms the bias demonstrated by Amvrosiadi et al. [23] with the popular Google-cluster traces. A similar situation appears for fields, but more tempered, as Fig. 3 (bottom) indicates. A large portion of the traces have their domains and fields as “uncategorized” (14 and 31 percent, respectively) which is unhelpful when determining if the proposed solution works in a certain environment.

Overall, the results reveal that the community has a strong bias for one domain (scientific) and favors scientific fields (especially bioinformatics). We conjecture the large amount of open-access data from these fields facilitates this bias. This is consistent with our findings O-4 and O-5, and with the assumption of people selecting traces with equal probability. An alternative is that the domains and fields whose data are used more, share artifacts that are more easily reused and rerun. An example of a well-known initiative for reproducibility in the scientific domain is the MyExperiment repository [24]. To overcome such biases, and to further reduce the large fraction of uncategorized traces evident in both plots of Fig. 3, we posit the community should require that open-access and diverse traces be used in articles claiming the generality of their techniques and indicate the domains and fields of the workflows used.

### 3 The Workflow Trace Archive

In this section, we outline the design of the WTA, the unified trace format used, tools to support consumers with the trace selection according to their use-case, and give a summarized overview of the current contents of the archive. Furthermore, that facilitates the continuous growth of the archive, we provide tools for trace anonymization and a collection of trace parse scripts for different trace sources.

Similar to how the design of experiments is now commonly described in publications in our field, as the setup leading to experimental results, we include an overview of the design process that led to the design presented in this section. Outlining the design and the process that led to the design is important for understanding how the final design came to be and how it fits the intended goal [25].

We started by listing initial requirements (see Section 3.1) that the WTA has to fulfill, and co-evolved the requirements with the development of the solution (the archive). For example, we added explicitly the requirement to provide scripts and datasets to aid users in building their own tools, as we discovered how difficult it was to engineer them from scratch (see Section 3.6). Next, we defined an initial format, centered around a number of unique features, such as non-functional requirements (NFRs) that are missing in other workflow trace formats. We improved this format iteratively, to meet the requirements and/or to pass various thought experiments. For the latter, whenever we encountered a new data-format that was not fully covered by our format, we discussed which properties and/or objects should be added to the format (see Section 3.4). We assessed the trade-off between format comprehensiveness (what to include?) and brevity (what is too much or too complex?) based on personal experience, on the perceived importance of data-fields in literature, and on their frequency of use in other archives. Finally, we designed the analysis tools iteratively, including in them initially our own ideas and then aspects highlighted by other archives, literature reports, and perceived shortcomings.

#### 3.1 Use Cases and Requirements

We foresee four direct use cases for the WTA. First, trace characterization and workload analysis for understanding and tuning systems. As workloads evolve it is important to characterize the changes in, e.g., structure and resource consumption to see if schedulers require change, can be improved or if these changes can be exploited. Such characterizations can provide interesting insights (see Section 4).

Second, experimentation using emulation or simulation. As discussed in Section 1, emulations and simulations may

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**TABLE 1**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Total</th>
<th>FGCS</th>
<th>CCCGrid</th>
<th>TPDS</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>104</td>
<td>37</td>
<td>17</td>
<td>17</td>
<td>33</td>
</tr>
<tr>
<td>R</td>
<td>40 (38%)</td>
<td>13 (35%)</td>
<td>8 (47%)</td>
<td>6 (35%)</td>
<td>13 (39%)</td>
</tr>
<tr>
<td>R+O</td>
<td>14 (13%)</td>
<td>6 (16%)</td>
<td>2 (12%)</td>
<td>3 (18%)</td>
<td>3 (9%)</td>
</tr>
</tbody>
</table>

The venues with > 5 hits have their individual column. The column “Other” shows combined results for conferences with ≤ 5 hits: ATC, CLOUD, CLUSTER, e-Science, Euro-Par, GRID, HPDC, JSSPP, IC2E, ICDCS, ICPE, IPDPS, NSDI, OSDI, SC, SIGMETRICS, and WORKS. Percentages are computed from the total in the corresponding column, e.g., 13 out of 37 for the cell corresponding to row R and column FGCS.
be the only viable option for specific scenarios (e.g., what if?, long-term operational analysis). Having an archive that offers diverse, heterogeneous traces allows for more diverse testing scenarios. Especially when a new scheduler is developed for multiple domains or scenarios, it is important to experiment with diverse workloads covering the scenarios and domains targeted (see Section 5, C-1).

Third, workload and operational models can arise from the characterization and simulation results. In turn, these models can lead to new insights or to new variations to experiment with.

Last, such data can be used for education and training. As systems grow more complex, education and training becomes more important for both students and employees [26]. Models and heterogeneous traces are useful in education, to demonstrate scenarios and to provide hands-on experience.

To meet these use cases, we identify five key requirements for the structure, content, and operation of a useful archive for workflow traces.

R-1: Diverse Traces for Academia, Industry, and Education. Trace archives, such as Google’s and Alibaba’s, offer only workloads from a single domain, e.g., industrial workloads.

We identify as requirement that an archive must include a diverse set of traces to cover a broad spectrum of workflow sizes, structures, and other characteristics, including both general characteristics to many domains and fields, and idiosyncratic characteristics corresponding to only one domain or field. This requirement is based on the conjectures that different traces can have workflows with significantly different characteristics (tested in Section 4) and such differences impact system performance (tested in Section 5, C-1).

Addressing this requirement is important for academia to demonstrate the generality and applicability of a novel approach, for industry to test production-ready systems or to validate techniques proposed by academia [27], and for education to train employees on more complex systems.

R-2: A Unified Format for Workload of Workflows Traces.

To improve the reusability of diverse traces and to support the reproducibility of experimental results, long-term, we identify as a requirement the use of a unified trace format for workloads of workflows. The format must cover a broad set of data about the workloads and about the workflow management systems including: workload metadata; workflow-level data including NFRs; task-level data including per-task NFRs and operational metadata; inter-dependencies between tasks and other operational elements such as data transfers; system-level information including resource provisioning, allocation, and consumption; etc.

Addressing this requirement simplifies trace exchange and integration effort, prevents redundant work for other users, and supports the development of dataset independent tools (expressed as R-3).

R-3: User level adapted insights into Trace Properties.

To improve trace discovery, the archive must provide detailed trace insights adapted to the level of the broad audience, from beginner to expert, as implied by R-1. Broad insights include extrinsic properties, such the number of workflows and tasks, and intrinsic properties, such the workflow arrival patterns and the resource consumption per-task. In contrast, detailed expert-level insights include analysis of single traces at workload-, workflow-, and system-level, and collective analysis across all traces or traces filtered by a feature (e.g., all traces of a domain or field). These properties must be accessible through readily available tools (see R-4) and, possibly, through interactive online reports. Addressing this requirement helps to correlate information across different traces, resulting in better quantitative evidence, intuition about otherwise black-box applications, and understanding that helps avoiding common pitfalls [28].

R-4: Tools for Trace Access, Parsing, Analysis, Validation.

The most important tool is the online presence of the archive itself. The archive must further provide tools to parse traces from different sources to the unified format (see also R-2), to provide insight into traces (see also R-3), and to validate common properties (e.g., the presence of and correctness of properties). An absence of such tools would lead to users unable to select appropriate traces, validate their properties, and compare them.

The archive should further aid users in building more sophisticated tools. Newly built tools can then be added to the selection of tools so more parties can make use of them (contributing to R-5).

R-5: Methods for Contribution.

The archive must reflect the continuous evolution of workflow use in practice, by increasing the coverage of different scenarios. We make a distinction between two types of contribution: (1) traces from a new domain or application-field, and (2) traces, introducing new properties. To facilitate the former contribution, the archive must provide a method for the upload and (basic) automated traces verification. To facilitate the latter, the format must integrate specific provisions that enable upgrades and long-term maintainability, such as adding a version to each component of the format.

Addressing this requirement encourages new and existing contributors to submit new traces. In particular, tools to add new domains are of particular importance, to support emerging paradigms with realistic data.

3.2 Overview of the WTA

We design the WTA as a process and set of tools helping a diverse set of stakeholders. We consider three roles for the WTA community members, outlined in Fig. 1. The contributor supplies, as the legal owner or representative, one or more traces to the WTA. A workflow trace contains historical task execution data, resource usage, NFRs, resource description, inputs and outputs, etc. To fulfill R-5, the WTA team assists the contributor in parsing, anonymizing, and converting the traces into the unified format (Section 3.4), minimizing the risk of competitive disadvantage, and verifying their integrity. WTA fulfills R-1 as it incrementally expands with contributors of traces from different domains with different properties.

The user represents non-expert or expert trace consumers. Non-expert users often need to rely on generic domain or trace properties, whereas the expert users have detailed knowledge of their system and require fine-grained details for selecting the correct trace. In addition, expert users may comment on (missing) properties and may develop new tools, models or other techniques to further compare and rank the traces. Both user types require assistance in selecting
3.3 Workflow Model

There are numerous types of workflow models used across different communities. A 2018 study by Versluis et al. finds DAGs are the most commonly used formalism in computer system conferences [29]. Popular formalisms such as CWL [30] and Condor DAG [31] are also DAG-based. Therefore, for the first design of this archive, we adopt DAGs as the workflow model.

A workflow constructed as a DAG in which nodes are computational tasks and directed edges depict the computational or data constraints between tasks. Entry tasks are tasks with no incoming dependencies and, once submitted to the system, immediately are eligible for execution. Similarly, end tasks are nodes that have no outgoing edges. A collection of workflows submitted to the same infrastructure over a certain period of time is considered a workload.

Although popular, we specifically do not focus in this work on BPMN and BPEL, Petri nets, hyper graphs, general undirected, or cyclic graphs. These formalisms either include business and human-in-the-loop elements [32] or add additional complexity due to having a large set of control structures such as loops, conditions, etc. [9] which we consider out of scope for this work.

Executable formalisms are meant to define what resources and software should be available before execution. Our formalism needs to capture the system state during execution. Both types of formalisms are needed, and are complementary to each other. For example, a person could use the CWL to define and run their workflow, then turn to our formalism and tools to analyze its execution and subsequently improve various operational aspects.

Given the different nature of these formalisms, if we were to extend an existing executable workflow formalism, e.g., CWL, several elements would not be used. This would lead to feature creep. Conversely, the additions made by our formalism could be regarded as feature-creep by the CWL community. This is emphasized by the CWL community currently developing CWLProv [33]. This formalism aims at fully reproducible workflows, including re-execution which is not a goal of the WTA. While promising, CWLProv is still a work in progress; elements such as capturing resource usage (e.g., CPUs and power consumption) are still lacking.

3.4 Unified Trace Format

Creating a unified format (R-2) requires from the designer a careful balance between limiting the number of recorded fields while supporting a diverse set usage scenarios for all stakeholders in Section 3.2. Modern logging and tracing infrastructure can capture thousands of metrics for each machine and workflow-task involved [34], from which the designer must select. We specifically envision support for common system and workflow properties found in the typical scenarios considered in the top venues surveyed in Section 2, such as engineering a workflow engine [35], characterizing the properties of workloads of workflows [36], and designing and tuning new workflow schedulers [37].

Our unified format attempts to cover different trace domains, while preserving valuable information, such as resource consumption and NFRs, contributing to fulfilling R-1 and 3. The full technical description of the format can be found in our technical report [38] and on the WTA website.  

By analyzing the raw data formats, we carefully selected useful properties to include in our unified format, omitting low-level details, such as cycles per instruction, page cache sizes, etc.

Answering RQ-2 and fulfilling R-2, our trace format is the first to support arbitrary NFRs both at task and workflow levels. For example, one of the LANL traces (introduced in Table 2) contains deadlines per workflow and the Google cluster data features task priorities, both are supported by the WTA unified format. Capturing these properties is important to test QoS-aware schedulers.

As depicted in Fig. 4, the WTA format includes seven objects: Workflow, Workflow, Task, TaskState Resource, ResourceState, and DataTransfer. Each of these objects contains a version field, updated whenever the set of properties is altered (R-5).
Each trace is a single workload, consisting of multiple workflows and their arrival process. Workload properties include the number of workflows, tasks, users, domain and field when available, authors list, and resource consumption statistics. Each workload belongs to one or more domains, and contains a description including its source, execution environment, etc.

Each workflow in the workload has a unique identifier, an arrival time, and contains a set of tasks and several properties, including scheduler used, number of tasks, critical path length, NFRs, and resource consumption. Each workflow also has the name of its field of study, when possible. Different related fields constitute a domain.

Each Task has a unique identifier and lists its submission and waiting time, runtime and resource requirements, including required (compute) resource type, memory, network, and energy usage. Additionally, each task provides optional dictionaries for task-specific execution parameters and NFRs. To model dependencies between tasks, the WTA format maintains for each workflow its topology by specifying parents and children per task. Similarly, data dependencies are recorded as a list of data transfers.

Resource objects cover various resource types, such as cloud instances, cluster nodes, and IoT devices. A resource has a unique identifier and contains several properties, such as resource type (e.g., CPU, GPU, threads), number, processor model, memory, disk space, and operating system. An optional dictionary provides further details, such as instance type or Cloud provider. The ResourceState event snapshots periodically the resource state, including availability and utilization. Analogous to the ResourceState, the TaskState records periodically the resource consumption of the task (the Task object records the resource demand).

Each DataTransfer describes a file transfer from a source to a destination task, which can be a local copy on the same resource or a network transfer from a remote source, etc. To support bandwidth analysis, a data transfer introduces submission time, transfer time, and data size. Each data transfer also provides an optional dictionary with detailed event timestamps (e.g., pause, retry).

3.5 Mechanisms for Trace Selection

We address R-3 by assisting archive users in retrieving appropriate traces for their scenarios, using filter and selection mechanisms. The website is the most important such filter and mechanism, containing an overview of all traces, including scheduler used, number of tasks, critical path length, NFRs, and job and task arrival times and runtime distributions (see Section 4). The metrics featured in the report are important by prior studies [39], [40] and enable developers to select traces appropriate for their intended use-case.

3.6 Tools for Analysis and Validation

We provide, online and as separate tools, a detailed report for each trace. Each report includes automatically generated statistics, such as the number of workflows and tasks, then resource properties such as compute, memory, and IO, and job and task arrival times and runtime distributions (see Section 4). The metrics featured in the report are important by prior studies [39], [40] and enable developers to select traces appropriate for their intended use-case.

3.6 Tools for Analysis and Validation

We implement the unified trace format using the Parquet file format and the Snappy compression algorithm. Parquet is a binary file format that is supported by many big data tools such as Apache Spark, Flink, Druid, and Hadoop [41]. Many programming languages also have libraries to parse this format, such as PyArrow for Python and parquet-avro for Java. Snappy	extsuperscript{3} compression reduces the size of the dataset significantly and has low CPU usage during extraction.

Beside trace selection support and to address R-4, the WTA offers several tools to facilitate and incentivize the continuous growth of the archive. Most of these tools required significant engineering effort to develop, due to the typical challenges of big data processing (high volume, noisy data, diverse input-formats, etc.). The WTA simplifies the upload of new traces by providing a set of parsing scripts for different trace sources, such as Google, Pegasus, and Alibaba. Parsing traces can become non-trivial, once they grow both in complexity and size. Such traces require big data tools, such as Apache Spark, and enough resources, a cluster, to compute. Noisy data raise another non-trivial issue: both Google’s and Alibaba’s cluster data contained either anomalous fields, undocumented attributes, and non-DAG workflows. Some of these issues were never discovered by their respective communities and were corrected in our parsing tools. Debugging, filtering, and correcting noisy big data requires significant compute power and detailed engineering.

Because traces may contain sensitive information, the WTA offers a trace anonymization tool, which supports users to automatically replace privacy and security-related information, to avoid an accidental reveal of proprietary information. Specifically, to remove sensitive information from trace files, we use two common techniques [42], culling and transforming. Culling is done during trace conversion, by omitting parts of the raw trace data which do not match our workflow trace format. For the transformation, as presented in Table 3, our anonymization tool automatically scans the

<table>
<thead>
<tr>
<th>Trace Anonymization Methods Used in WTA Tools</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP Encode IPv4 addresses</td>
<td>IP</td>
</tr>
<tr>
<td>Mail and host Obfuscate mail and host names</td>
<td>Mail and host</td>
</tr>
<tr>
<td>File paths Hide file paths in Linux and Windows format</td>
<td>File paths</td>
</tr>
<tr>
<td>Executable files Encode executable file names, e.g., py, sh, exe, jar</td>
<td>Executable files</td>
</tr>
<tr>
<td>All files Hide all file names, ending with 2, 3, or 4 letters</td>
<td>All files</td>
</tr>
<tr>
<td>Keywords Anonymize a list of custom keywords</td>
<td>Keywords</td>
</tr>
<tr>
<td>All Apply all obfuscation methods listed above</td>
<td>All</td>
</tr>
</tbody>
</table>

3. https://github.com/google/snappy
Several technological each workload is uniquely identified by a combina-

5.0

W

= partially available, blank = not available, and Task details = individual task information.

<table>
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<th>Network</th>
<th>Energy</th>
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</tr>
</tbody>
</table>

Legend: ✓ = available, ~ = partially available, blank = not available, and Task details = individual task information.

3.7 Current Content

Having a diverse set of traces available is necessary to use in experimentation. When using traces in experimentation, different traces should be used to prove generality of the proposed approach (see Section 5). Gathering and parsing raw logs and other traces requires significant computing effort. Using 16 nodes (32 eight-core Xeon E5-2630 v3 and 1TB RAM) from the Dutch DAS5 super computer [44], several traces require up to a day to compute using big data tools such as Apache Spark. In total, the WTA team spent more than two person months on converting traces to the unified trace format. By offering these parse scripts and the data, we contribute to R-4.

The WTA features currently 96 workloads from 10 different sources, with over 48 million workflows and 2 billion CPU core hours. All of them are available on the WTA archive website. Each workload is uniquely identified by a combination of the following properties if available: source, runtime environment, application, and application parameters [45].

These tools help combating perceived barriers to share data described by Sayogo et al. [43]. Several technological barriers are addressed by using a unified format and validation (data architecture, quality, and standardization), Legal and policy barriers are more difficult to address. Our anonymization tool aids in overcoming the data protection barrier, yet legal and other enforced policies may require tailored solutions.

Besides offering these tools, the WTA also hosts the trace data, addressing logistic and economical barriers. The increasing focus on sharing data artifacts by the community, is lowering the barrier regarding competition for merit and reputation for quality and bolsters the culture of open sharing. Finally, each trace has its own DOI by also uploading it to Zenodo which can be cited and thus provides authors with the appropriate credits (incentive barrier).

patterns. Our characterization reveals significant differences between workloads from different domains and sources. Such differences further support our claim that the community needs to look beyond just scientific workloads, and consider a wider range of domains and sources for experimental studies when developing workflow management systems aimed at multiple domains or for general applicability.

We present in this section only detailed insights that lead to new observations for the community. We include in our technical report other types of analysis, such as task and workflow inter-arrival times, task and workflow runtimes, and their breakdown per domain and source [38].

4.1 Structural Patterns

O-7: Scientific, industrial, and engineering workflows exhibit various structural patterns, but at least 60 percent of tasks in a domain match the dominant pattern of that domain.

O-8: Industry workflows stand out by exhibiting primarily scatter patterns, as opposed to pipeline operations.

This characterization quantifies five structural patterns in workflows often used by researchers [46]: scatter (data distribution), shuffle (data redistribution), gather (data aggregation), pipeline, and standalone (process). Investigating these structural patterns is important to understand the types of applications being executed and tune a system’s performance. We exclude from this analysis the LANL, Two Sigma, and Google traces, which lack structural information, that is, task parent-child relationship information.

Fig. 5 depicts the structural patterns found per domain. From this figure, we observe that in each domain a dominant pattern emerges that accounts for 61–85 percent of tasks. In the scientific and engineering domains, the majority of tasks are simple pipelines. Interestingly, the industrial workflows include primarily scatter operations. This observation matches known properties of the Alibaba trace, which accounts for over 99 percent of tasks with structural information we analyzed in this domain. In particular, the Alibaba trace includes MapReduce jobs, each consisting of many “map” tasks (scatter operations) and a smaller number of “reduce” tasks (gather operations).

4.2 Arrival Patterns

O-9: From all domains, industrial traces show on average orders of magnitude higher rates of task arrival.

O-10: Scientific traces can show high variability in task arrival rates, unlike industrial and engineering traces.

O-11: Two Sigma shows a typical workday diurnal pattern.

To investigate the weekly trends that may appear in workload traces, we depict in Fig. 6 for several traces the average number of tasks that arrive per day of the week. We omit the Askalon new source from the hourly task-arrival plot as they contain 4 or 5 data points, which is too few to plot a trend. We observe that traces have significantly different arrival rates and patterns. The Alibaba trace features the highest task arrival rates, peaking at over 10,000,000 tasks per hour. Google and the Two Sigma workloads follow with 100-10,000 tasks per hour. This shows that industrial workloads included in this work have significantly more tasks per hour than the other compute environments, which agrees with companies such as Alibaba and Google operating at a global scale. The non-industrial traces show significant fluctuations throughout the week, whereas both Alibaba and Google do not. This might be due to the global, around-the-clock operation of Alibaba’s and Google’s services, which can lead to a more stable task arrival rate.

To observe differences in daily trends, we depict the average task rate per hour of day in Fig. 7. This figure reaffirms our observation that the two largest traces–Alibaba and Google–have a relatively stable arrival pattern throughout the day. In contrast, the Two Sigma 1 trace exhibits a
typical office hours pattern; task arrival rates increase around hour 7 and start dropping around 17. The same pattern occurs to a lesser extent in the Two Sigma 2 trace. The highly variable arrival rates of tasks in the LANL traces, as observed in Fig. 6, are also evident in our analysis of daily trends. We study this in more depth in Section 4.3.

4.3 Burstiness

O-12: Most traces investigated exhibit bursty behavior within small window sizes.

O-13: The LANL trace exhibits maximum burstiness at medium window sizes.

O-14: The largest traces (Alibaba and Google) exhibit uniquely bursty behavior: low burstiness at small and high burstiness at large, window sizes.

To investigate if workloads expose bursty behavior, a special kind of arrival pattern, the Hurst exponent $H$ is used. $H$ quantifies the effect previous values have on the present value of the time series. A value of $H < 0.5$ indicates a tendency of a series moving in the opposite direction based on the previous values, and thus exhibit jittery behavior (sporadic burst). A value of $H > 0.5$ indicates a tendency to move in the same direction, and thus towards well defined peaks (sustained burst). When $H = 0.5$, the series behaves like a random Brownian motion.

In this experiment, we inspect bursty behavior by computing the Hurst exponent for task arrivals. The results of this experiment are visible in Fig. 8. From this figure, we observe most traces depict bursty behavior at least for one of small, medium, and large window size. They are also not bursty for at least one window size. This is expected, as in most systems task arrivals vary at (sub-)second interval. Interestingly, LANL traces exhibit most bursty behavior at medium window sizes. This might be due to national laboratories workflows being submitted in batches. A batch of tasks is submitted all at once, leading to a burst. But, the batch itself is processed at a constant rate. The workload is also stable over longer time periods as evidenced by $H \approx 0.5$ for larger windows. Finally, the two largest traces in this work, Alibaba and Google, exhibit increasingly burst behavior for larger windows. This indicates that for larger arrival times, the workloads (in absolute numbers) vary more than for the other sources. This matches the observations in Section 4.2.

4.4 Parallelism in Workflows

O-15: Task parallelism per workflow can differ significantly between workload domains and sources.

O-16: Industrial workflows exhibit the highest level of parallelism.

O-17: Out of all sources, Alibaba workflows have the highest level of parallelism, followed by Pegasus and WorkflowHub.

With the structural patterns observed, we investigate if the large occurrence of the pass-through patterns expresses in a high level of parallelism. The level of parallelism indicates how many tasks can maximally run in parallel for a given workflow, provided sufficient resources. Fig. 9 depicts the approximated level of parallelism per domain. The approximation algorithm used produces results very close to the true level of parallelism as demonstrated by Ilyushkin et al. [47]. From this figure, we observe the industrial domain exhibits the highest level of 99th percentile parallelism, up to hundreds of thousands of tasks. This is likely a consequence of the many MapReduce workflows, which are highly-parallel by nature, that are present in the Alibaba trace. Alibaba also contains bag of tasks workflows, which by nature have a high parallelism. Scientific workflows exhibit low median parallelism but high 99th percentile parallelism, featuring levels of parallelism up to thousands of tasks. Engineering traces exhibit a moderate amount of
median parallelism, between industry and scientific, with at most 1000 concurrent running tasks.

Fig. 10 shows the level-of-parallelism per source. From this figure, we observe that Alibaba exhibits the highest levels of parallelism, as discussed previously. Second are the Pegasus and WorkflowHub workflows. These sources contain a variety of scientific applications, commonly known for their parallel structures, as observed in Section 4.1. Other traces demonstrate less parallelism, with up to 1000 concurrent running tasks. As Shell exists entirely of sequential pipelines, the source does not exhibit any variation.

4.5 Limits to Parallelism in Workflows

O-18: Workflows from the scientific domain have significantly different critical-path lengths.

O-19: The amount of tasks on the critical path is the highest for engineering workflows.

O-20: Although highly parallel, industrial workflows exhibit longer critical paths than scientific workflows.

The critical path (CP) refers to the longest sequence of dependent tasks in a workflow, from any entry task to any exit task. By quantifying the CP length, we investigate if workflow runtimes are primarily dominated by a few heavy tasks, or by many small tasks. Fig. 11 presents the results of this characterization per workload domain. From this figure we observe the CP length for engineering workflows is the highest. This matches with the parallelism observations in Sections 4.1 and 4.4. Interestingly, even though industrial workflows are often highly parallel, their critical paths are often longer than those of scientific workflows. This indicates that industrial workflows are bigger in size than scientific workflows, which our data supports.

Fig. 12 presents the results of CP characterization per workload source. From this figure we observe the CP length differs significantly per trace. Based on the prior findings, the engineering traces are expected to show longer critical paths. As we can observe, the Askalon old traces contain workflows with the longest critical path. Alibaba workflows also exhibit long critical paths, indicating their workflows next to being highly parallel, also contain a lot of tasks with stages. More concentrated, the other traces exhibit lower critical path lengths, yet the traces are still clearly distinct. As the Shell trace contains solely sequential workflows, the critical path length is one.

5 Addressing Challenges of Validity

In this section, we discuss challenges to the validity of this work. We address the challenges through either trace-based simulation (the first) or argumentation (the others).

Challenge C-1. Trace diversity does not impact the performance of workflow schedulers. As outlined in Sections 3.7 and 4, the WTA traces are diverse. However, what is the impact of trace diversity?

To demonstrate the impact of trace diversity on scheduler performance, we conduct a trace-based simulation study. The simulator used is an optimized version of DGSim [48] which we publish as open-access artifact. We simulate workloads from five sources using two scheduler configurations. We equip the simulated scheduler with either the first-come first-serve (FCFS) or the shortest job first (SJF) queue sorting policy. For both scheduler configurations, we further use a best-fit task placement policy. We do not use a fixed resource environment to prevent bias when sampling or scaling traces [28]. Instead, we tailor the amount of available resources for each trace to reach roughly a 70 percent resource utilization on average, based on the amount of CPU (core) seconds of trace and its length. Although ambitious, 70 percent resource utilization is achievable in parallel HPC environments [49] and can be seen as a target for cloud environments. To evaluate the performance of each scheduler, we use three metrics commonly used to assess schedulers’ performance [50], [51]: task response time (ReT), bounded task slowdown (BSD, using a lower bound of 1 second), and normalized workflow schedule length (NSL, the ratio between a workflow’s response time and its critical path). The entire experiment, including software and data, can be reproduced on Code Ocean.

We report the performance of each simulated scheduler in Table 6 per source. From this table we observe significant differences between schedulers and trace sources. In particular,
we find that the relative performance of schedulers differs between trace sources. For example, SJF outperforms FCFS on the normalized scheduled length metric by up to two orders of magnitude on traces from Askalon Old and Pegasus. In contrast, on traces from Askalon New and Shell, the scheduling policies perform similarly. For other metrics, these differences are present, but less pronounced. SJF performs better than FCFS on response time and slowdown for each trace source, but the differences in performance between the schedulers vary greatly across traces.

Overall, we kept the working environment fixed per trace, yet obtained significantly different results depending on the scheduler and input trace. Thus, our trace-based simulations give practical evidence that researchers require experimenting with different traces to claim generality and feasibility of their proposed approaches.

C-2. Limited venue selection in the survey. Besides omitting venues that yielded no results on our initial query, we made sure that journals, workshops, and conferences were covered at various levels in terms of quality. The selected venues are highly ranked in several of the available rankings, including CORE, Google Scholar, and AMiner. As these rankings use different metrics to define the top-ranking, we made a selection that covers different types of venues that also match our experience in terms of quality, see the list in Table 1. We believe this covers the field of systems community to a degree where conclusions can be drawn from. We specifically focus on articles published in the systems communities as specialized communities, e.g., bioinformatics, focus on systems that solve domain-specific problems, but rarely conduct in-depth experiments, including trace-based, to test the system-level capabilities and behavior.

C-3. Level of data anonymization. The Google team published interesting workflow data [42], but their anonymization approach, of normalizing values of both resource consumption and available resources, reduces significantly the usability of traces and the characterization details they provide. We argue this type of anonymization is not preferred. When available resources per machine, e.g., available disk space, memory, etc., and resource consumption numbers are normalized, reusing traces for different environments becomes difficult. Researchers then need to make assumption on what kind of hardware the workflows were executed as done in the work of Amvrosiadis et al. [15] or need to assume a homogeneous environment. Instead, obfuscation techniques, such as multiplying both consumption and resources by a certain factor, allow for relative comparisons and the possibility to replay scheduling the workload on the resources while still concealing the original data.

C-4. The Workflow Trace Format. A fourth challenge is the properties included in the workload trace format. For each encountered property in other formats, we carefully decided whether to include it or not. Low-level details such as page caches are omitted to not complicate unnecessarily the traces. If future work demands change, the versioning schema per object will allow for these additions. In defining the fields of our trace format, we also looked at a variety of workflow specification languages and formalisms, from the very generic (e.g., BPMN/BPEL and Petri net) to the executable workflow formalisms (e.g., CWL and DAX).

6 RELATED WORK

We survey in this section the relevant body of work focusing on trace archives and on characterizing workloads. Differently from other archives, the WTA focuses on workflows of workflows, preserving workflow-level arrival patterns and task inter-dependencies not found in other archives. Differently from other characterization work, ours is the first to reveal and compare workflow characteristics across different domains and fields of application.

Open-Access Trace Archives. Closest to this work is WorkflowHub [14], which archives traces of workflows executed with the Pegasus workflow engine and offers them in a unifying format containing structural information. WorkflowHub also provides a tool to convert Pegasus execution logs to traces, similar to our parsing tools. Different from this work, WorkflowHub’s traces include a single workflow and thus not a workload with a job-arrival pattern. WorkflowHub also does not provide statistical insights per trace and thus, they do not meet requirements R-1 and R-3, and only partially meet R-4.

Also relatively close to this work, the ATLAS repository maintained by the Carnegie Mellon University [15] contains two traces (the S3 traces in this work), with other two traces announced but not yet released (as announced, the S7 traces in this work). None of their published traces contains task-interdependency data, so, although overlapping with our S3 and S7, the ATLAS work is different in scope and in particular does not address workflows. Further, they do not consider different domains nor fields, and their archive lacks a unified format, statistical insights, selection mechanisms, and tooling—thus, they do not meet our requirements R1–4.

Other trace-archives with similarities to this work include the MyExperiment archive (ME) [24], the Parallel Workloads Archive (PWA) [52], and the Grid Workloads Archive (GWA) [53]. ME stores workflow executables, and semantic and provenance-data, but not provide execution traces as WTA does and thus has different scope. The PWA includes

<table>
<thead>
<tr>
<th>Source of Trace</th>
<th>Policy</th>
<th>Asklon Old</th>
<th>Asklon New</th>
<th>Pegasus</th>
<th>Shell</th>
<th>SPEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. ReT FCFS</td>
<td>2.02 · 10^3 s</td>
<td>167 s</td>
<td>2.43 · 10^3 s</td>
<td>9.76 s</td>
<td>491 s</td>
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<td>2.12 · 10^3 s</td>
<td>9.52 s</td>
<td>248 s</td>
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</tr>
<tr>
<td>Avg. BSD FCFS</td>
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<td>61.5 s</td>
<td>1.31 · 10^3 s</td>
<td>1.13 s</td>
<td>47.4</td>
<td></td>
</tr>
<tr>
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<td>0.14 · 10^4 s</td>
<td>11.6 s</td>
<td>0.10 · 10^3 s</td>
<td>1.06 s</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>Avg. NSL FCFS</td>
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<td>2.50 s</td>
<td>2.35 · 10^3 s</td>
<td>1.12 s</td>
<td>13.9</td>
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</tr>
<tr>
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<td>0.06 · 10^3 s</td>
<td>1.07 s</td>
<td>1.78</td>
<td></td>
</tr>
</tbody>
</table>

Lower values are better.
traces collected from parallel production environments, which are largely dominated by tightly-coupled parallel jobs and, more recently, by bag-of-tasks applications. The GWA includes traces collected from grid environments; differently from this work, these traces are dominated by bag-of-tasks applications and by virtual-machine lease-release data.

Workload Characterization, Definition, and Modeling. There is much related and relevant work in this area, from which we compare only with the closely related; other characterization work does not focus on comparing traces by domain and does not cover a set of characteristics as diverse as this work, leading to so many findings. Closest to this work, the Google cluster-traces have been analyzed from various points of view, e.g., [54], [55], [56]. Amvrosiadis et al. [15], [23] compare the Google cluster traces with three other cluster traces, of 0.3-3 times the size and 3-60 times the duration, and find key differences; our work adds new views and quantitative data on diversity, through both survey and characterization techniques. Bharathi et al. [46] provide a characterization on workflow structures and the effect of workflow input sizes on said structures. Five scientific workflows are used to explain in detail the compositions of their data and computational dependencies. Using the characterization, a workflow generator for parameterized workflows is developed. Juve et al. [36] provide a characterization of six scientific workflows using workflow profiling tools that investigate resource consumption and computational characteristics of tasks. The teams of Feitelson and Lospup have provided many characterization and modeling studies for parallel [57], grid [58], and hosted-business [59] workloads; and Feitelson has written a seminal book on workflow modeling [60]. In contrast, this work addresses in-depth the topic of workloads of workflows.

7 CONCLUSION AND ONGOING WORK

Responding to the stringent need for diverse workflow traces, in this work we propose the Workflow Trace Archive (WTA), which is an open-access archive containing workflow traces.

We conduct a survey of how the systems community uses workflow traces, by systematically inspecting articles accepted in the last decade in peer-reviewed conferences and journals. We find that, from all articles that use traces, less than 40 percent use realistic traces, and less than 15 percent use any open-access trace. Additionally, the community focuses primarily on scientific workloads, possibly due to the scarcity of traces from other domains. These findings suggest existing limits to the relevance and reproducibility of workflow-based studies and designs.

We design and implement the WTA around five key requirements. At the core of the WTA is an unified trace format that, uniquely, supports both workflow- and task-level NFRs. The archive contains a large and diverse set of traces, collected from 10 sources and encompassing over 48 million workflows and 2 billion CPU core hours.

Finally, we provide deep insight into the WTA traces, through a statistical characterization revealing that: (1) there are large differences in workflow structures between scientific, industrial, and engineering workloads, (2) our two biggest traces— from Alibaba and Google—have the most stable arrival patterns in terms of tasks per hour, (3) industrial workflows tend to have the highest level of parallelism, (4) the level of parallelism per domain is clearly divided, (5) engineering workloads tend to have the most tasks on the critical path, (6) the three domains inspected in this work show distinct critical path curves, (7) in order to claim generality of an approach, one should test a system with a variety of traces with different properties, possibly from different domains.

In ongoing work, we aim to attract more organizations to contribute real-world traces to the WTA, and to encourage the use of the WTA content and tools in educational and production settings. One of our goals is to develop a library system administrators can integrate into their systems to generate traces in our format. Our preliminary experience with this learns that developing such a library, even for a single system, requires significant engineering effort and is thus left for future work. We aim to support other formalisms in the future, including directed graphs, BPMN workflows, etc. based on the community’s needs. Investigating if formalisms such as CWLProv can be used to further enhance the archive’s content, possibly by merging, is another interesting item for future work. Finally, we aim to improve the trace format and statistics we report for each trace, based on community feedback.

REPRODUCIBILITY STATEMENT

We support reproducible science. A full description on how to reproduce our findings can be found in our technical report [38]. The WTA datasets are available online on the archive’s website https://wta.atlarge-research.com/. The WTA tools, simulator, and parse scripts and survey data are available as free open-source software at https://github.com/atlarge-research/wta-tools, https://github.com/atlarge-research/wta-sim, and https://github.com/atlarge-research/wta-analysis, respectively. The experiment conducted in Section 5, C-1 can be reproduced using our Code Ocean capsule available at https://doi.org/10.24433/CO.8484557.v1.

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For more information on this or any other computing topic, please visit our Digital Library at www.computer.org/csdl.
Boosting the Performance of SSDs via Fully Exploiting the Plane Level Parallelism

Congming Gao®, Liang Shi®, Kai Liu®, Chun Jason Xue, Jun Yang, and Youtao Zhang®, Member, IEEE

Abstract—Solid state drives (SSDs) are constructed with multiple level parallel organization, including channels, chips, dies, and planes. Among these parallel levels, plane level parallelism, which is the last level parallelism of SSDs, has the most strict restrictions. Only the same type of operations that access the same address in different planes can be processed in parallel. In order to maximize the access performance, several previous works have been proposed to exploit the plane level parallelism for host accesses and internal operations of SSDs. However, our preliminary studies show that the plane level parallelism is far from well utilized and should be further improved. The reason is that the strict restrictions of plane level parallelism are hard to be satisfied. In this article, a from plane to die parallel optimization framework is proposed to exploit the plane level parallelism through smartly satisfying the strict restrictions all the time. In order to achieve the objective, there are at least two challenges. First, due to that host access patterns are always complex, receiving multiple same-type requests to different planes at the same time is uncommon. Second, there are many internal activities, such as garbage collection (GC), which may destroy the restrictions. In order to solve above challenges, two schemes are proposed in the SSD controller: First, a die level write construction scheme is designed to make sure there are always \( N \) pages of data written by each write operation. Second, in a further step, a die level GC scheme is proposed to activate GC in the unit of all planes in the same die. Combining the die level write and die level GC, write accesses from both host write operations and GC induced valid page movements can be processed in parallel at all time. To further improve the performance of SSDs, host write operations blocked by GCs are suggested to be processed in parallel with GC induced valid page movements, bringing lesser waiting time cost of host write operations. As a result, the GC cost and average write latency can be significantly reduced. Experiment results show that the proposed framework is able to significantly improve the write performance without read performance impact.

Index Terms—SSD, parallelism, storage, scheduling, performance improvement

1 INTRODUCTION

Solid state drives (SSDs) are widely adopted in modern computer systems, ranging from embedded systems, personal computers, to large servers in data centers. SSDs have many advantages, such as shock resistance, high random access performance, and low power consumption [1]. An SSD usually consists of multiple channels with each channel having multiple chips, each chip having multiple dies, and each die having multiple planes [2], [3]. To achieve high performance, the prior studies strive to exploit the parallelism at channel/chip/die/plane levels so that multiple accesses, such as reads, writes, and erases, can be processed in different parallel units simultaneously [4], [5], [6].

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Nevertheless, the parallelism at the last level, referred to as plane level parallelism, exhibits strict restrictions—for two operations that can be issued simultaneously to two different planes, they not only need to be of the same type (i.e., read or write) but also need to have the same in-plane address (i.e., the same offset within each plane), making it challenging to explore as shown in recent studies [7], [8], [9], [10], [11], [12]. For example, to concurrently write two planes, their write points need to be aligned. Unfortunately, a host often sends uneven numbers of write requests to different planes [9] and the activities originated from SSDs (e.g., garbage collection operations) are often imbalanced across different planes [9], [13]. Such asynchronicity leads to sub-optimal exploration of plane level parallelism and prevents modern SSDs from achieving further performance improvement.

To exploit plane level parallelism, Tavakkol et al. proposed TwinBlk to write data to the different planes in a die in a round-robin fashion [11] such that concurrent writes can be issued to different planes at the same time. However, the write points from different planes may be mis-aligned due to (1) single-page write operations; (2) GC or wear leveling activities originated inside the SSD [9], [13], [14], [15], disabling the concurrent writes in these cases. To reduce GC-induced write point mis-alignment, Shahidi et al. proposed ParaGC to activate GC from all planes of the same die at the same time [9], which opportunistically exploits the plane level parallelism when all the pages at the same address of
different planes are valid. TwinBlk can concurrently activate multiple GCs by choosing multiple blocks with the same offset in different planes. It cannot process all valid page movements in parallel when not all paired pages are valid. Superpage enabled SSDs [1], [16], [17] strip requests to all planes in a die, which increases the number of sequential writes as well as concurrent write opportunities. However, activating GC in one plane introduces mis-aligned free blocks so that subsequent requests cannot be processed in parallel. In summary, a major limitation of existing studies is that they explore plane level parallelism passively, making it difficult to satisfy the access restrictions all the time. In particular, it is challenging to construct multi-plane command after GC and/or wear leveling mis-align the write points in different planes.

In this paper, we propose SPD, an SSD from plane to die parallel optimization framework, to fully exploit the plane level parallelism of SSDs for performance improvement. We summarize our contributions as follows.

- We propose SPD to treat all planes (e.g., \(N\) planes) in a die as a single unit so that a die write results in \(N\) page writes while a die read fetches \(N\) or fewer pages. Similarly, internal activities, e.g., GC, get triggered for \(N\) blocks from different planes that have the same in-plane block address. To our best knowledge, this is the first work on actively maintaining aligned write points for multiple planes in a die combining writes from both host and internal activities for all the time;
- We then propose die level write construction and die level GC schemes to fully exploit the plane level parallelism enabled by SPD. The write construction scheme is to construct write operation with \(N\) pages of data and issue them to a die at once; The die level GC scheme is to process valid page movements, aligning the write points of all planes in the same die.
- To further improve the write performance, we propose SPD+, which is designed to process host write operations blocked by die level GCs in parallel with GC induced valid page movements. Therefore, host write operations are able to be completed with lesser waiting time.
- We evaluate the proposed approach using a significantly extended SSDSim [10] and compare it to the state-of-the-arts. The experimental results show that proposed approach is able to significantly improve write performance of SSDs without read performance impact.

The rest of this paper is organized as follows: In Section 2, the background is presented. In Section 3, the problem statement is presented. In Section 4, the SPD framework is presented. In Sections 5 and 6, the experiment setup and evaluations are presented. In Section 7, related works are discussed. Finally, the work is concluded in Section 8.

2 BACKGROUND

In this section, we briefly discuss the background, including SSD organization, advanced SSD commands, parallelism, and garbage collection (GC).

2.1 SSD Organization

A modern SSD usually consists of multiple channels with each channel containing multiple flash chips. Within each flash chip, there are multiple dies with each die containing multiple planes. Fig. 1 illustrates the organization of a typical SSD that has 4 channels, 2 chips per channel, 2 dies per chip, and 2 planes per die. The SSD parallelism can be exploited at channel/chip/die/plane levels, which have one major focus of previous studies for performance improvement [2], [13], [18]. To manage the flash memory as well as to explore the parallelism, an SSD controller comprises several components, including flash translation layer (FTL), data allocation (DA), wear leveling (WL), garbage collection (GC).

The FTL is to manage the mapping between logical addresses and physical addresses. Based on the operation granularity, there are three types of mapping schemes, i.e., page mapping [4], block mapping [19], and hybrid mapping [20] [21], [22]. In this work, we assume the widely adopted page mapping as it tends to have its better performance. The data allocation is to determine the allocations of channel, chip, die and plane for write operations. The wear leveling is to distribute written data evenly to flash pages for prolonging the SSD lifetime [23], [24]. Since flash memory cannot reprogram a programmed flash page before executing an erase operation to reclaim the whole block, modern SSDs widely adopt out-of-place-update scheme for data updating. To reclaim invalid pages, GC is activated while the number of free pages drops below a predefined threshold.

In addition, modern SSDs widely equip a built-in Random Access Memory (RAM), referred to as the SSD buffer, within SSD controller for temporarily storing hot data and metadata. Since the access latency of RAM is much smaller than that of flash memory, buffer-equipped-SSDs can provide much better performance for data hit in the buffer [25], [26], [27], [28].

2.2 Parallelism and Advanced Commands

The hierarchical SSD architecture provides four level parallelism, from channel, chip, die to plane. For channel and chip level parallelism, data can be processed in different chips in parallel. The parallelism of these two levels is naturally supported by SSDs while that of the rest two levels are supported by advanced commands [9], [10], [12], [18], [29]. The die and plane level parallelism is also referred to as internal parallelism [3].

For die level parallelism, operations issuing to the same chip but different dies can be processed in parallel with
interleaving command [9], [10]. There is no restriction on when to use the interleaving command. For the last level parallelism, plane level parallelism may be exploited to further improve performance through processing operations concurrently on different planes of the same die. Due to circuit restrictions [7], as shown in the open NAND flash interface (ONFI) standard specification [8], the plane level parallelism can be exploited when satisfying the two operation type and in-plane address restrictions of multi-plane command. A multi-plane command improves plane utilization as it operates multiple planes within the same die in parallel and only takes the time to finish one operation. However, when the restrictions cannot be met, it processes different planes sequentially to the requested operation. In particular, an operation processed on one plane blocks other planes of the same die from servicing other operations.

2.3 Garbage Collection

Within flash memory, pages can not be updated in place [7]. In order to solve this issue, data is always updated out of place by programming updated data in another block and invalidating original version. Invalid pages can not be reused until they are erased. With the increasing of invalid pages’ number and reduction of free pages’ number, garbage collection (GC) is triggered for reclaiming invalid pages [1], [13]. The process of GC can be described as follows: First, a victim block is selected; Second, valid pages in the victim block are read and wrote to free pages in other blocks; Third, the victim block is erased. During this process, valid page movement is performed page by page, which is able to introduce significant time cost of SSD system while the incoming host requests are blocked and delayed [13].

To solve such a problem, a simple and effective approach, termed greedy GC, has been proposed and widely used to reduce the time cost of valid page movement [22], [30]. Greedy GC is designed to minimize the cost of valid page movement by selecting block with minimum number of valid pages. Thus, the total GC induced time cost can be minimized so that fewer host requests are blocked and delayed, increasing the performance of SSDs. In this work, greedy GC also is considered as a typical GC algorithm while other GC algorithms [31], [32], [33] also can be applied in proposed approach without loss of generality.

3 Problem Statement

In this section, we study the challenges in exploiting the plane level parallelism, which comes mainly from the restrictions of the multi-plane command [9], [18]. For clarity, we focus on write operations as they are much slower than read operations and thus have larger impact on the overall performance.

We next conduct a study on the operations to an SSD with each die consisting of two planes. Without losing generality, the non-GC operations that access the same die may be categorized to the following four cases.

Case 1: The operations are issued to one plane only (Single Write). Such write operations introduce unaligned write points across different planes;

Case 2: Two different types of operations are issued to the two planes of the same die. Due to the operation type restriction, the operations are not allowed to be processed in parallel;

Case 3: Two same type operations with unaligned in-plane addresses are issued to two planes of the die (Unaligned Writes). Due to the address restriction, the operations cannot be processed in parallel either;

Case 4: Two same type operations with aligned in-plane addresses are issued to two planes (Parallel Writes), which can be processed in parallel with the support of multi-plane command.

Among these four cases, Case 2 can not be avoided due to the circuit restriction of multi-plane command while mixed types of operations being issued to different planes of the same die. Then, the numbers of operations falling in Case 1, Case 3 and Case 4 are collected and reported in Fig. 2. The experiment setting details can be found in the experiment section. We have two observations from the results: (i) plane level parallelism is far from well utilized; (ii) a large percentage of write operations issued to the die are unaligned write operations, which can be exploited for performance improvement.

To solve the issue of unaligned write points across planes, a naive solution is to write data at the aligned points greedily [10]. However, if the current write points are unaligned, writing data at the aligned points lead to wasted space. For example, we assume there are two planes per die, one block per plane, and six pages per block, as shown in Fig. 3a. In Fig. 3a-(1), the current write points are unaligned. Traditionally, if two write operations, W1 and W2, are issued to the two planes in the same die, they will be processed sequentially. If they are written to the aligned pages, a free page in Plane 1 would be wasted, as shown in Fig. 3a-(2). In this work, we strive to design a write construction scheme to align the write points in each die.

Apart from host requests, internal SSD activities, e.g., GC, also introduce non-negligible performance impact from the unaligned write points across planes [13], [14]. Given a die with multiple planes, if one plane activates GC, the other planes cannot be accessed before this GC finishes. To solve this problem, Shahidi et al. proposed to activate GCs in all planes at a time so that GC induced time cost can be overlapped [9]. To avoid significant parallel GC induced write amplification, ParaGC proposed to select a block containing most invalid pages in a plane first. Then, if the number of invalid pages in the paired block resides in the paired plane is large too, these two blocks can be reclaimed by GCs simultaneously. Otherwise, only one block is processed by GC. However, such a solution faces two issues: First, since the
number of valid pages in paired blocks are different, ParaGC may lead to unaligned write points across different planes after valid page movements. For example, in Fig. 3b, after moving valid pages in each plane in Fig. 3b-(1), new write points (WP in the figure) become unaligned, as shown in Fig. 3b-(2). Second, if only one block is occupied by GC, write points will be unaligned while this block is erased and switched as a free block and its paired block still has not been reclaimed. That is, to maintain aligned write points at all time, we need to construct multi-plane oriented writes for both of host requests and GC induced operations.

4 SPD: FROM PLANE TO DIE PARALLELISM EXPLORATION

4.1 Overview
To maximize plane level parallelism, the access addresses of writes on all planes in the same die should be aligned at all time. In this work, we propose SPD, an SSD from plane to die framework, to exploit the plane level parallelism for performance improvement by smartly maintaining aligned write points across multi-planes in each die at all time.

Basically, SPD takes the following strategies to achieve the objective, as shown in Fig. 4. The basic SPD design adds three new components — a die level write construction, a die level GC and a combination scheme. The die level write construction is designed to maintain aligned write points for host writes. The die level GC is designed to maintain aligned write points for GC induced page movements. Note that for other activities, such as WL, they also can adopt the same design principle of GC. For simplicity, only GC is taken as an example in this paper due to its non-negligible performance impact on SSDs. The combination scheme is proposed to process write operations and GC related valid page movements in parallel so that write operations from host system can be processed with lesser waiting time. We will elaborate the details of these three components in following sections.

4.2 Die Level Write Construction
Given that multi-plane commands would be disabled if the in-plane addresses are mis-aligned, the basic idea of die level write construction is to maintain aligned write points all the time by write the same amount of data synchronously to all planes in the same die. That is, (1) the amount of data issued to a die should be a multiple of \( N \) pages, assuming there are \( N \) planes in a die; and (2) the starting locations of data should be aligned for all the planes in the same die. With this scheme, whenever there are multiple write operations issued to a die, they can be processed in parallel.

SPD exploits SSD buffer to assist die level write construction. An SSD buffer evicts a multiple of \( N \) dirty pages from one die at a time such that these pages can be written using Die-Write. For data allocation, we adopt a round-robin plane allocation scheme within a die [11], which evenly distributes \( N \) dirty pages to different planes at each cycle. The data allocation at higher levels can either be static or dynamic, as discussed in Section 2.1. In the following discussion, we assume static allocation at the channel, chip, and die levels, which is simple and has been widely equipped in real SSD devices.

4.2.1 Buffer Supported Die-Write
Fig. 5 illustrates how the SSD buffer assisted Die-Write works. Fig. 5a shows how the SSD buffer is organized. It maintains a die queue that keeps a list of dirty pages for each die in the system. The pages in each list are linked together
using LRU algorithm. The data evicted from the buffer are written to their corresponding dies. To balance the number of writes sent to different dies, SPD adopts round-robin to choose the next die from which its LRU pages are evicted.

For the example, in Fig. 5b, the SSD has four dies, each die has two planes, and the current turn is Die 0. When the SSD buffer is full and there is a host requirement for inserting five dirty pages to the buffer, SPD chooses the victim dies with at least two dirty pages (i.e., two is the number of planes in a die) and evicts the dirty pages from each selected die. In the example, it first chooses Die 0 and then skips Die 1 as the latter does not have enough dirty pages. It continuously chooses Die 2 and Die 3 and then evicts two pages from Die 0, 2, and 3, respectively.

From this example, the write points of all planes are effectively aligned. The proposed scheme may evict one more dirty page than the number of dirty pages from the host. Since one Die-Write takes the same amount of time as one page write, the scheme is able to speed up the storage access by reducing the most timing cost. We elaborate the details as follows.

Since the addresses of requested data are fixed, die level read operations cannot be constructed in the same way as that for Die-Write. In this work, Die-Read only read the requested data, i.e., if there exist read operations with aligned access locations, they can be issued to the die in parallel; otherwise, only single page read gets processed next. The goal of Die-Read is to maximize the number of multi-plane command supported read operations without introducing additional time cost. In addition, since all Die-Writes operations can be scheduled in parallel by leveraging the parallel architecture of SSDs, SPD avoids the access conflicts on the same die [3], [18]. Due to that we always evict the pages at LRU positions, the write amplification also can be minimized.

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process scans the states (valid/invalid/free) of all pages in a block and finds a victim block which contains maximal number of invalid pages, proposed Die-GC is realized by summing the number of valid pages of blocks in each aligned block. Based on the summed number, victim aligned block is selected. Second, SPD uses Die-Read (in Section 4.2.1) to read the valid pages to the SSD buffer, where $N$ page slots are required to store valid pages from victim blocks. Third, after reading $N$ pages of valid data, SPD groups the $N$ pages of data to construct a Die-Write operation and then writes the valid data back to the die. Finally, when all the valid pages are written back, the $N$ aligned blocks can be erased in parallel. Given SPD reclaims $N$ blocks from one GC invocation, the GC gets triggered less frequently than that of the traditional one. However, since each GC erases and reclaims two blocks after one invocation, the total number of erase operations during the whole lifetime of SSDs can be increased while the frequency of triggering GC can not be significantly reduced, causing lifetime degradation of SSDs. In the experiment, the impact on lifetime is going to be evaluated and presented.

For the example shown in Fig. 6, let us assume the two aligned blocks 0 from two planes are selected as the victim blocks. According to Die-GC, the valid pages in these two blocks are read and written with Die-Read and Die-Write, respectively.

**Step1:** Read page 0 from plane 0 and page 1 from plane 1 to the SSD buffer. Since they are not aligned, they are read sequentially.

**Step2:** Group the two valid pages together to construct a Die-Write operation and written them back to the current aligned write point of both planes at block $j$. The current write points are marked using red arrows in the figure.

**Step3:** Then, read page 2 from plane 0 and plane 1 to the SSD buffer. These two pages are read in parallel as they have aligned addresses.

**Step4:** Repeat step (2) for the last two valid pages.

**Step5:** Then, erase the two victim blocks in parallel. From the above discussion, Die-GC significantly reduces GC cost because it maintains aligned write points in the die such that many strip reads and writes can operate in parallel.

An exception for the above scheme happens when the total number of valid pages in the victim aligned blocks is odd. In this case, the last Die-Write cannot be constructed due to the lack of one more valid page, causing the write points of different planes misaligned after GC while the last Die-Write is carried out with only one valid page inside. To address this issue, the last Die-Write operation is constructed by the remaining valid page in victim block and one dirty page in the write buffer (as discussed in Section 4.2).

### 4.3.2 Implementation, Analysis, and Discussion

We next elaborate the implementation overhead of SPD. We identify the construction of Die-Write as the most critical component in SPD. Given that SPD transfers more data to the write buffer in the controller, it demands larger data storage. Considering the worst that all dies are activated with the die level GC, each die needs at least $N$ pages in the write buffer. For a typical SSD setting as presented in Section 4.2, the required buffer size for Die-GC is 256 KB for a 512 GB two-plane SSD and 512 KB for a 512 GB four-plane SSD at least. In summary, the storage requirement is modest for modern SSDs.

### 4.4 Combining Die Level Writes and GC

Both of Die-Write and Die-GC can be used to improve the write performance by fully exploiting the plane level parallelism. However, if there is a die being occupied by Die-GC, constructed Die-Writes towards this die will be delayed and wait for the completion of Die-GC. As shown in Fig. 7, there are four Die-Writes accessing current die, where a Die-GC is being processed in two blocks. In this case, these four Die-Writes have to be delayed until the completion of Die-GC. The bottom of Fig. 7 shows the time flow of Die-GC and Die-Writes. All these four Die-Writes suffer from a long waiting time caused by Die-GC. Such a GC induced conflict between Die-GC and Die-Write is regarded as access conflict in this work. In order to minimize the impact from access conflict between Die-Write and Die-GC, the most straightforward method is to process Die-Write with higher priority and postpone Die-GC. However, such a method may make Die-GC induced writes starve. Therefore, to avoid the starvation of Die-GC induced writes, we propose a combination scheme, which aims to reduce the waiting time of Die-Write without destroying the aligned write points.

#### 4.4.1 Combination Scheme

As shown in top part of Fig. 8, there are eight valid pages in victim blocks (light blue boxes), and eight dirty page (gray boxes) generated write operations, which can be processed in parallel by constructing Die-GC and Die-Writes, respectively. Totally there are three steps (Step 0 to Step 2), which are processed in sequence order. For Die-Writes, which arrive during the process of Die-GC, they have to be delayed before the completion of Die-GC, causing write performance degradation.

To minimize the waiting time of Die-Writes, Die-GC is divided into two parts, including valid page movements and erase operations, where valid page movements are processed by constructing new Die-Writes. To solve this problem, in the bottom of Fig. 8, valid pages in victim blocks and dirty pages from buffer are combined to construct new Die-Writes. In this case, we assume there are three and five valid pages in two victim blocks, respectively. The whole combination scheme can be divided into three steps (Step 3 to Step 5 in Fig. 8). For Step 3, one victim block is selected to execute valid page movements while the paired...
block is used to service write operations. Totally, three write operations from the buffer can be written in parallel with valid page movements of GC as three Die-Writes. Similarly, in Step 4, valid page movements in another victim block are realized while additional five dirty pages from the buffer are being wrote to the paired block. Since write points of all planes in the same die are aligned, the new Die-Writes are constructed and processed in parallel via accessing aligned write points. Lastly, in Step 5, two erase operations are executed in parallel as a Die-GC that does not contain valid page movements. For the time cost of this combination scheme, although the total time cost is the same as the original scheduling case (the top part in Fig. 8), the time cost of write operations from host system can be reduced while they are being processed in parallel with GC process.

4.4.2 Pseudocode Analysis

To explain more details, a combination scheme based algorithm is presented in Algorithm 1. Initially, we assume there are two planes in a die. Before evicting dirty pages to a corresponding die, the following algorithm is used to check whether these evicted dirty pages can be processed in parallel with valid page movements in GCs. If there is a Die-GC being processed in a die and some dirty pages are going to be evicted from buffer (Line 1), the combination scheme is activated. Otherwise, proposed Die-GC and Die-Write are processed as presented in above sections (Line 11-14). For the combination scheme, the numbers of valid pages and evicted pages should be larger than 0 (Line 2, 6) so that at least one valid page in victim block can be read out and constructed as a new Die-Write with an evicted dirty page from the buffer (Line 3, 7). After that, this new Die-Write containing valid page and dirty page is written back to the aligned write points (Line 4, 8). This process is repeatedly executed until all valid pages have been moved out or there is no more dirty pages being evicted from buffer. After that, Die-GC is resumed (Line 10). If there still exist some valid pages in victim blocks, paired valid pages are read out and written back in parallel based on the design of Die-GC. Otherwise, two erase operations are processed in parallel, such as the Step 5 in Fig. 8.

4.4.3 Implementation and Analysis

To implement proposed combination scheme, the Die-GC required buffer space is used to maintain one dirty page in the buffer, and then another valid page from victim block is transferred into this buffer space (two page space is required while a die contains two planes), where two pages can be constructed as a Die-Writes. Based on the implementation of Die-Write and Die-GC, proposed combination scheme can be realized without introducing additional implementation overhead.

Algorithm 1. Optimizing Access Conflict between Die-GC and Die-Writes

Input: Assume that there are two victim blocks in a die, Blk_0 and Blk_1;
Die_GC: indicates a die is occupied by Die-GC;
Die_Write: indicates a write operation distributed to a die with Die-GC;
Blk1_VP and Blk2_VP: indicates the number of valid pages in two blocks.

Output: 

1: if Die_GC ≠ NULL and Die_Write ≠ NULL then
2: while Blk0_VP ≠ 0 and Die_Write ≠ NULL do
3: DieWrite_Generation(Valid Page, Dirty Page);
4: Processing_New_DieWrite;
5: end while
6: while Blk1_VP ≠ 0 and Die_Write ≠ NULL do
7: DieWrite_Generation(Valid Page, Dirty Page);
8: Processing_New_DieWrite;
9: end while
10: Processing_DieGC;
11: else if Die_GC ≠ NULL then
12: Processing_DieGC;
13: else if Die_Write ≠ NULL then
14: Processing_DieWrite;
15: end if

5 EXPERIMENT SETUP

5.1 Simulated SSD Devices

Due to that the proposed scheme needs firmware support of SSDs, in this work, we use a popular trace driven simulator, SSDsim [10], to evaluate the effectiveness of the proposed framework. In order to simulate a state-of-the-art SSD, SSDsim is significantly extended based on ONFI [8]. During the evaluation, a 512 GB SSD is simulated, and page mapping and greedy based GC scheme are adopted [3], [10]. The threshold value for GC activation is set to 7 percent [9]. To triggering GC process, SSD is warmed up by filling SSD with valid and invalid data ahead. The warming up process contains two steps: first, each plane of the SSD is randomly filled with data from 93 to 95 percent to trigger GC immediately, of which 80 percent are valid; second, the evaluated workload is pre-processed in the SSD to validate read data [13]. The over-provisioning ratio is set to 25 percent, which complies with the setting in previous work [9]. For the data allocation scheme, the most widely used Channel-Chip-Die-Plane scheme is adopted. The experiment settings represent an aged state-of-the-art SSD. Other details are presented in Table 1.

During the evaluation, a DRAM buffer is configured in the SSD. We set the buffer size to be 1% of the footprint of the evaluated workload [25], [40], which helps to prevent setting a large buffer from generating biased results in evaluation.
The default data organization of die lists in the buffer is designed based on the scheme of the Element-Level Parallelism Optimization (EPO) [41]. EPO evicts dirty pages from buffer based on its die location so that the utilization of die level parallelism can be maximized. The data are organized in LRU for each die list of the buffer.

### 5.2 Evaluated Workloads

The workloads studied in this work include a subset of MSR Cambridge Workloads from servers [42]. These workloads are widely used in previous works for studying SSD performance [9], [14], [18], [43]. The characteristics of workloads are presented in Table 2. Each workload is characterized by three metrics: \( W/R \) Ratio, \( FP \), \( R_V \), \( W_V \), \( R_S \) and \( W_S \). \( W/R \) Ratio represents the write and read operation ratios, \( FP \) is the footprints of each workload, \( R_V \) is the total amount of read data, \( W_V \) represents the total amount of written data, \( R_S \) represents the average size of read requests, and \( W_S \) is the average size of write requests.

### 5.3 Evaluated Schemes:

Seven schemes are implemented to show the effectiveness of SPD.

**Baseline-D.** This scheme is implemented to represent the traditional SSD design [10]. The buffer management of Baseline-D adopts EPO to exploit die level parallelism through adding dirty pages to different die lists based on their die locations [41]. With this organization, dirty data evicted from write buffer can be distributed to different dies so that die level parallelism can be exploited.

**Baseline-P.** This scheme is similar to Baseline-D. The difference is that Baseline-P evicts dirty data based on their plane locations to further exploit plane level parallelism. In this case, dirty pages accessing different planes within the same die are evicted at a time. Baseline-P evenly distributes dirty pages to different planes to better exploit plane level parallelism, which is similar to the previous studies [18], [44]; twinBlk. This scheme is designed based on the work proposed by Tavakkol et al. [11], which aims to align write points of all planes in a die via round-robin policy. In this case, several host requests can be processed in parallel when write points are aligned. During GC process, the adopted round-robin policy is designed to align write points of active blocks in victim blocks as well, aiming to move valid pages with the support of multi-plane command;

**SuperPage.** This scheme is implemented based on [17], [45], which groups pages into one super page that is set as the smallest access granularity of flash memory. Such a large-granularity accessing approach can fully exploit plane level parallelism by writing pages to all planes in a die at any time. Differing from our proposed work, if there exist an update operation, the paired data in other planes should be read out first if they are valid. Later, the read data and updated data are constructed as one new super page, and then are written to the die with the support of multi-plane command.

**ParaGC.** This scheme is designed by Shahidi et al. [9], which aims to align valid page movement during GC to minimize the GC cost. Differing from TwinBlk, ParaGC aligns write points of active blocks through sequentially moving valid pages to one active block until write points of all planes are aligned. After that, with cache assistance, all valid pages can be written back to active blocks with the support of multi-plane command;

**SPD.** This is the proposed framework, which includes Die-Write and Die-GC.

**SPD+.** This is the proposed framework, which includes Die-Write, Die-GC and the combination scheme.

### 6 Experiment Results and Analysis

In this section, basic SPD is evaluated with two scenarios based on whether GC is triggered. For the first scenario without triggering GC, it is evaluated to show the advantages of the proposed Die-Write scheme. For the second scenario with triggering GC, it is evaluated to show the effectiveness of SPD, including Die-Write and Die-GC. In addition, the Die-GC is also evaluated in terms of its cost and lifetime impact. Then, proposed SPD+ is evaluated and compared with basic SPD scheme to identify its effectiveness. Since SPD+ is designed to solve the access conflict between Die-Write and Die-GC, there is only one scenario with triggering GC. Finally, the impact of different buffer sizes and results on SSD with 4 planes per die are presented.

#### 6.1 Experiment Results Without GC

1) **Write Latency Evaluation:** Fig. 9 shows the results of write latency for the six schemes. Note that, since ParaGC is designed to optimize GC process, the results of ParaGC in this part are same to that of Baseline-D. The results show that SPD achieves write latency reduction for all evaluated workloads. For example, for HM_0, PRN_0, PROJ_3, SRC2_0 and PRXY_0, the write latency is reduced by more than 15 percent.
compared with Baseline-D. These results show that deploying Die-Write to maintain aligned write points for the multiple planes in a die is important in improving the access performance. In Fig. 10, we collected the percentages of write operations processed by multi-plane command. The results show that the proposed Die-Write is able to maintain aligned write points for all write operations. However, this is not a promise for the other schemes.

To obtain more details, we compare SPD with other three schemes, Baseline-P, TwinBlk and SuperPage. Two observations can be concluded from the results: First, compared with these three schemes, SPD achieves the best write performance. Baseline-P is proposed to distribute the same type requests to all planes evenly. However, the address restriction is not taken into consideration. As a result, Baseline-P only achieves little write latency reduction, which is only up to 1.4 percent. TwinBlk aims to align write points of all planes in the same die as well. However, the write points still may be unaligned due to the unaligned accesses on planes of the same die. On average, TwinBlk achieves 7.8 percent write latency reduction compared with Baseline-D. As shown in Fig. 10, the percentages of write operations processed by multi-plane command for Baseline-P is similar to that of Baseline-D. For TwinBlk, the percentage is largely increased compared with Baseline-D. SuperPage also can fully exploit plane level parallelism at any time by constructing super page on a die. However, such a super page would introduce write amplification and cause write traffic. Before constructing an update operation related super page, read operations are required to read out all valid pages resided in original super page. Therefore, compared with Baseline-D, although SuperPage can achieve 10.4 percent write latency reduction, SPD still outperforms SuperPage at the performance improvement.

Second, for several workloads, TwinBlk only achieves similar performance improvement to that of Baseline-D, such as RSR_0, STG_0, TS_0 and PRXY_0. This can be explained from the results in Fig. 10, where the percentage of write operations supported by multi-plane command is limited. The reason is that TwinBlk cannot guarantee aligned write points for all planes all the time.

For read latency, the average read latency improvement compared with Baseline-D is presented in Table 3. The results show that read latency is similar among the six schemes. The key reasons are from two aspects: first, read requests of all evaluated schemes are processed with highest priority [3], [31], [46], [47]; second, Die-Read is designed to only read requested pages, which are barely located in the same in-plane addresses. In conclusion, the proposed Die-Read is same to that of normal read operations without introducing read amplification. Note that, in SuperPage, read operations always are executed with the support of multi-plane command, but only required data is transferred out for time cost saving.

2) Plane Utilization: Plane Utilization is defined to present the average number of planes being occupied in parallel. In order to obtain plane utilization, the number of planes being accessed is counted when each buffer eviction process is completed. Fig. 11 shows the plane utilization (Bars) and the maximal number of planes being accessed in parallel (Dots +Line) for the six schemes. The results have a matching pattern with the write performance improvement in Fig. 9. SPD can significantly increase the plane utilization through doubling the number of parallel planes with satisfying the restrictions of multi-plane command. On average, the plane utilization is increased by 34.5 percent compared with Baseline-D. For the maximal number of planes accessed in parallel, all planes of the SSD can be accessed in parallel for most workloads. Similarly, SuperPage also can achieve the maximal plane level parallelism, whose plane utilization is averagely increased by 32.9 percent compared with Baseline-D. However, for Baseline-D, Baseline-P and TwinBlk, there still
exists a large gap compared with SPD. In conclusion, Die-Write is not only able to increase plane utilization, but also can make a full use of all planes of the SSD.

3) Buffer Hit Ratio: Differently from previous work, Die-Write may need to evict more data from the buffer to align the write points. In this case, it may have impact to the hit ratio of buffer. Fig. 12 presents the results of buffer hit ratios for the six schemes. The results show that SPD has little impact to the hit ratio of buffer. The average buffer hit ratio is reduced by only 1.92 percent, which is negligible. The reason for the slight reduction is that Die-Write is designed with following principles: first, it always only need to evict one more dirty page, which is critical in aligning write points; second, the buffer is designed to only evict the cold dirty data from the LRU position.

6.2 Experiment Results With GC

Fig. 13 shows the results of write latency with GC triggered. The results show that SPD is able to significantly reduce the write latency for all workloads. The write latency is reduced by 48.61, 47.65, 42.05, 28.19, and 28.58 percent compared with Baseline-D, Baseline-P, TwinBlk, SuperPage, and ParaGC, on average. The significant improvement comes from three aspects: First, SPD constructs aligned write access to reduce write latency, which has been verified in Section 6.1. Second, the GC cost is further reduced through moving all valid pages with the support of Die-Write. Third, total GC count is noticeably reduced by reclaiming two planes at once time, and write amplification is avoided compared with SuperPage. To understand more details, the total GC costs are presented in Fig. 14.

For read latency, results of read latency improvement with considering GC are presented in Table 4. Similarly, the read latency is similar among each scheme. The reason also comes from the highest priority of read request, which can be processed without delay [31], [46]. The results show that SPD has no impact to read access with significant write performance improvement.

6.3 GC Evaluation

In this part, Die-GC is evaluated. First, the average GC cost and the number of triggered GC in different schemes are

SuperPage and ParaGC are reduced by 22.4 and 30.8 percent while TwinBlk only reduces the total GC cost by 6.9 percent. For SuperPage, it reclaims two blocks at once time to reduce GC cost, but the total GC count can be increased when update operation would read other valid pages for constructing a new super page, consuming free space more rapidly. More details about GC cost is presented in Section 6.3. For ParaGC, it activates GCs in paired planes only when the number of free pages in the other plane is smaller than 7 percent. In this case, it can avoid introducing high GC cost while moving valid pages. In addition, ParaGC proposed to align write points during the process of valid page movement so that valid pages in the same position of paired planes can be read and written in parallel. However, for TwinBlk, it activates paired GCs without considering the number of valid pages in the paired planes. In this case, more valid pages from paired planes may be moved during GC process. In addition, TwinBlk adopted round-robin policy. If current write points are not aligned, valid pages having same position in different planes still can not be read and written in parallel. Therefore, for some workloads, the total GC cost of TwinBlk is larger than Baseline-D. Second, even though SPD also activates GC at the all planes at the same time, it is proposed to regard the whole die as the smallest access unit and all the write operations during GC are processed via Die-Write. Moreover, parallel access in SPD is constructed without introducing write amplification, avoiding triggering more GCs. As a result, the total GC cost is reduced by 36.4 percent, on average. In conclusion, SPD achieves the best write performance compared with all other related works.
evaluated. Second, the number of erase operations induced by GC is collected to show its impact on the lifetime of SSDs.

1) Average GC Cost: Average GC costs are collected in Fig. 15. In the Figure, the average GC cost is broken into four parts: read cost, write cost, transfer cost and erase cost. Read cost is the cost in reading valid pages from the victim block; write cost is the cost in writing the valid data to free pages; transfer cost is the cost in transferring the valid data among planes or between controller and chips; and erase cost is the time cost in erasing the victim block. The results show that the write cost takes the dominate part of the total cost [18]. This is because write latency of flash memory is several times of read latency. In addition, there are always a large number of valid page movement during GC. There are three observations from the results:

First, SPD has the less GC cost compared with TwinBlk and ParaGC. Clearly, the reduced GC cost is from the $\text{Die-Write}$ used in $\text{Die-GC}$, which is triggered to write dirty pages back to the multiple planes in parallel. For TwinBlk, it also triggered GC in the paired planes. However, TwinBlk adopted round-robin policy for write operations among planes, which is not able to always align the write points. In this case, many valid pages written back may be processed sequentially. Second, the GC cost of SPD is similar to that of Baseline-D and Baseline-P. As presented in the technique part, $\text{Die-GC}$ is designed to reclaim several blocks in one GC. Several block reclaiming costs are similar with single block reclaiming cost in Baseline-D and Baseline-P due to that we carefully select victim blocks among planes as a single unit and use $\text{Die-Write}$ to speed up the process. Third, SuperPage can achieve the minimal GC cost compared with other schemes. The reason is that, SuperPage can aggressively invalidate pages in a block, reducing the total number of valid pages in a block. This is because each update operation can invalidate all paired pages for maintaining consistent states (valid or invalid) of all pages in a super page.

2) GC Count: Fig. 16 shows the total number of triggered GCs during runtime. We can find that $\text{Die-GC}$ highly reduces the number of GCs. Therefore, the frequency of triggering GC is reduced. The results show that GC count is reduced in the range of 32.9 to 50.1 percent, compared with Baseline-D. As a result, the total GC cost during whole runtime can be highly reduced as well so that the performance of SSDs can be improved. For related works, the number of triggered GCs in Baseline-P is similar to Baseline-D. Both TwinBlk and ParaGC can reduce the number of triggered GCs as well. This is because that TwinBlk and ParaGC erase more blocks in each GC process as well. But for TwinBlk, it selects victim blocks inefficiently so that its GC counts are slightly higher in most cases. For a exception, PROJ_0, since SPD may slightly increase write operations, the total triggered GC count of SPD may be slightly increased. For SuperPage, since it can cause significant write amplification, total number of GCs is increased as well.

3) GC Induced Erases: Fig. 17 shows the number of erase operations for the six schemes. Since TwinBlk, SuperPage, ParaGC and $\text{Die-GC}$ are designed to erase more blocks in each GC process, the number of erase operations are larger than that of Baseline for most workloads. The reason is that, reclaiming blocks from different planes at once time may trigger premature GCs [48], [49]. However, the results show that the number of erase operations of $\text{Die-GC}$ is much smaller than TwinBlk, SuperPage and ParaGC. For example, TwinBlk, in the worst case, introduces more than 102.2 percent erase operations for PRXY_0, compared with Baseline-D. ParaGC, introduces more than 65.8 percent erase operations compared with Baseline-D. Worst of all, SuperPage triggers 177.7 percent erase operations compared with Baseline-D. Compared with these three related works, SPD introduces fewer erase operations in most cases. On average, the number of erase operations is reduced by 13.43, 34.23 and
10.04 percent compared with TwinBlk, SuperPage and ParaGC. The reason comes from that Die-GC is triggered with regarding the whole die as the smallest unit without introducing additional valid page movements. In addition, the write amplification is also avoided to reduce total GC count.

### 6.4 Experiment Results of SPD+

In order to evaluate the effectiveness of proposed SPD+, the impact of access conflict between Die-Write and Die-GC is measured first, which is the potential of SPD+. In SPD scheme, write requests in the constructed Die-Write are identified as blocked write requests while there is a Die-GC in the accessing die. To indicate the impact of access conflict, the percentages of blocked write requests’ time cost are evaluated and presented in Fig. 18. On average, total time cost of write requests blocked by GCs accounts for 18.2 percent. The results presented in Fig. 18 show a matching pattern with Fig. 13, where the write performance improvements of PRN_0, PRN_1 and TS_0 are slight, accounting for 0.5, 0.2 and 5.1 percent, respectively. On the contrary, for PROJ_3, of which the total time cost of write requests is highly affected by GCs, the achieved write latency decrease reaches 28.7 percent.

To elaborate the reason of achieved significantly write performance improvement, the results of average time cost of write requests blocked by GCs are evaluated and presented in Fig. 20, where the results of SPD+ are normalized to SPD. For SPD+, while write requests can be processed in parallel with valid page movements during GC process, the waiting time of write requests can be reduced, causing the average time cost drop. In this figure, one can see that the average time cost of write requests blocked by GCs is reduced by 27.1 percent. Similarly, for PRN_1, which is least affected by GC induced access conflict, the achieved average time cost reduction of write requests blocked by GCs is the maximal one, reaching 31.2 percent. In conclusion, the proposed combination scheme can be applied to significantly reduce waiting time of write requests blocked by GCs, making write performance to be further improved.

For GC evaluation, the total GC time cost, average GC time cost and GC count of SPD+ are similar to SPD, which are increased by 0.012, -0.015 and 0.027 percent, respectively. Since proposed SPD+ is designed to schedule write requests for reducing waiting time, the GC process will not be affected, and then the metrics of evaluated GC process are similar to SPD.

For read latency, SPD+ still executes read requests with highest priority, making the achieved read latency be similar to SPD. On average, the read latency of SPD+ is reduced by 0.33 percent compared with SPD.

### 6.5 Sensitive Studies

1) **Buffer Size Impact:** In this part, the write intensive workload, RSR_0, is selected for buffer size sensitivity study. Buffer size is different within different devices. Its impact on SPD is presented. Fig. 21 shows the results of the normalized write latencies of the six schemes by varying buffer size from 256 KB to 16 MB. During the evaluation, GC is not triggered to only understand the impact from different buffer sizes. Two observations can be concluded from the results. First, with larger buffer size, the write latencies of all schemes can
be further reduced. This is because that more dirty pages can
be stored and higher hit ratio can be achieved. Second, com-
pared with other schemes, stable write latency reduction is
achieved by SPD with different buffer sizes. The proposed
framework is designed to align the write point of planes all
the time. It has benefit once there are multiple write opera-
tions issued to a die.

2) Four-Plane SSD Evaluation: In this part, SSD with four
planes per die is evaluated for SPD. For the four planes of a
die, each paired planes can be accessed in parallel with the
support of multi-plane command [1], [10]. The results of write
latencies for Baseline-D, Baseline-P, and SPD are presented
in Fig. 22, where GC is not triggered to evaluate the single
influence from more planes per die. First, Baseline-P has
similar write latency to that of Baseline-D. Four-plane SSD
requires that only paired plane 0&1 or 2&3 can be processed
in parallel. Only a few write operations can be processed
with the support of multi-plane command. Second, for SPD
with four-plane SSD, the write latency is further reduced.
This is because that all four planes are regarded as one unit
in Die-Write. Therefore, more dirty pages can evicted and
written back as Die-Write at the cost of one write opera-
tion when the number of planes in a die increases. On aver-
age, compared with Baseline-D, SPD achieves 43.9 percent
write latency reduction, on average.

3) Various Mapping Schemes based SSDs: In this part,
demand-based selective caching of page-level address map-
pings, termed DFTL [4], and hybrid mapping scheme are
implemented to show the effectiveness of proposed SPD.
Similar to the settings of above evaluations, GC is not trig-
gerated in this evaluation. For DFTL based SPD, mapping
entries evicted from RAM buffer are organized as Die-Write
as well. When mapping entries are supposed to be read from
flash memory, Die-Read is generated to read required
mapping entries as well. For hybrid mapping, FAST is imple-
mented as an example [50]. In FAST based SPD, paired log
blocks resided in paired planes are reserved. That is, write
requests can be written into paired log blocks in the form of
Die-Write as well. While merge operation is required,
valid pages in selected paired log blocks and corresponding
data blocks are read out and written to new paired data
blocks by Die-Write.

Take RSR_0 as an example, Fig. 23 shows the write laten-
tics of pure page-level mapping, DFTL and FAST based eval-
uated schemes. Herein, the write latencies of DFTL based
schemes are worse than that of pure page-level mapping
based schemes. For SPD, the write latency of DFTL based
SPD is increased by 22.47 percent compared with pure page-
level mapping based SPD. For FAST based schemes, since
there are additional merge operations caused by FAST map-
ping, the write latencies of FAST based schemes are highly
larger than that of other mappings based schemes respecti-
vely. But for FAST based SPD, it still can achieve substantial
write performance improvement compared with other
schemes equipped with FAST. This is because that, not only
host write requests can be processed in parallel, but also
each merge operation can reclaim two log blocks at the time
cost of one merge operation. On average, compared with
Baseline-D equipped with FAST mapping, FAST based SPD
can reduce write latency by 23.43 percent.

4) Hot/Cold Separation based GC: To further reduce GC
time cost, a widely adopted method is to distribute data to
different blocks according to their hotness. Inside SSD, valid
pages in victim block are cold enough, since only hot data
are evicted from buffer and only relatively hot data in flash
blocks tend to be invalidated. To further improve the effi-
ciency of GC process, cold and relatively hot valid pages in
victim blocks are written to different blocks according to
their hotness as well in the revision [51], [52]. To realize
SPD with considering hot/cold separation in GC process,
each plane maintains two active blocks, cold block and hot
block. To separate hot and cold data, the update count
within a fixed time interval is recorded, which is set to 120
minutes according to [53]. Within 120 minutes, most data in
evaluated workloads are updated. Except for valid pages
from GC, host data evicted from buffer are considered as
hot data and written into hot block as well.

The writ latencies of evaluated schemes with and without
hot/cold data separation are collected and presented in
Fig. 24. In this figure, two observations can be concluded.
First, after applying hot/cold data separation, proposed SPD and SPD+ still can noticeably reduce write latency. On average, compared with Baseline-D, SPD+ can reduce write latency by 57.64 percent. Second, hot/cold data separation can significantly reduce write latency. Averagely, take Baseline-D as an example, scheme applied with hot/cold data separation can reduce write latency by 23.83 percent. The reasons come from two aspects: first, GC efficiency is improved while fewer valid pages are moved during GC process; second, total GC count can be reduced while cold pages are barely moved.

7 RELATED WORKS

In this section, related works on improving the plane level parallelism and reducing GC impact on performance are presented, respectively.

(1) Plane Level Parallelism Exploration: In order to improve plane level parallelism, several previous works have been proposed. Gao et al. [18] and Jung et al. [44] proposed to increase the potential of using multi-plane command through distributing requests belonging to different planes at one time. Similarly, Abdurrab et al. [29] proposed DLOOP to modify mapping policy to evenly distribute data across planes based on a fixed location calculation. However, the achieved performance is limited since they highly depend on the access patterns of workloads to match the limitations of multi-plane command. On the other hand, Tavakkol et al. [11] and Hu et al. [10] proposed to align writing points of planes. Tavakkol et al. [11] proposed to maintain the write points to distribute writes among planes in round-robin fashion. However, due to the above mentioned unaligned access problem, plane level parallelism still can not be fully exploited. Hu et al. [10] proposed a greedy multi-plane command. They proposed to allocate new writing points in the same position. However, this will waste space. Caulfield and Seong et al. [17] and [45] proposed a new design principle for fully exploiting plane level parallelism, which groups all pages residing in the same in-chip location as a large logical page, termed super page. Such design can boost the performance and bandwidth of SSDs at the penalty of lifetime.

Different from all these works, SPD is the first on proposing to align the write points in an active way. Die-Write is designed to align the write point all the time. In this case, all write operations issued to multiple planes in a die can be processed in parallel. Also, SPD can minimize the impact on the lifetime of SSDs by grouping pages together more flexible.

(2) Garbage Collection Impact Minimization: Previous works aiming at reducing GC impact on performance can be classified into two groups: The first group proposed to reduce the time cost of GC activity [13], [54]; For example, Gao et al. [13] proposed to reduce the time cost of valid page movement through migrating valid pages to idle chips. Park et al. [54] proposed a new hotness identification method for accurately capturing the recency and frequency of data. The second group proposed to schedule requests or GCs to reduce the impact on performance of SSDs [12], [14], [55]. For example, Wu et al. [14] used cache to store requests conflicted by GC. Jung et al. [55] proposed to advance or delay GC through moving the time-consuming activity from busy period to idle period. Choi et al. [12] proposed to combine host I/O operations with valid pages migration. However, the aforementioned GC optimization methods still have not taken unaligned access problem of plane level parallelism into consideration.

There are two works proposed to reduce GC impact resulted from unaligned access problem. Shahidi et al. [9] proposed ParaGC to select paired planes, where GC activities can be processed in parallel. However, if the paired planes can not be found, unaligned access problem still exist. Tavakkol et al. [11] proposed TwinBlk, which can minimize the unaligned access induced impact on GC. TwinBlk is designed to trigger GCs on all planes of the same die simultaneously so that symmetric victim blocks on planes can be reclaimed in parallel. During this process, valid pages are evenly moved to all planes in round robin policy for aligning write points of all planes.

Different from these works, SPD uses Die-GC to speed up the GC process and reduce the GC cost. Die-GC is designed to select multiple blocks in the unit of die and adopt Die-Write to speed up the GC process.

8 CONCLUSION

In this work, a from plane to die optimization framework is proposed to exploit the plane level parallelism, which is the last level parallelism of SSDs. Three components are designed in the framework: die level write construction, die level GC and combination scheme. Different from previous work, this work is the first which is able to maintain the aligned write points for the multiple planes for each die at the time. There are two components designed to align the write points of all planes in the same die all the time. In this case, the last level parallelism, plane level parallelism, is fully exploited to improve the performance of write requests and internal activities. In addition, the combination scheme is used to construct new die level write containing dirty page evicted from cache and valid page in victim block. The combination scheme can largely reduce the waiting time of write requests blocked by GCs, bringing write latency decrease. Experiment results show that SPD and SPD+ achieve significant write performance improvement and much smaller lifetime impact compared with state-of-the-art works.

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Towards Higher Performance and Robust Compilation for CGRA Modulo Scheduling

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Abstract—Coarse-Grained Reconfigurable Architectures (CGRAs) is a promising solution for accelerating computation intensive tasks due to its good trade-off in energy efficiency and flexibility. One of the challenging research topic is how to effectively deploy loops onto CGRAs within acceptable compilation time. Modulo scheduling (MS) has shown to be efficient on deploying loops onto CGRAs. Existing CGRA MS algorithms still suffer from the challenge of mapping loop with higher performance under acceptable compilation time, especially mapping large and irregular loops onto CGRAs with limited computational and routing resources. This is mainly due to the under utilization of the available buffer resources on CGRA, unawareness of critical mapping constraints and time consuming method of solving temporal and spatial mapping. This article focus on improving the performance and compilation robustness of the modulo scheduling mapping algorithm for CGRAs. We decomposes the CGRA MS problem into the temporal and spatial mapping problem and reorganize the processes inside these two problems. For the temporal mapping problem, we provide a comprehensive and systematic mapping flow that includes a powerful buffer allocation algorithm, and efficient interconnection & computational constraints solving algorithms. For the spatial mapping problem, we develop a fast and stable spatial mapping algorithm with backtracking and reordering mechanism. Our MS mapping algorithm is able to map loops onto CGRA with higher performance and faster compilation time. Experiment results show that given the same compilation time budget, our mapping algorithm generates higher compilation success rate. Among the successfully compiled loops, our approach can improve 5.4 to 14.2 percent performance and takes x24 to x1099 less compilation time in average comparing with state-of-the-art CGRA mapping algorithms.

Index Terms—CGRA, modulo scheduling, temporal mapping, spatial mapping

1 INTRODUCTION

Recent years, Coarse-Grained Reconfigurable Architectures (CGRAs) has gained many attentions from both academia and industry. Based on our knowledge, at least 40 CGRAs have been developed to adapt diverse applications in the past decades of years. These CGRAs are either positioned as accelerators or standalone processing units, and target on improving the efficiency of running applications that cover mobile computing [1], [2], [3], media processing [4], [5], [6], [7], [8], image processing [9], [10], digital signal processing (DSP) [11], [12], [13], [14], [15], [16], [17], [18], ultra-low power processing [19], [20], [21], machine learning [22], [23], [24], data or computational intensive domains [25], [26], [27], [28], [29], [30], [31], [32], and even general purpose computing [33], [34], [35], [36], [37], [38]. However, building the software ecosystem around CGRAs is challenging due to the diverse CGRA hardware design flavors and application purposes. Many compilers for CGRAs are hardware specific (works such as [28], [39], [40], [41], [42], [43]).

Therefore, in terms of generality and programmability, a generalized CGRA execution model is important for the compilers to formulate a uniform compilation problem. ADRES [44], CRC [27] and CGRA-ME [45] are representative works which provide generalized CGRA template of modeling CGRA loop accelerators. These templates define the basic components inside CGRA computing system, which includes reconfigurable computing unit (RPU) that consists of one or multiple arrays of processing elements (PE), hierarchical on-chip buffers, and interconnect networks. The template model is not specific to single CGRA design flavor and the components inside the CGRA template can be parameterized. In this way, the compiler is able to formulate a uniform problem of mapping the dataflow graph (DFG) extracted from the application program onto this generalized CGRA template, and the goal of the mapping problem depends on the CGRA execution schemes.

One of the popular execution scheme is to software pipeline the innermost loops on CGRA by overlapping the execution of successive loop iterations [46]. Operations within the same and different iterations are executed in parallel and both instruction-level parallelism and loop-level parallelism can be exploited. The interval between two consecutive iterations of the loop is referred as Initiation Interval (II), which is the most important metric to evaluate the performance of mapping, the smaller the better. In general, compilers use modulo scheduling [46] based mapping algorithm to deploy loops onto CGRA and this mapping problem has been proven to be NP-Complete [47], [48].

Earlier work [49] has formulated the CGRA module scheduling (MS) problem as mapping the DFG onto an abstracted modulo extended 3D-CGRA (CGRA is extended...
II in the third dimension), a space-time graph that models the computational resources and routing resources of CGRA, and their goal is to minimize the II. The mapping problem consists of multiple processes such as assigning time, PE placement, PE routing and allocating buffers (registers). Hamzeh [50] classified the patterns of organizing these processes into the integrated policy or decomposed policy. With the integrated mapping policy, the process of assigning time, buffer allocating, and PE placement and routing are taken place at once in a node-by-node fashion [50]. Whereas the decomposed mapping policy handles these processes separately, each process can be formulated to a problem which is independent with others with a well-defined objective. Based on the decomposing style of existing mappings [47], [51], [52], [53], [54], [55], [56] and our mapping in this paper, we conceptually name two basic decomposed processes as the temporal mapping and spatial mapping for all the mappings using the decomposed mapping policy.

Existing CGRA MS algorithms using different mapping policies may suffer from different or same challenges:

1. **Underutilizing the buffer resources in both policies**: Most of the works do not explicitly provide optimization algorithm using all types of the available buffer resources in CGRA. Some works use single buffer types, for example, REGIMap [52] use local register buffer (LRB) distributed in PEs, MA-Map [54] leverages on-chip memory buffer (OMB). Even though RAMP [55] use both LRB and OMB, it only use them separately. The combinational utilization of PE, global register buffer (GRB), LRB and OMB is important to keep the performance competitive when there are insufficient buffer resources on CGRA.

2. **Good performance but long compilation time in algorithms using integrated mapping policy**: Even though classic optimization heuristics such as simulated annealing [57], particle swarm optimization [58] or linear integer programming (ILP) [59] are able to generate competitive mapping performance within a reasonable time budget. They still take relative long time to converge comparing with other heuristics, especially for large and irregular DFGs [52], [57], [60].

3. **Fast but sacrifice performance in algorithms using integrated mapping policy**: Other heuristics such as EMS [61], Resource-Aware [62] and GraphMinor [48] focus on some specific aspects during mapping, but lacks generality. This makes their approach faster but lower mapping efficiency over some irregularities that they do not cover.

4. **Unawareness of the interconnection and computational resources constraints in algorithms using decomposed mapping policy**: Decomposing the temporal and spatial mapping has been shown to be effective [43], [50]. However, if the temporal mapping cannot realize the insufficiency of the interconnection and routing resources in CGRA which leads to the failure of spatial mapping, the spatial mapping algorithm will perform redundant and useless searching.

5. **Time consuming spatial mapping algorithm in decomposed mapping policy**: Existing mappings such as EPIMap [47], REGIMap [52], Conflict-free [53], Mem-Aware [54] and RAMP [55] integrate buffer allocating and PE placement and routing processes into spatial mapping (Fig. 2a). They formulate spatial mapping problem of finding maximum clique, which is also an NP-Complete problem. They select a time consuming way to perform spatial mapping, which incurs the waste of time if problem 4 appears.

These motivations push us to think about a new mapping methodology of solving the CGRA MS problem towards higher performance and more robust compilation. The contributions of this paper are summarized as follows:

1. We give a comprehensive analysis on existing CGRA mapping algorithms, propose a mapping design philosophy of decomposing the temporal mapping and spatial mapping. We argue that the temporal mapping should be paid more attention than spatial mapping for the purpose of higher performance and robust compilation (Fig. 2b). Therefore, we develop a systematic temporal mapping flow which comprehensively analyzes and reschedules the DFG to help improving the successful rate of spatial mapping and the performance of the generated code.

2. In the temporal mapping flow, we formulate a buffer allocating problem by building the constraints and rules for different buffer resources and assigning the routing paths to these buffer resources according to their corresponding constraints and rules in order to maximize the released computational resources. We solve the problem by proposing a buffer allocating heuristic which is capable of using both same and hybrid buffer resources to buffer value during their lifetime. This makes our algorithm have strong adaptability over CGRAs with limited buffer resources.

3. Also in the temporal mapping flow, we propose the interconnection and computational constraints solving heuristics to foresee the incapability of the spatial mapping, and reschedule the operations in DFG to guarantee the successful rate of the following spatial mapping process.

4. We propose a fast and efficient spatial mapping heuristic which combines backtracking and reordering mechanism to guarantee the success rate of the spatial mapping. In the forward process, we use the greedy based algorithm to fast generate the optimal mapping with minimal II. If it fails, the backtracking and reordering algorithm will guarantee the successful mapping.

Experiment results show that our mapping strategy guarantees the stability of the compile time, all the selected loop kernels are able to be mapped within certain time budget. Among the successfully mapped loops, our mapping improves the performance from 5.4 to 14.2 percent and compilation time from $x_{24}$ to $x_{1099}$ faster in average comparing with state-of-the-art competitive CGRA mapping algorithms.

2 **Architecture Overview of CGRA**

CGRA is a kind of spatial architecture which is able to explicitly execute dataflow graphs. CGRA contains computational resources and routing resources. The computational
resources are referred as processing elements (PEs). The architecture of PE is shown in Fig. 1. The PE is able to receive input data from neighbor PEs (including itself), local register buffer (LRB), global register buffer (GRB) and on-chip memory buffer (OMB). The function unit (FU) inside PE performs fixed-point arithmetic and logic operations and write the computation result to the output register, LRB, GRB or OMB.

In CGRA, data is communicated through the routing resources. The routing resources includes the interconnections between PEs which we refer as the interconnection resources, and all the available on-chip buffer resources including PE, LRB, GRB and OMB. The produced data can be directly consumed through the interconnections between PEs. However, there exists situation that the produced data can not be consumed immediately due to the different latency between input values. For example, in Fig. 4a, the data produced by operation (or node) A is able to be consumed immediately by operation B, but not by node E and F. This forms a routing path from A to E and A to F. The routing path inside the DFG is a path from producer to one or multiple consumers where the producer can not be immediately consumed by the consumers.

In real world, there are many large and irregular DFGs that contain many routing paths. In this way, the compiler must use additional routing resources to route the value along the routing path. CGRA provides four buffer resources for routing, which are PE, LRB, GRB and OMB (highlighted with grey color in Fig. 3).

1. **PE**: When we use PE to be the routing media, data is buffered in its output register (OReg). However, when the output register is used to buffer the data in a certain time, the PE can not perform any other operation during the same time. In this way, the routing node inside the routing path will be assigned to PEs and the compiler use PEs and interconnections between PEs to route value. Fig. 4b shows the example of using PE to route data produced by node A to node E and F.

2. **Local Register Buffer (LRB)**: Unlike the previous buffering choice, when the value is buffered in LRB of a PE, PE can perform the other operations. However, any operation who consumes the data in LRB must be mapped to the producer PE, which brings additional mapping constraints onto the compiler, we called it the **LRB constraints**. All the routing paths inside the DFG must satisfy LRB constraints if they use LRB to perform routing. The detail of the LRB constraints will be illustrated in Section 4. Fig. 4c is an example of using LRB to route value A. One of the LRB constraints is that operation A, E, F must be executed by the same PE (same color means same PE).

3. **Global Register Buffer (GRB)**: GRB can be accessed by any PEs. Any operation who consumes the data in GRB can be mapped to any PEs. However, as the overhead of the GRB scales super-linearly with its number of ports [49], the GRB resource is usually limited. Thus, GRB must be carefully used for the optimization purpose. Fig. 4d shows the example of using GRB to route value.

4. **On-chip Memory Buffer (OMB)**: At last, data can be buffered into on-chip memory. The OMB can be accessed by all PEs and has relative more read/write ports than GRB because of its multi-bank designing style. However, when the compiler choose to buffer the value into OMB, the additional store/load operations will be added (Fig. 4e), and both the accessing latency and energy are larger than all the previous buffers.

One distinguished feature of our mapping over existing mappings is the combinational usage of different buffer
resources for single routing path. Fig. 4c to (i) plots the combinations of using different buffers for the shared routing path from A to E and F (red edges specified in Fig. 4b). The combinational usage of buffer resources makes our algorithm able to keep the competitive performance when certain type of buffer resource is limited in some buffer-hungry CGRAs for the purpose of power or area saving.

These examples shows that, the effective utilization of different buffer resources releases PEs from routing data so that these PEs are able to perform other valid computations. We formulate this buffer allocating problem and provide a heuristic to solve it in Section 4.2

3 CGRA Compilation Techniques

The compilation flow of CGRA starts with extracting the loops from the application and transform the corresponding code regions to the data flow graph (DFG) structure. The DFG, $D(V, E)$, is consisted of set of nodes $V$ and edges $E$, where each node $v \in V$ represents a valid operation such as arithmetic, logic or even memory access, and each edge $e \in E$ represents the data dependency between the producer and consumer. The CGRA modulo scheduling technique is to find out a mapping relationship between DFG, $D(V, E)$ and time-space CGRA resources graph, $R_{II}(V_R, E_R)$.

Since 2002, there have been many CGRA modulo scheduling heuristics. We summarize these heuristics and our mapping heuristic in Table 1 from five prospectives including patterns of organizing the mapping processes, buffer resources that are explicitly leveraged for the optimization purpose during mapping, each mapping distinguished features, their placement and routing method and the 3D-CGRA model they select to use. We analysis these mappings according to the aforementioned integrated and decomposed mapping policy they use and analysis their characteristics as follows.

3.1 Mappings Using the Integrated Mapping Policy

There are pioneers who solve the CGRA MS mapping using classic optimization heuristics such as simulated annealing, particle swarm optimization and integer linear programming solving. For example, DRESC [57] use simulated annealing (SA) based mapping algorithm. PSOMap [58] use particle swarm optimization which mimics social behavior of the bird flocks. These works first formulate the problem

![Fig. 4. Different routing methods for single routing path: (a) the original DFG, (b) only use PEs, the dash circle represent an routing operation in PE, (c) only use LRB, the colored circles must be mapped onto the same PE, (d) only use GRB, (e) only use OMB, (f) use PE and LRB, (g) use PE and GRB, (h) use LRB and GRB, (i) use OMB and LRB, (j) use LRBs of different PEs.](image)

<table>
<thead>
<tr>
<th>Mapping</th>
<th>Pattern</th>
<th>Leveraged buffers</th>
<th>Distinguished features</th>
<th>P&amp;R method</th>
<th>3D-CGRA format</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRESC [60]</td>
<td>Integrated</td>
<td>PE+LRB</td>
<td>Simulated annealing (SA)</td>
<td>Cost Function</td>
<td>MRRG</td>
</tr>
<tr>
<td>PSOMap [58]</td>
<td>Integrated</td>
<td>PE+LRB</td>
<td>Particle swarm optimization</td>
<td>Cost Function</td>
<td>MRRG</td>
</tr>
<tr>
<td>AA-ILP [59]</td>
<td>Integrated</td>
<td>PE+LRB</td>
<td>Integer Linear Programming (ILP)</td>
<td>ILP Solver</td>
<td>MRRG</td>
</tr>
<tr>
<td>MGE [63]</td>
<td>Integrated</td>
<td>PE+LRB</td>
<td>Skewing the scheduling space</td>
<td>Cost Function</td>
<td>MRT</td>
</tr>
<tr>
<td>EMS [61]</td>
<td>Integrated</td>
<td>PE+LRB</td>
<td>Edge-centric</td>
<td>Cost Function</td>
<td>MRT</td>
</tr>
<tr>
<td>RA [62]</td>
<td>Integrated</td>
<td>PE+LRB</td>
<td>Backtracking</td>
<td>Cost Function</td>
<td>MRRG</td>
</tr>
<tr>
<td>GM [48]</td>
<td>Integrated</td>
<td>PE</td>
<td>Graph-minor+Routing paths sharing</td>
<td>Cost Function</td>
<td>TEC</td>
</tr>
<tr>
<td>Bimodal [64]</td>
<td>Integrated</td>
<td>PE+LRB+CU</td>
<td>Adaptive cost function</td>
<td>Cost Function</td>
<td>MRRG</td>
</tr>
<tr>
<td>SPR [51]</td>
<td>Decomposed</td>
<td>PE+LRB</td>
<td>Simulated annealing (SA)</td>
<td>Cost Function</td>
<td>MRRG</td>
</tr>
<tr>
<td>EPIMap [47]</td>
<td>Decomposed</td>
<td>PE</td>
<td>Epimorphism+Re-compute</td>
<td>MaxClique Finding</td>
<td>TEC</td>
</tr>
<tr>
<td>REGIMap [52]</td>
<td>Decomposed</td>
<td>PE+LRB</td>
<td>LRB aware+Reordering</td>
<td>MaxClique Finding</td>
<td>TEC</td>
</tr>
<tr>
<td>MA [54]</td>
<td>Decomposed</td>
<td>PE+LRB+OMB</td>
<td>OMB aware</td>
<td>MaxClique Finding</td>
<td>TEC</td>
</tr>
<tr>
<td>RAMP [55]</td>
<td>Decomposed</td>
<td>PE+LRB+OMB</td>
<td>Re-schedule</td>
<td>MaxClique Finding</td>
<td>TEC</td>
</tr>
<tr>
<td>Ours</td>
<td>Decomposed</td>
<td>PE+LRB+GRB+OMB</td>
<td>All buffers aware</td>
<td>Cost Function</td>
<td>TEC</td>
</tr>
</tbody>
</table>
of mapping DFG onto the Modulo Routing Resource Graph (MRRG), which is one of the representation form of 3D-CGRA. Each operations in DFG are randomly assigned with time and resource, and resource conflict are checked. The resource conflict can be regarded as different operations executed on the same PE and in the same cycle. Their compilers iteratively do this until they either find a valid mapping and the performance can not be further improved (converge), or the mappings reach the compilation time budget and return sub-optimal solution.

DRESC and PSOMap integrate timing, placement and routing during mapping, and the searching space for operations covers both time and space dimensions. All these mappings perform buffer allocating in the last priority, which incurs large rescheduling overhead when the local register buffer spills.

The mappings such as [59] and [65] use 0-1 integer linear programming to solve the mapping problem between DFG and MRRG. Their goal is to find out the set of values of all the variables for the objective function. The ILP based mappings also suffers from relative long compilation time of mapping large DFG onto CGRA with limited computational and routing resources. This is because the time of finding the optimal solution critically hinges upon the power of linear programming solver. The time complexity of the ILP solver increases exponentially with the number of variables and constraints. Thus, ILP based mapping is also challenging for large loops. For example, based on the data in [59], there exists 2 kernels whose operation number in DFG is no more than 30, but the compilation time is larger than 24 hours. Whereas in our test bench, the operation number in DFG can reach 154 at most, using ILP may suffer from extremely long compilation time.

EMS [61] foresees the importance of the routing dependencies between operations. Instead of using the node-centric approach which takes the PE placement as its first priority, EMS takes an edge-centric approach which focus on the routing problem as its primary objective. The EMS lacks the backtracking or remapping mechanism when the mapping is failed, it gains faster compilation speed at the cost of performance. Bimodal [64] scheduler improve EMS by performing back-tracking and using adaptive priority functions and cost functions over different loops to make a better trade-off between performance and compilation time.

Resource-aware [62] mapping use node-centric approach to map each operation according to cost function. When mapping fails, it uses the backtracking method to remap the previous operations under the same II in order to prevent the performance from reduction.

GraphMinor [48] formalizes the mapping problem into finding a subgraph of MRRG whose minor graph is isomorphic to DFG. GraphMinor proposes a path sharing technique to keep data in the shared PE during its life time from producer to multiple consumers. This mechanism can significantly result in better PE utilization. However, GraphMinor lacks the optimization of using buffer resources in CGRA.

In summary, DRESC, PSOMap and AA-ILP are relative generalized heuristics of solving CGRA mapping for any DFG formats. Theoretically, given enough time, they are able to converge to the optimal solution. However, this time is unpredictable and heavily relies on the tunable parameters within the algorithm. In terms of compilation time, they have to adjust the tunable parameters to compromise on the performance, for example, set time budget parameter. Otherwise, the compilation time may become unacceptable, especially when compiling large and irregular loops.

Whereas heuristics such as EMS, Resource-Aware and GraphMinor are dedicated optimization heuristics which capture some specific problems during mapping process, and provide one or multiple dedicated optimizations to help fast search for the optimal solution. Their approach is fast comparing with mappings using classic heuristics, but sacrifice the performance and generality.

### 3.2 Mappings Using the Decomposed Mapping Policy

SPR [51] decomposes mapping into time scheduling process and placement & routing process. SPR permits the operations to be rescheduled beyond slack windows to meet data movement latency, but its mechanism is nearly the same with DRESC [57].

EPIMap [47] formulates the graph epimorphism problem with the additional feature of re-computations. A systematic approach is used to schedule (perform timing) each operation and transform the DFG into a modified graph that satisfies several constraints. It converts the placement and routing process to finding a maximum common subgraph of a time-extended CGRA (TEC) graph which is isomorphic to the modified DFG. To find the subgraph, the mapper build a compatible graph between the modified DFG and TEC where each node represents a possible mapping pair (between operation and PE in certain time slot), and each edge reflects a possible valid mapping. Some edges will be eliminated due to the mapping constraints. Finally, the compiler needs to find a maximum clique whose number of nodes equals to the number of the operations in the modified DFG. For values with long life time, EPIMap prefers to use PE as the routing engine, which occupies the computational resources. Whereas REGImap [52] explicitly utilizes the LRB to buffer the value. It also combines buffer allocating with placement and routing, which is able to effectively avoid the large remapping overhead due to register spilling. Memory-aware [34] is an algorithm that explicitly use OMB during mapping process. Instead of using OMB as the final option that tackling the register spilling, memory-aware mapping put OMB allocating into temporal mapping process. RAMP [55] is an extension of the REGImap, it provided a stronger re-scheduling heuristic to tackle the mapping failure which aims to prevent the compiler from large searching time. RAMP provides a comprehensive choices to solve the routing problem. Right now, it is a very competitive mapping in both performance and compilation time comparing with other mappings. However, the robustness of the compilation time is still questionable in EPIMap, REGImap and RAMP, as the compatible graph is a large graph with dense connectivity, finding maximum clique is also an NP-complete problem, the time complexity of the heuristic they use in their paper is O(N^2) [55]. The critical problem is that these mappings can not judge whether the maximum clique exist and trapped into time consuming searching. Although RAMP provides some strong heuristics of rescheduling, the fetal problem still remains.
In conclusion, for mappings which decompose the temporal and spatial mapping process. The key is to build a reliable mechanism of organizing temporal and spatial mapping phases. It should be able to foresee the impossibility of performing spatial mapping over DFG after temporal mapping and the compiler is able to realize this impossibility without taking too much time.

Thus, our mapping use a totally different philosophy of designing the temporal and spatial mapping. We think temporal mapping should perform efficient optimization and more comprehensive analysis over DFG to guarantee the success of spatial mapping. It should reach the following goals:

- The temporal mapping should decide the schedule time for each operation in DFG and provide the routing solution for all the routing paths inside DFG according to the available routing resources inside CGRA.
- All the buffer resources including PE, LRB, GRB and OMB should be leveraged during temporal mapping for the optimization purpose.
- A powerful routing constraint solving algorithm should be used to check whether the interconnections in CGRA are enough to perform spatial mapping over DFG after temporal mapping.
- A powerful computational constraint solving algorithm should be used to check whether the PEs in CGRA are enough to support the parallelism generated in the temporal mapping phase and re-schedule the time for some operations.

On the other hand, spatial mapping algorithm should be fast and efficient instead of time consuming.

The details of the temporal mapping flow is introduced in Section 4, and the spatial mapping algorithm is introduced in Section 5

### 3.3 Other Related CGRA Compiling Techniques

There is also CGRA mapping work [66] uses dynamic compilation techniques to generate efficient code with lower compilation time. RL-Map [67] leverages the deep reinforcement learning model to build methods that learn to map DFGs onto spatially programmed CGRAs directly from experiences. This approach currently only covers the 2D spatial mapping. The mapping approach like conflict-free [53] combines modulo scheduling and memory conflict optimization to reduce the conflicts when multiple PEs access concurrently access shared data memory. In this way, the synchronization overhead inside CGRA can be efficiently reduced.

Despite the novelty of these works, their execution models and optimization goals are not purely the modulo scheduling problem, therefore, are different with the work in this paper.

### 4 Temporal Mapping Flow

The temporal mapping flow gets the original DFG as the input and outputs a modified DFG that is ready for spatial mapping. In the modified DFG, all the nodes will have a certain schedule time and their data will also be assigned to buffer into a specific buffer resource. If the data is buffered into LRB, there will be additional LRB constraints. Fig. 5d is one example of the modified DFG. It specifies the routing solution, that is, data produced by operation A is buffered in GRB and then consumed by E, F and G. The data produced by B is buffered in LRB and consumed by F, B and F must be mapped onto the same PE.

Fig. 7 shows the organization of the temporal mapping flow. There are six basic algorithms inside our temporal
mapping flow, which are time assignment, graph balancing, routing paths sharing (RPS), buffer allocating, interconnection constraints solving (ICS) and computational constraints solving (CCS). The flow iteratively goes through these algorithms until it thinks the modified DFG is ready for spatial mapping. We will briefly introduce the former three algorithms which are mainly based on existing works [47], [48], [68] and emphasis on introducing the latter three novel algorithms. Before going deeper into each algorithm, we list all the symbols that will appear in the following texts and their descriptions in Table 2.

### 4.1 Time Assignment, Graph Balancing, and Routing Path Sharing

**Time assignment:** The time assignment algorithm statically calculates the schedule time for all the nodes in DFG. The goal of the time assignment is to keep the life time of every variable as short as possible while maintaining the data dependency between operations. The insight of minimizing the lifetime of value is to release the routing resources for the other valid operations as much as possible. Therefore, we use life time sensitive scheduling algorithm [68] to decide the schedule time for each node. The details of the scheduling algorithm can be referred in paper [68]

**Graph Balancing.** The concept of graph balancing is first proposed in EPIMap [47]. Essentially, graph balancing is a technique to guarantee every node gets all of their input data at the same time. When there exists $v_i, v_j \in V$ and $(v_i, v_j) \in E$, if $t(v_i) - t(v_j) > 1$, where $t(v_i)$ denotes the schedule time of node $v_i$, the DFG is not balanced. In this way, a routing node should be inserted between $v_i$ and $v_j$. The number of the routing node added into the path is equal to $t(v_i) - t(v_j) - 1$. Routing node can be regarded as virtually assign a PE to buffer the value for one cycle. The PE keep the value until it is consumed by the successor node. Figs. 5a and 5b reflects the graph balancing example. The dash circles in Fig. 5b represent the routing nodes which keep the result of operation $A$ and $B$. Actually, recall Section 2, all the routing nodes are added into the routing paths.

**Routing Paths Sharing (RPS).** The RPS optimization is first proposed in Graph Minor [48]. The motivation of RPS is to save PEs for routing data. If there exists routing nodes scheduled at the same time and keep the same data, they can share the same predecessor to be a shared routing node. Formally, the RPS is to merge all the routing paths who share the same producer into a single routing path. For example, in Fig. 5b, there are three routing nodes keeping the same data of operation $A$ in time $T_1, T_2$ and $T_3$. Three routing paths from $A$ to $E, F$ and $G$ can share the same data during its lifetime by combining their routing nodes in $T_1, T_2$ and $T_3$ into a single routing node $A^1, A^2, A^3$ and forms a shared routing path (Fig. 5c). In this way, the DFG after RPS saves six routing nodes and edges. We use $sl$ to represent the shared_level of each routing node. The shared_level of the routing node reflects how many successors will consume the same data.

### 4.2 Buffer Allocating

The buffer allocating algorithm is an optimization leveraging PE, GRB, LRB and OMB to buffer values during their lifetime so that the PE is able to be saved for the other valid computations.

#### 4.2.1 Problem Formulation

**Definition 4.1 (Routing path).** Let $P_i$ be the set of all the routing paths in DFG, $D_m(V_m, E_m)$, after routing paths sharing. Let $P_i(V_p, E_p)$ be the routing path $(0 \leq i < |\text{RPS}|)$ inside $P_i$, where $V_p$ and $E_p$ are the node and edge set inside the routing path. $P_i(V_p, E_p)$ is a acyclic weakly connected graph whose $V_g$ must contains one unique producer $v_0^p \in V_p$. $J_i$ routing nodes $v_1^p, v_2^p, \ldots, v_j^p$ ($J_i \geq 1$), and $K_i$ consumers $v_{K_i}^p, v_{K_i-1}^p, \ldots, v_{1}^p$ ($K_i \geq 1$). Let $v_j^p \in V_p^i$ ($1 \leq j \leq J_i$) be the routing node which is $j$ cycles latency relative to the producer $(t(v_j^p) - t(v_0^p) = j)$.

The routing paths in DFG after routing paths sharing have 4 features:

1. Every routing paths has its unique producer and routing nodes.
2. The routing node may have multiple successors, but have only one predecessor.
3. Except for the routing node $v_j^p$, all the successors of the producer $v_0^p$ do not belong to routing path.
4. The life time of routing path $P_i$, $LT(P_i) = J_i + 1 = \max_{v_j^p \in V_p} t(v_j^p)$, where $0 \leq k < K_i$.

Essentially, assigning a routing path to GRB, LRB or OMB is to assign the routing nodes along the path to these buffers. Different buffer assignment over the routing path will lead to different DFG modification rules and constraints.

DFG modification rules of buffer allocating over $P_i$:

1. **GRB assignment rule:** eliminate all the routing nodes in $V_g$ and all the incoming and outgoing edges of these eliminated routing nodes.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$N_pe$</td>
<td>Number of PEs in CGRA</td>
</tr>
<tr>
<td>$N_l$</td>
<td>Size of local register buffer (LRB)</td>
</tr>
<tr>
<td>$N_g$</td>
<td>Size of global register buffer (GRB)</td>
</tr>
<tr>
<td>$D(V, E)$</td>
<td>The original DFG</td>
</tr>
<tr>
<td>$D_m(V_m, E_m)$</td>
<td>The Modified DFG during scheduling</td>
</tr>
<tr>
<td>$R_{GI}(V_R, E_R)$</td>
<td>The time-extended CGRA (TEC)</td>
</tr>
<tr>
<td>Pre$(v)$</td>
<td>Node set which are the predecessor/</td>
</tr>
<tr>
<td>suc$(v)$</td>
<td>successor of node $v$</td>
</tr>
<tr>
<td>I$(S)$</td>
<td>If $S = {v_1, \ldots, v_n}$,</td>
</tr>
<tr>
<td>L$(S)$</td>
<td>If $S = {v_1, \ldots, v_n}$,</td>
</tr>
<tr>
<td>U$(S)$</td>
<td>If $S = {v_1, \ldots, v_n}$,</td>
</tr>
<tr>
<td>T$(S)$</td>
<td>Node set scheduled at time $t$ in node set $S$,</td>
</tr>
<tr>
<td>M$(S)$</td>
<td>Node set in set $S$ scheduled at modulo time</td>
</tr>
<tr>
<td>R$(S)$</td>
<td>Set of all PEs in set $S$ of time slot $j$ in $V_p$,</td>
</tr>
<tr>
<td>C$(S, n)$</td>
<td>The set of combination of selecting $n$ elements from set $S$</td>
</tr>
</tbody>
</table>

**Table 2**

List of Symbols
2. **LRB assignment rule**: same as (1), additionally, same PE constraints that the producer \(u_p^i\) and all the \(K_i\) consumers of \(u_p^i\) should be mapped onto the same PE.

3. **OMB assignment rule**: same as (1), additionally, add store node \(w_s^i\) and edge \(e(u_p^i, w_s^i)\), if there exists consumer \(v_k^j\) (\(0 \leq k < K_i\)), where \(t(v_k^j) - t(u_p^i) > 2\), add load node \(w_l^{i-1}\) and edge \(e(w_l^{i-1}, w_l^i)\).

In order to trigger the rules of assigning the routing path to GRB, LRB and OMB, their corresponding constraints must be satisfied.

**GRB constraints**:

1. The GRB must be available to buffer all the routing paths that is assigned to GRB under \(II\) according to:

\[
N_{req}^i = \left\lfloor \frac{LT(G_i)}{II} \right\rfloor + 1 \quad (1)
\]

\[
\sum_{i=0}^{|GRPs|-1} N_{req}^i \leq N_p,
\]

where \(GRPs\) denotes the set of routing paths which are assigned to GRB, \(G_i(V_p, E_p)\) is the GRB routing path \(i\) and \(N_{req}^i\) denotes the GRB requirement for routing path \(G_i\).

2. In order to get benefits from GRB assignment, there should be at least one routing node eliminated from \(G_i\) according to GRB assignment rule, that is, \(LT(G_i) > 2\).

**LRB constraints**:

We use \(LRPs\) to denote the set of routing paths assigned to LRB. For each routing path \(L_i(V_j, E_j)\) (\(0 \leq i < |LRPs|\)) in \(LRPs\), let \(S_j^i \subset V_j^i\) be the set of nodes which contains only producer \(u_p^j\) and consumers \(v_k^j\). For each routing path \(L_i\), the LRB constraints is performing over its corresponding \(S_j^i\).

1. The performance should not be worse after LRB assignment, that is, \(|S_j| \leq II\).

2. All the nodes inside \(S_j\) can be assigned to the same PE without conflict, they should be distributed in different modulo time, that is, \(\forall j \in [0, II], |M_j(S_j)| \leq 1\).

3. There are enough interconnection resources to support the routing between nodes outside \(V_j\) and same PE nodes in \(S_j\). Let \(V_j\) contains all such node \(v \in V_m\) and \(v \notin V_j\), \(\exists u \in S_j, e(u, v) \in E_m\) but \(e(u, v) \notin E_j\) or \(e(u, v) \notin E_j\). \(\forall j \in [0, II], |M_j(V_j)| \leq \max_{0 \leq j < N_p} |Succ(PE_j)|\),

\[
\leq \max_{0 \leq j < N_p} |Succ(PE_j)|, \quad (3)
\]

where \(|Succ(PE_j)|\) is the number of the interconnections of \(PE_j\).

4. LRB has enough space to buffer the value for \(L_i\) according to:

\[
\frac{LT(L_i)}{II} + 1 \leq N_l, \quad (4)
\]

5. Same as GRB constraint 2, \(LT(L_i) > 2\).

**OMB constraints**:

We use \(ORPs\) to represent the set of routing paths assigned to OMB. In order to benefit from OMB assignment, the latency of any routing path \(O_i(V_o, E_o)\) in \(ORPs\) (\(0 \leq i < |ORPs|\)), \(LT(O_i) > 4\).

**Definition 4.2 (Sub-routing path)**. Routing path \(G_i(V_p, E_p)\) is the sub-routing path of \(P_j(V_p, E_p)\) iff: 1. \(u_p^i \in V_p^j\) is the producer of routing path \(G_i\), for \(\forall v \in V_p\), if \(1 < t(v) - t(u_p^i) \leq LT(G_i)\), \(v \in V_p^j\), 2. if \(v \in V_p\) and \(t(v) - t(u_p^i) = 1\), \(v \in V_p^j\) if \(v\) is the routing node in \(P_j\), 3. for \(\forall u, v \in V_p\), if \(e(u, v) \in E_p\), \(e(u, v) \in E_{p'}\). The relationship between \(G_i, P_j\) can be represented by \(G_i \subseteq P_j\).

Given the routing path set \(RPs\), the buffer allocating problem is to decide the \(GRPs\), \(LRPs\) and \(ORPs\) such that: 1. \(\forall G_i(V_p, E_p) \in GRPs\), \(\exists G_i \subseteq P_j\) and \(G_i; \forall L_i(V_i, E_i) \in LRP_s\), \(\exists L_i \subseteq P_j\); 2. \(\forall O_i(V_i, E_i) \in ORPs\), \(\exists O_i \subseteq P_m\) (\(0 \leq j, k, m < |RPs|\)). 2. All the routing paths in \(GRPs\) satisfy GRB constraints, every routing path in \(LRPs\) satisfies the LRB constraints and every routing path in \(ORPs\) satisfies the OMB constraints. \(D_m(V_m, E_m)\) is the DFG generated by modifying \(D_m\) according to the corresponding rules for every routing path in \(GRPs, LRP_s\) and \(ORPs\). The goal is to maximize the number of eliminated routing nodes, \(|V_m - V_o|\).

**Algorithm 1. Buffer Allocating**

**Input**: \(D_m(V_m, E_m)\), \(II\) and \(BI\)

**Output**: \(D_b(V_b, E_b), SPT\)

1. FinishAllocation ← Failed;
2. \(GRPs, LRP_s, ORPs \leftarrow \emptyset\);
3. while FinishAllocation = Failed do
4. \(D_b \leftarrow D_m\);
5. \(RPs \leftarrow GenerateRoutingPaths(D_b)\);
6. for each \(P_i \in RPs\) of \(D_b\), \(0 \leq i < |RPs|\) do
7. if \(LRB\text{-constrains}\).checking \((S_j^i, II, BI) = True\) then
8. \(LRPs \leftarrow LRP_s \cup P_i\);
9. \(RPs \leftarrow RemovePath(RPs)\);
10. end
11. end
12. \(GRPs, LRP_s, RPs \leftarrow AssignGRB(RPs, BI, GRPs, LRP_s)\);
13. for each \(P_i \in RPs\) of \(D_b\), \(0 \leq i < |RPs|\) do
14. \(LRPs, ORPs \leftarrow CombAssign(P_i, II, BI)\);
15. end
16. \(LRPs \leftarrow MergeLRBPaths(D_b)\);
17. if \(|V_o|/N_p \geq II + 1\) then
18. \(II \leftarrow II + 1\);
19. continue;
20. else
21. \(D_b \leftarrow GRBAssignRule(GRP_s)\);
22. \(D_b, SPT \leftarrow LRBAssignRule(LRP_s)\);
23. \(D_b \leftarrow OMBAssignRule(ORPs)\);
24. FinishAllocation = Success;
25. end
26. end

**4.2.2 Buffer Allocating Overview**

The buffer allocating heuristic is shown in Algorithm 1, the input of the algorithm is the modified DFG \(D_m\) after routing paths sharing, \(II\), and buffer information \(BI\) such as \(N_p, N_l\)
removes C from itself.

II False e to P E Þ i

by inserting one routing path in P, if and only if sl Þ¼ and return over each routing path in

¼ i]u RPs ;E RPs Þg from GRPs; RPs P i are sorted in decreasing order i level of their routing node

¼ where is assigned to two LRBs from different

max of their routing node and i j level from 1 to do

checking ;E constraint u Þ when its ;E and \ if

e nd end

LRB end is the flattened complemented routing path

= f end is the flattened complemented routing path

;E for col-

LRB

each

GRPs

is one of the input to the spatial mapping process which

mapped to the same PE due to the LRB constraints. The SPT (SPT) structure which specifies the operations that must be

must be merged or decomposed if some of them shares the

combinational buffer assignment scheme (line 14) and

and ORPs. All the routing paths in LRP

be merged or decomposed if some of them shares the common node or nodes (line 16). If buffer allocating is not able to release enough PEs so that the theoretical performance can reach II, II will be increased by 1 and the allocating process is repeated until the performance reaches II (line 17 to 19). Otherwise the algorithm finally assign paths in

GRPs, LRP

and ORPs to GRB, LRP and OMB according to their corresponding DFG modification rules (line 21 to 24).

Algorithm 2. AssignGRB

Input: RPs, BI, GRPs, LRP
Output: GRPs, LRP

1 RPs ← SortRoutingPaths(RPs);
2 GRPs ← InitializeGRPs(RPs);
3 max_sl ← getMaxSharedLevel(RPs);
4 for max_sl > shared_level do
5 for each routing path Pi(Vi, Ei) in ordered RPs do
6 for each u, sl in Vi, j from 1 to Ji do
7 if u, sl = shared_level then
8 Gi ← extendPath(Gi, u, j);
9 Li ← OCRP(Gi, Pi);
10 if GRB constraints(GRPs, II) = False then
11 Gi ← remove(Gi, u, j);
12 RPs ← UpdateRPs(GRPs);
13 return GRPs, RPs;
14 end
15 if LRB constraints checking(Si) = True then
16 LRP s ← LRP s ∪ Lj;
17 RPs ← RemovePath(Pi);
18 end
19 end
20 end
21 RPs ← updateRPs(GRPs);
22 return GRPs, RPs;

4.2.3 GRB Assignment

Our GRB assignment approach is shown in Algorithm 2. All the routing paths in RPs are sorted in decreasing order according to the maximum shared_level of their routing node (line 1). At first, GRPs is initialized by setting each path Gi(Vi, Ei) where Vi = {u, j} and Ei = {c(u, j)} (line 2). We gradually extend the path Gi by inserting one routing node in Pi to be the routing node in Gi according to the decreasing order of their shared_level (line 8) and checks the GRB constraints over GRPs until GRP do not have enough space (line 10 to line 12). The algorithm will stop the extension of Gi when its overlapped complemented routing path (OCRP) Li satisfies the LRB constraints.

Definition 4.3 (Overlapped complemented routing path).

Given routing path P±1(V±1, E±1) ⊆ P±1(V±2, E±2) and u±1 = u±2, P±2(V±1, E±2) is the overlapped complemented routing path of P±1, if and only if P±1 ⊆ P±2, V±1 ∩ E±1 = ∅, E±1 ∪ E±2 = E±2, V±1 ∪ V±2 = V±2 and V±1 ∩ V±2 = u±1 where j = LT(P±1).

The insight of this heuristic has two perspectives:

1. Assigning routing node with higher shared_level to GRB in priority is because we want to assign the highly shared routing paths to GRB. Usually these paths are difficult to be efficiently assigned by LRB and OMB such as Fig. 8a.
2. Using the fine-grained approach which assign one routing node each time instead of one routing path is because routing paths with same shared level should have equal opportunity to utilize the GRB resource.

4.2.4 Combinational Buffers Assignment

Definition 4.4 (Flattened complemented routing path). Given routing path P±1(V±1, E±1) ⊆ P±1(V±2, E±2) and u±1 = u±2, P±2(V±1, E±2) is the flattened complemented routing path (FCRP) of P±1, if and only if P±1 ⊆ P±2, V±1 ∩ V±2 = V±2, V±1 ∩ V±2 = ∅, E±1 ∪ E±2 = E±2 = c(u±1, u±2), where j = LT(P±1).

The algorithm of the combinational buffers assignment is shown in Algorithm 3. For each remaining routing path Pi in RPs, we perform the exhausted searching of all the combinations of assigning Pi to LRB and OMB. The routing path Pi is decomposed to two complemented routing paths Pi and Pi. And we evaluate the total saving number of the routing nodes under different combinational buffer assignment schemes and select the one which can save maximum number of the routing nodes (line 9). It should be noticed that if routing path Pi is assigned to two LRBs from different PEs, it is decomposed to two LRB routing paths Pi and Pi (line 7), and Pi is the flattened complemented routing path.
of \( P_i \). This is because if \( P_i \) can not be assigned using LRB from single PE, at least two LRBs from different PEs should be leveraged, which needs an routing node from one PE to another. Recall example in Fig. 4j, there is additional routing from A^2 to A^3.

**Algorithm 3. CombAssign**

**Input:** \( P_i, BI, II \)
**Output:** LRP\( s, ORPs \)
1. for each \( P_i \in \text{LRPs of } D_0 \), \( 0 \leq i < |\text{LRPs}| \) do
2. for \( LT(P_i^0) = 0 \) to \( LT(P_i) \), \( P_i^0 \subseteq P_i \), and \( w_i^{P_i} = w_i^P \) do
3. \( P_i^0 \leftarrow \text{OCRP}(P_i^0, P_i) \);
4. \( P_i^0 \leftarrow \text{FCRP}(P_i^0, P_i) \);
5. \( \text{save}[] \leftarrow \text{calculateLRB}(P_i^0) + \text{calculateOMB}(P_i^0) \);
6. \( \text{save}[] \leftarrow \text{calculateLRB}(P_i^0) + \text{calculateLRB}(P_i^0) \);
7. \( \text{save}[] \leftarrow \text{calculateLRB}(P_i^0) + \text{calculateLRB}(P_i^0) \);
8. end
9. LRP\( s, ORPs \leftarrow \text{SelMaxSave}([\text{save}[]]); \)
10. end
11. return LRP\( s, ORPs \);

**Algorithm 4. LRB Paths Merging and Decomposing**

**Input:** LRP\( s, BI, II \)
**Output:** LRP\( s \)
1. \( \text{CCs} \leftarrow \text{BuildConnectedComponents}(\text{LRPs}); \)
2. for each \( \text{CC}, \in \text{CCs}, 0 \leq i < |\text{CCs}| \) do
3. while \( \text{LRB} \text{constraint checking}(\text{S_merge}) = \text{False} \) do
4. \( \text{CC} \leftarrow \text{MaxMerge}(\text{CC}); \)
5. \( \text{CC}, \text{LRPs} \leftarrow \text{Decompose}(\text{CC}^1, \text{CC}^1); \)
6. end
7. end
8. return LRP\( s; \)

### 4.2.5 LRB Paths Merging and Decomposing

Recall the LRB constraints, for each LRB routing path \( L_i \in \text{LRPs} (0 \leq i < |\text{LRPs}|) \), the LRB constraints are performed over \( S_i \). We construct an undirected graph \( C(V_c, E_c) \), where each routing path \( L_i \) is abstracted to a vertex \( v_{L_i} \in V_c \), and for any two LRB routing paths \( L_i, L_j \) \( (i \neq j) \), the undirected edge \( e(v_{L_i}, v_{L_j}) \in E_c \) if and only if \( S_i \cap S_j \neq 0 \). This abstracted graph \( C \) contains a set of multiple weak connected components CCs (line 1). For each weak connected component \( \text{CC}^1(V_{\text{CC}}, E_{\text{CC}}) \in \text{CCs}, (0 \leq i < |\text{CCs}|) \) the LRB constraints performs over the merged set \( S_{\text{merge}} \), where \( S_{\text{merge}} \) is the union set of all the \( S_i \) corresponding to \( L_i \), such that \( v_{L_i} \in V_{\text{CC}} \).

For example in Fig. 8b, three LRB paths, \( L_1, L_2 \) and \( L_3 \) share the common node \( C \), the connected component is shown in dash frame, and the LRB constraints should perform over \( S_{\text{merge}} = S_1 \cup S_2 \cup S_3 \). When \( S_{\text{merge}} \) can not pass the LRB constraints, strategy should be used to decide which paths can be merged to pass the LRB constraints and which path should be decomposed with the other paths. We refer these two steps as LRB paths merging and decomposing. The \( \text{MaxMerge} \) function performs the combinatorial search for the maximum sub-connected component \( \text{CC}^1 \) from \( CC^1 \) so that: 1. All the routing paths within \( CC^1 \) can be merged so that the \( S_{\text{merge}} \) satisfies the LRB constraints. 2. After \( CC^1 \) is cut from \( CC^1 \), the remaining graph \( CC^m \) is also a connected component. The \( \text{Decompose} \) function implements the process of cutting \( CC^1 \) from \( CC^1 \). Every routing path in \( CC^m \) removes the node, which is the common with the merged paths in \( CC^1 \), from its own path. We iteratively performs merging and decomposing until all the merged routing paths are able to pass the LRB constraints. In Fig. 8c, after LRB path merging and decomposing, \( P_1 \) and \( P_3 \) are merged and \( P_2 \) is decomposed from these paths.

**Algorithm 5. Interconnection Constraints Solving (ICS)**

**Input:** \( D_m, R_{II} \) and II
**Output:** \( D_m \)
1. for \( L - 1 \geq i \geq 0 \) then
2. if \( 1 \leq n \leq \text{Thresholddo} \)
3. for every \( S(i) \in C(T_i, n_0) \) do
4. while \( I_C \text{ checking}(S(i), II) = \text{Failed} \) do
5. if \( n = 1 \) then
6. if \( v \in S(i) \text{ is OriginalNode} \) then
7. \( D_m \leftarrow \text{RescheduleNode}(D_m); \)
8. end
9. else
10. if \( I_{\text{pre}}(S(i)) \text{ has routing node} \) then
11. \( D_m \leftarrow \text{SplitSRNode}(D_m); \)
12. end
13. else
14. if \( I_{\text{pre}}(S(i)) \text{ doesn’t have routing node} \) then
15. \( D_m \leftarrow \text{RescheduleNode}(D_m); \)
16. end
17. end
18. end
19. end
20. end
21. end
22. end
23. end
24. \( II_{\text{new}} \leftarrow \left[D_m \bmod N_{\text{new}} \right]; \)
25. return \( II_{\text{new}}, D_m \)

### 4.3 Interconnection Constraints Solving (ICS)

The ICS checks whether there is enough interconnection resources to support the spatial mapping from \( D_m \) to TEC and then reschedule operations if necessary. Performing ICS can effectively prevent compiler from trapping into time consuming spatial mapping process caused by unawareness of the interconnection constraints.

Our ICS not only checks the interconnection constraints of every single node, for example, the fanout constraints evaluated in most existing mappings [47], [61]. It also checks the interconnection constraints between multiple nodes scheduled at the same time. The algorithm of ICS is shown in Algorithm 5. We start with checking the interconnection constraint for single node (\( n = 1 \)), then for any two nodes (\( n = 2 \)) scheduled at the same time, then for three and so on. Due to the interconnection limitation of the hardware model, the threshold of \( n \) is no larger than 5 under mesh or torus topology. For any combination of selecting \( n \) nodes from all the nodes that are scheduled at time \( i \) (\( S(i) \) in line 3) the \( I_C \text{ checking} \) function checks the interconnection constraints over \( S_i \) according to Eq. (5). The meaning of each symbol can be referred from Table 2. The routing constraints can be illustrated in this way, \( \forall i, m = (i \mod II) \) and \( (1 \leq n \leq |T(i)|)\),
∀S(i) ∈ C(T(i), n) must satisfies Eq. (5). Furthermore, when 
\( \frac{I}{2} \leq 2 \), Eq. (6) must be satisfied. The meaning of the symbols 
in the equation can be referred from Table 2.

For any operation does not pass the interconnection con-
straints checking, we perform RescheduleNode on the ori-
ignal node or SplitSRNode on the routing node. If there is 
routing node added in the critical path, the schedule time of 
all the node must be re-calculated and our compiler will go 
back to the start of the temporal mapping flow and go 
through these sub-algorithms again (Fig. 7).

There are two ways of rescheduling in RescheduleNode 
function, one is re-compute [47], another one is creating a 
routing operation. For example, in Fig. 9a, operation B has 
four fanout nodes. Existing work such as EPIMap [47] is 
able to perform ICS over single node. According to Fig. 9 
the maximum number of PEs that have interconnection 
with single common PE is 3. Whereas operation B has four 
fan-outs, which means the interconnection resources in 
CGRA is not enough to directly map this DFG and B is re-
computed.

Our work is able to find out the interconnection con-
strains between multiple nodes, in Fig. 9c A and B are 
scheduled at the same modulo time, and they have three 
common successors. Accordingly, in CGRA, there should 
exist two PEs which have interconnection with three com-
mon PEs. However, the maximum number of PEs that con-
nects to any two common PEs is 2. The ICS will solve this 
problem by re-computing B 3 times.

Re-computing is not always the correct option for ICS, if 
a node has multiple inputs, re-computing will lead to dou-
ble scale of the input edges. We provide another way by cre-
ating routing node for this operation. An example is shown 
in Fig. 9b, as operation D has three inputs, routing operation 
D (dash circle) is created.

\[
|I_{pred}(S(i))| \leq \max_{\forall S_R(m) \in C(R(m), n)} |I_{pred}(S_R(m))| \\
|I_{succ}(S(i))| \leq \max_{\forall S_R(m) \in C(R(m), n)} |I_{succ}(S_R(m))| \\
\]

(5)

It should be noticed that in LRB constraint checking 
method, we also check the interconnection constraint for 
LRB constraint, this is not included in the ICS phase.

4.4 Computational Constraints Solving (CCS)

Finally, the CCS algorithm checks whether there are eno-
ugh PEs to support the mapping of operations scheduled at 
the same time. Under the modulo scheduling scheme, the 
way of checking computational constraints should be 
∀j ∈ \{0, II\}, M_j(V_m) ≤ N_{pe}. The number of operations that is 
scheduled in a specific modulo time should not be larger 
than the available computational resources. When there 
exists modulo time j, where M_j(V_m) > N_{pe} we select node 
with higher schedule time mobility and re-assign their 
schedule time from modulo time j to i, where i ≠ j and 
M_i(V_m) < N_{pe}. If there is no node with schedule mobility 
larger than one, additional routing node will be added to 
the critical path. In this way, the schedule time of all the 
operations should be analyzed again, our compiler will go 
back to the start point of the temporal mapping flow and go 
through these sub-algorithms again. For example, in the 
middle DFG of Fig. 9a, there are five operations in M0 (A, C, 
D, E, F), but there are only 4 PEs. The CCS add routing node 
B so that E and F are scheduled one cycle later from modulo 
time 0 to 1, and both the number of operations in M0 and 
M1 are 4 (right most DFG in Fig. 9a).

5 Spatial Mapping

5.1 Mapping Overview

The problem of spatial mapping can be formulated as find-
ing the subgraph of II extended TEC, \( R_{(V, E)} \), which is 
isomorphic to \( D_m(V_m, E_m) \), so that each node \( v \in V_m \) 
scheduled at time \( t(v) \) must be mapped on \( P\in E_m(v) \) in \( V_m \).

Algorithm 6 shows the complete mapping process. The 
input of the spatial mapping is the modified DFG \( D_m \) gen-
erated through the temporal mapping flow, II, and same PE 
table (SPT). The SPT is a table structure recording the set of
operations that must be mapped to same PE according to LRB constraints generated from temporal mapping. Fig. 10f is an example of SPT generated from the modified DFG in Fig. 6b. It shows operation A1 and G, B and B2 must be mapped to same PE. If the operation do not have same PE mapping constraint, its same PE set is empty in SPT.

Our mapping algorithm starts with calculating a minimal $II$ (line 1). Then, it iteratively performs the temporal mapping (line 7) and spatial mapping (line 12 to line 57) until the mapping success. When the mapper is not able to map all the nodes of $D_m$, it repeats the temporal and spatial mapping process with a compromised $II$ (line 51 to 54).

5.2 The Spatial Mapping Algorithm

The spatial mapping algorithm places and routes each operation from $D_m(V_m, E_m)$ to $R_{II}(V_r, E_r)$ according to the topology node order list (line 10). When the algorithm is to find a PE slot that is able to be mapped for a node, it first find out all the PE slot candidates and sort them according to the priority (line 25). The principle of selecting the candidate is that, if the mapping node is $v$ in $V_m$, and for any node $u$ in the set of operations that have already been mapped to $PE(u) \in V_r$, the candidate PE slot of node $v$, $PE(v)$, must have interconnection with $PE(u)$:

If $e(u, v) \in E_m$, then $e(PE(u), PE(v)) \in E_r$; if $e(v, u) \in E_m$, then $e(PE(v), PE(u)) \in E_r$.

From the candidates, we select the PE slots with the highest priority, create a mapping pair between this PE and node $v$, and save all the candidates into the failure table (line 27, 28). After the node $v$ is successfully mapped, it then check SPT to see whether there exist the other node that must be mapped onto the same PE of node $v$ according to LRB constraint, and map them onto the same PE if $SPT[v] \neq \emptyset$ (line 29).

The priority function is based on Eq. (7), where $N_r$ denotes the number of PE nodes in TEC, $R_{II}$, that the candidate PE node $v$ can reach through the interconnection between $v$ and the placed PE nodes, and $N_d$ is number of the unplaced predecessors and successors of the mapping node $v$. If $N_r$ is smaller than $N_d$, then placing $v$ on $v_r$ will cause some of $v$’s unplaced predecessors or successors failing to place, due to the lack of interconnection resources. If $N_r$ is equal to or larger than $N_d$, then we choose slot $v_r$ whose $N_r$ is the largest of all. The insight behind this selection is to leave resources as many as possible for the rest unmapped nodes.

$$\text{Priority of candidate } PE = \begin{cases} 0 & \text{if } N_r < N_d \\ \frac{N_d}{N_r} & \text{if } N_r \geq N_d \end{cases}$$

(7)

Backtracking. When mapping fails on a certain node $v$, our algorithm is able to backtrack to the previous node $u$ that may influence the mapping of the failed node (line 31). It erases the saved information in FT and mapping pair (MP) of all the operations after $u$ in $V_{order}$ and select another PE candidates in FT for operation $u$ (line 35 to 39). When there is no node to backtrack, the mapping under $V_{order}$ fails. To further improve the mapping successful rate, we also develop the second insurance plan, which is to remap the DFG according to another topology node order list (line 48, 49).

### Algorithm 6. Mapping Algorithm

| Input: Original data flow graph $D$, and CGRA size. |
| Output: Mapping pairs MP |
| $MII$ ← Calculate_MII($D, CGRASize$); |
| $II$ ← $MII$; |
| $FT$ ← Initialize_FailureTable(); |
| $MP$ ← Initialize_MappingPairs(); |
| $V_{order}$ ← $\emptyset$; |
| repeat |
| $D_m, II, SPT ← Temporal_Mapping(D, II);$ |
| $R_{II} ← Construct_TEC(CGRASize, II);$ |
| if $V_{order} = \emptyset$ then |
| $V_{order} ← OrderNode(D_m);$ |
| end |
| while $|MP| = |V_m|$ do |
| pointer ← $V_{order}.begin();$ |
| while pointer ≠ $V_{order}.end()$ do |
| $v ← pointer;$ |
| if $MP.already\_mapped(v) =$ True then |
| pointer ← pointer.forward.to_next_node(); |
| continue; |
| end |
| if $FT[v] \neq \emptyset$ then |
| $MP ← Insert_MP(v, FT[v][0]);$ |
| pointer ← pointer.forward.to_next_node(); |
| continue; |
| end |
| $v.candidates ← OrderSlotsByPriority(R_{II}, v);$ |
| if $v.candidates \neq \emptyset$ then |
| $MP ← Insert_MP(v, v.candidates[0]);$ |
| $FT[v] ← v.candidates;$ |
| $MP ← Map\_nodes\_in\_SPT(SPT[v]);$ |
| else |
| $v_{bk} ← Node\_To\_Backtrack(v, V_{order}, FT);$ |
| if $v_{bk} = NULL$ then |
| break; |
| else |
| $FT ← Erase\_FT\_Behind\_Node(v_{bk}, FT);$ |
| $FT ← Erase\_MP\_Behind\_Node(v_{bk}, MP);$ |
| $FT[v_{bk}] ← FT[v_{bk}] − FT[v_{bk}[0]];$ |
| pointer ← pointer.backward.to_next_node(); |
| continue; |
| end |
| end |
| pointer ← pointer.forward.to_next_node($V_{order});$ |
| end |
| if $|MP| = |V_m|$ then |
| return $MP;$ |
| else |
| if $ reorder\_time \leq |V_{order}|$ then |
| $V_{order} ← Update\_Node\_Order(D_m);$ |
| reorder\_time ← reorder\_time + 1; |
| else |
| $II ← II + 1;$ |
| Clear\_FailureTable(); |
| Clear\_MappingPairs(); |
| break; |
| end |
| end |
| until $|MP| = |V_m| or II > L$ |
Reordering. The order of mapping each node is an important factor that influence the successful rate of spatial mapping. This is because the modified DFG may contain some special nodes that is hard to be mapped. For example, the node with high degree. These nodes have heavy requirement on the interconnection resources. Another special set of nodes is those mapped onto the same PE, because these nodes have special requirements on the modulo time of PE slot. If we map these special nodes in the latter order, there will be high possibility that the mapping fails at these nodes. In this way, when backtracking also fails to generate the valid mapping, we will generate a new topology order which is different with previous orders and remap the DFG under the same II. We generate the different topology orders of nodes by performing BFS over different source nodes inside DFGs. If all the node order is tried and there is no valid mapping, we increase II by 1, clear the previous mapping status and restart the temporal and spatial mapping process (line 51 to 54).

5.3 Spatial Mapping Example

We use an example of mapping the modified DFG in Fig. 6b onto TEC with II=3, (Fig. 10a). In this TEC, we use PE\textsuperscript{m}\textsubscript{i} to represent the PE slot of PE\textsubscript{i} in modulo time m. The node order list is generated from the BFS topology order starts from operation F, which is the same as GraphMinor [48]. In Figs. 10c, 10d, and 10e, in node order, red color denotes the nodes that have already been mapped, green color is the mapping node, and black color ones are the unmapped nodes. The mapping fails at operation B which has the same PE mapping constraint with B\textsuperscript{2}, after B\textsuperscript{2} is mapped to PE\textsubscript{3}, B should be mapped onto the same PE, PE\textsubscript{3}. However, it is conflict with the already mapped operation F. According to the backtracking algorithm, it backtrack to operation D, all the saved candidates of the operations after D in the node order list are erased from the failure table. The status of the failure table changes from Figs. 10c and 10d. The operation D is remapped to another candidate PE\textsubscript{0} and then, operation B is successfully mapped onto PE2. The final mapping result is shown in Fig. 6c.

In conclusion, we use a fast greedy based approach to map each operation in forward order of the order list. If mapping fails, we use backtracking and reordering to guarantee the mapping successful rate.

6 Evaluation

6.1 Methodology

To evaluate the performance and compilation time of our mapping algorithm, we implement a full stack compiler framework based on LLVM [69] platform. The compiler extracts the loops from the program of different applications and compiles them into the context of CGRA. A runtime simulator is build to evaluate the efficiency of the compiler. We select two state-of-the-art CGRA mapping algorithms as the baseline, REGIMap [52] and RAMP [55]. These two mapping algorithms are two representative ones which decomposes the temporal and spatial process, and have been proven to be competitive in both performance and compilation time over the other CGRA MS algorithms, especially the representative simulated annealing based algorithm DRESC [60]. Our compilation flow is able to deploy the extended C language based loop kernel onto the real energy-efficiency dynamic reconfigurable fabric [70] which target on Berkeley 13 draws [71].

Compilation. We modify the Clang (front-end of LLVM) by adding keywords to specify the loop kernels in the program. Then, we split the loop kernel from the main program in LLVM IR level and the DFG of the extracted loop is generated from LLVM IR. Our mapping algorithm is implemented and integrated into LLVM backend as passes. We also port the open-source code of REGIMap and RAMP (https://github.com/cmlasu/ccf) into the same LLVM framework with our mapping algorithm so that they are able to map the same DFG and their performance and compilation time can be compared fairly. Experiments are conducted on CGRAs with 4x4 PEs and different GRB and LRB sizes. Since PEs are connected in a 2D torus network, all PEs can route data to and access data from neighboring PEs or themselves. The first PE and last PE at each column or row are also neighbors. All experiments are carried out on the same Intel Core i5 machine with CPU frequency of 3.60 GHz.

Test Suite. We take 28 computational intensive loop kernels from different benchmark suites and applications. Some of loops are directly extracted from existing benchmark suite such as EEMBC, MediaBench [72], MiBench [73], MachSuit [74] and Polybench [75]. Others are modified from different application domains, including digital signal processing (DSP), graph searching (Graph), dynamic programming (DP) and computer vision (CV). The size of their loop body varies from 17 to 154 operations. In order to reflect the regularity of the data flow graph, we calculate the number of routing paths (RPs) and number of high degree nodes inside DFG. The RPs shows the number of producer-consumer relationship whose lifetime is larger than 1. More RPs exists in DFG means higher pressure for the compiler to manage the buffer resources. The degree of each operation means the input edge and output edge number of a node. Typically, the high degree nodes are those who have three inputs (sel instruction) or multiple outputs (larger than 1). More high degree nodes inside DFG means higher for the compiler to manage the interconnection resources. Table 3 shows the features of all the loops we select.

6.2 Performance

We evaluate the performance of mapping algorithms over CGRA with different LRB sizes from 2 to 8, and GRB size is 4. For the compilation time, we set one week as the upper bound. If the mapper cannot finish mapping within one week, it will stop the mapping even though the mapper may find the valid mapping without time limitation. The reason of selecting 2, 4 and 8 as the LRB size is because for most of the loop kernels we select, there is almost no difference in performance when LRB is increased from 4 to 8 when using our baseline mapping. GRB size is set to 4 is because we mainly want to test the performance of our mapping when GRB buffer resource is limited.

6.2.1 LRB Size Influence

Fig. 11 shows the performance of REGIMap, RAMP and our mapping approach. The Y axis is the ratio between
minimum II and actual II, which reflects the percentage of the theoretical best performance. Kernels specified with red rectangle frame are those cannot be compiled within one week time budget.

**REGIMap versus Our.** We can observe from Table 4 that when LRB=2, REGIMap gets 0.84 normalized performance among the successfully compiled loops in average, whereas our mapping (Our complete) is able to reach 0.95 in average when GRB=0, and 0.96 when GRB=4. When LRB grows from 2 to 4 and 8, the average performance of REGIMap increase from 0.84 to 0.89, and our approach increase to 0.97 and 0.98 respectively. When LRB grows from 2 to 4 and 8, the average performance of REGIMap increase from 0.84 to 0.89. Out of 28 loop kernels, REGIMap is able to map 15 kernels with optimal performance. 13 kernels cannot be mapped with optimal performance, and 3 of them cannot be mapped within the given compilation time budget. As LRB size grows, the performance of 6 out of 10 kernels are improved. This means LRB resource is important for these 6 loop kernels, but is not useful for the rest of 4 kernels.

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| **TABLE 3** Loop Kernel Features |

Fig. 11. The performance comparison between different mapping algorithms over 4x4 CGRA with different LRB sizes.
These kernels contain irregular routing paths which cannot be routed using LRB. REGIMap choose PE as the routing media, this is why the performance of these 4 kernels do not increase as the LRB grows. Another reason is REGIMap does not use GRB as the first routing choice but position it and OMB as the final solution for LRB spilling. This is also one of the reasons why REGIMap fails the compilation for 3 kernels no matter how large LRB is.

**RAMP versus Our.** We then observe from Table 4 that when LRB=2, RAMP fails map four loop kernels. Among all the successfully mapped loops, RAMP is 0.93 of optimal performance whereas our mapping algorithm is 0.95 when GRB = 0 and 0.99 when GRB = 4. When LRB size grows, RAMP is able to get two more mapped loops *karatsuba* and *unstructured*, but still not able to get the mapping for *harris* and *MD5*. The average performance among all the mapped kernels is also increase slightly. Comparing with REGIMap, RAMP shows stronger mapping ability, if we compare them with the successfully mapped kernels, RAMP is better than REGIMap. This make sense because RAMP improves REGIMap by providing more choices of re-scheduling when map fails, and OMB is also be explicitly used during the temporal mapping phase. However, RAMP only select one strategy when it tries to assign buffer resources for routing paths. It does not consider the hybrid use of these routing choices, especially the rational utilization of the valuable GRB resource.

**6.2.2 GRB Size Influence**

Fig. 13 shows the average performance and compilation time of our mapping over 4x4 CGRA with same LRB size (LRB=2) in each PE and different GRB size (GRB=0, 4, 8, 16). From Fig. 13a we can observe the performance is improved slightly when GRB size increases. This data shows the strong adaptability of our mapping, because even when there is no GRB in CGRA, the performance can still reach 94 percent of the theoretical best performance. However, the shortage of GRB will lead to the increase of compilation time. This is because more routing paths will be assigned to LRB and OMB. Assigning routing paths to LRB brings same PE mapping constraint. This mapping constraint may increase the probability of failure in the forward mapping process and force the spatial mapping to perform backtracking and even reordering in order to keep the performance from decreasing.

**6.3 Performance Degradation Analysis**

Fig. 12 shows the performance degradation after temporal and spatial mapping process. We can observe that performance is mainly degrade during the temporal mapping process but not spatial mapping process. Except for the loop kernel *karatsuba*, the spatial mapping process is able to maintain the performance after temporal mapping. This means as long as all the constraints are find out during our temporal mapping process, our spatial mapping algorithm is able to place and route the modified DFG without performance degradation for most of the loop kernels. This data...
also prove our strategy that paying more effort on the temporal mapping phase and using fast spatial mapping algorithm is effective.

6.4 Compilation Robustness Analysis

Even though our algorithm and two baseline algorithms are all decomposing the temporal and spatial mapping process, but the strategy we use is totally different with theirs. REGIMap and RAMP lacks the process of checking important constraints before spatial mapping, and at the same time, they select a very time consuming spatial mapping heuristic (which is $O((V_m \times N_pe)^8)$ according to their paper). On the other side, we think the importance of these two process should be reversed. The temporal mapping process should be performed more carefully. More precisely, it should make a precise prediction on whether the temporal mapped DFG and be successfully spatially mapped onto CGRA and then make adjustment in time. And for spatial mapping, we do not need to select time consuming heuristic but fast and effective one instead.

Table 4 reflects the compilation successful rate under specific time budget over 4x4 CGRA with different LRB sizes. Our mapping is able to generate all the mappings within time budget. Whereas REGIMap gets 89.3 percent compilation successful rate when LRB = 2, 4 and 8, RAMP gets 85.7 percent when LRB = 2, 92.9 percent when LRB = 4 and 8.

The reason of the failure are different among these kernels. For example, REGIMap fails in fft, harris and unstructured, The reason that REGIMap fails to map is because the DFG of these kernels contains irregular routing paths which is one-producer to multiple consumers. These RPs do not satisfy the LRB constraint. As REGIMap lacks the LRB constraint checking process, it is able to generate the comparable graph but the spatial mapping process can not realize that it can not find the maximum clique no matter how many compilation time is given. RAMP has the same problem as REGIMap, even though it has more choices of selecting the buffer resource. However it only use OMB for long routing path when LRB size is not able to buffer the value during its life time. This makes RAMP still traps into infinite iteratively rescheduling and spatial mapping process for harris kernel. Fortunately, RAMP is still able to map MD5, karatsuba2 and unstructured2 when RAMP is given more LRB or more compilation time budget (larger than one week).

6.5 Compilation Time Analysis

Fig. 14 plots the compilation time of loops over 4x4 CGRA with different LRB size. Same as the way of evaluating performance, when comparing the compilation time with every baseline, we only evaluate the compilation time for those loop kernels that are able to be successfully mapped within given time budget (one week). Table 4 summarizes the average data of the compilation time. When LRB = 2, REGIMap takes 3061 seconds and our mapping takes 14 seconds among 25 successfully compiled loop kernels in average. Whereas RAMP takes 5536 and our mapping takes 14 seconds among
24 successfully compiled loops. Our mapping is x219 and x595 times faster than REGIMap and RAMP respectively.

For each loop, when LRB size grows, the compilation time will decrease. However, it makes sense that both of the average compilation time of the successfully compiled loops using RAMP when LRB=4 and 8 is shorter than that when LRB=2. This is because when LRB=2 RMAP fails in two more loop kernels and their compilation time is very large.

Another observation is our mapping approach is able to map most of the kernels within 1 seconds. However, when compiling large and irregular loops or the node number of the modified DFG is very close to the PE resource in time extended CGRA, our spatial mapping may take longer time because we try different topology orders to get the best mapping result.

7 Conclusion

In this paper, we provide a novel methodology of solving the CGRA modulo scheduling problem. We leverage the decomposed mapping policy and argue that temporal mapping flow is far more important than we expect for mappings using decomposed policy. We stress on developing the systematic temporal mapping flow which contains comprehensive buffer allocating heuristic and interconnection and computational resources constraint solving algorithms to improve the mapping performance and guarantee the success rate of the following spatial mapping. Together with the lightweight, fast and efficient spatial mapping algorithm, our approach is able to generate high mapping success rate within certain compilation time budget. Furthermore, the experiment results show that our mapping is able to generate higher performance and lower compilation time.

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References


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Lock-Free Parallelization for Variance-Reduced Stochastic Gradient Descent on Streaming Data

Yaqiong Peng®, Member, IEEE, Zhiyu Hao®, and Xiaochun Yun

Abstract—Stochastic Gradient Descent (SGD) is an iterative algorithm for fitting a model to the training dataset in machine learning problems. With low computation cost, SGD is especially suited for learning from large datasets. However, the variance of SGD tends to be high because it uses only a single data point to determine the update direction at each iteration of gradient descent, rather than all available training data points. Recent research has proposed variance-reduced variants of SGD by incorporating a correction term to approximate full-data gradients. However, it is difficult to parallelize such variants with high performance and accuracy, especially on streaming data. As parallelization is a crucial requirement for large-scale applications, this article focuses on the parallel setting in a multicore machine and presents LFS-STRSAGA, a lock-free approach to parallelizing variance-reduced SGD on streaming data. LFS-STRSAGA embraces a lock-free data structure to process the arrival of streaming data in parallel, and asynchronously maintains the essential information to approximate full-data gradients with low cost. Both our theoretical and empirical results show that LFS-STRSAGA matches the accuracy of the state-of-the-art variance-reduced SGD on streaming data under sparsity assumption (common in machine learning problems), and that LFS-STRSAGA reduces the model update time by over 98 percent.

Index Terms—Gradient descent methods, machine learning, parallel computing, multicore, streaming data

1 INTRODUCTION

1.1 Motivation

The core of machine learning problems is to learn a mathematical model from training datasets, in order to make predictions or decisions. The model is typically characterized by a decision variable, namely a vector of weights \( w \in \mathbb{R}^d \). Let us consider a set of \( n \) training data points \( S \). Fitting a model to \( S \) is usually cast as an optimization problem of minimizing the finite sum form:

\[
\mathcal{R}_S(w) = \frac{1}{n} \sum_{i=1}^{n} f_i(w),
\]

where \( f_i(w) \) is the loss of \( w \) on the \( i \)th data point in \( S \). \( \mathcal{R}_S(w) \) is often assumed to be convex. The goal of machine learning problems is typically to find the decision variable \( w^* \) that minimizes \( \mathcal{R}_S(w) \). Gradient descent methods are widely used to pursue \( w^* \).

There are two common gradient descent methods: Batch Gradient Descent (BGD) and Stochastic Gradient Descent (SGD) [1], [2]. BGD uses all training data points to compute the full gradient at each iteration of gradient descent. In BGD, the cost of a gradient computation increases with the size of the training dataset. In contrast, SGD selects a single data point randomly from the training dataset to compute the gradient at each iteration of gradient descent, and thus SGD is especially suited for a large dataset. While an iteration of SGD is cheaper than that of BGD, its variance tends to be high. Remarkable recent progress has been made to propose Variance-Reduced Variants of SGD (VR SGD) by incorporating a correction term to approximate a full-data gradient generally in two ways. The first variance-reduced method stores the last gradient computed at each data point and uses their average at each iteration of gradient descent [3], [4], [5]. Another method periodically computes a gradient based on a batch of data points [6], [7], [8], requiring more computation than the first method.

Meanwhile, parallelization is a crucial requirement for large-scale applications. In parallel setting, many asynchronous variants of SGD are proposed on both a single multicore machine and distributed systems. Although the variance-reduced improvements yield a more accurate model in general, it is more challenging to parallelize such variants with high performance and accuracy, especially on streaming data arriving as an endless sequence of data points, which is a common scenario in computer systems. The first variance-reduced method mentioned above requires using the average of historical gradients. In parallel setting, it is difficult to maintain the average on streaming data with low cost, because the size of training dataset changes over time. Section 2.3 analyzes the problem in detail. In addition, the second method is also not suitable for this scenario, because maintaining a model on streaming data is limited in processing time.

In this paper, we focus on the parallel setting in a multicore machine with shared memory, and explore how to parallelize VR SGD on streaming data with high performance and accuracy. Although large datasets are generally suggested to be processed on a cluster of machines, the size of the data necessary for statistical analysis may be a few terabytes or less after appropriate preprocessing in many problems [9]. Such problems can be processed on a single multicore machine, rather than an expensive cluster. In addition, multicore systems have significant performance advantages due to low latency and high throughput of shared main memory [9].
1.2 Key Ideas
Overcoming the challenges of parallelizing VR SGD on streaming data with high performance and accuracy, we present an algorithm named LFS-STRSAGA based on the state-of-the-art serial streaming data version of VR SGD named STRSAGA [10]. STRSAGA belongs to the first variance-reduced method mentioned in Section 1.1. It stores the last gradient computed for each data point in a global vector. To parallelize STRSAGA, LFS-STRSAGA embraces a lock-free concurrent data structure to process the arrival of streaming data. In addition, LFS-STRSAGA replaces the global vector of historical gradients with a local one for each thread to asynchronously maintain, and the model update rule based on the thread-local vector of historical gradients is sparse. To pursue high performance, LFS-STRSAGA updates the decision variable without using any locking like HOGWILD! [9], which is the foundation for the most parallel implementations of SGD and its variants. Our theoretical analysis shows that LFS-STRSAGA retains the linear convergence rate of STRSAGA to reach statistical accuracy in sparsity assumption. The sparsity assumption means that most update steps only modify small and disjoint parts of the decision variable. This phenomenon exists in most machine learning problems.

Like STRSAGA, LFS-STRSAGA is the choice in the settings where memory is not limited. Nowadays, the settings given the abundance of memory are common in many practical situations [10]. As mentioned in Section 7, we will explore how to restrict LFS-STRSAGA to use limited memory in the future work.

1.3 Experimental Evaluation
To show the competitiveness of LFS-STRSAGA in practice, we compare it to the original version of STRSAGA [10] and the natural streaming data version of HOGWILD! [9] (referred to as S-HOGWILD! in this paper) in terms of model update time and sub-optimality (defined in Section 4.2). We consider the support of all the evaluated algorithms for two popular machine learning problems, Logistic Regression and Matrix Factorization, on real-world datasets under various arrival distributions. The experimental results show that LFS-STRSAGA matches the accuracy of STRSAGA, and that it reduces the model update time by over 98 percent. In addition, LFS-STRSAGA is more accurate than S-HOGWILD!.

1.4 Contributions
In summary, this paper makes the following contributions:

- The presentation of LFS-STRSAGA: a lock-free approach to parallelizing VR SGD algorithm on streaming data.
- A theoretical analysis showing that LFS-STRSAGA matches the accuracy of STRSAGA in sparsity assumption.
- An experimental evaluation demonstrating the competitiveness of LFS-STRSAGA in practice.

1.5 Outline
The rest of the paper is organized as follows. Section 2 provides a foundation for LFS-STRSAGA. Section 3 presents the details of LFS-STRSAGA, followed by a theoretical analysis in Section 4. Section 5 provides a comprehensive evaluation of LFS-STRSAGA, and Section 6 overviews the most related work. Section 7 concludes the paper and discusses our future work.

2 Preliminaries
In this section, we first discuss the basics with respect to LFS-STRSAGA in more detail, and then look into the key ideas of typical SGD algorithms and the variance-reduced improvements. Finally, we discuss the challenges for parallelizing VR SGD on streaming data.

2.1 Basics
Empirical Risk Minimizer. The core of machine learning problems is to learn a mathematical model, which is drawn from a class of functions \(F\) and parameterized by a vector of weights \(w \in \mathbb{R}^d\) called decision variable. The expected risk of a function \(f \in F\) is defined as \(R(w) = \mathbb{E}[f(x)|w]\), where \(f(x)|w\) denotes the loss of \(w\) on input \(x\) and the expectation is taken over all \(x\) drawn from some underlying probability distribution \(P\) (unknown to the algorithm). For example, given a data point \((x, y)\), the corresponding loss in L2-regularized logistic regression problem is: \(f(x,y)(w) = \log(1 + e^{-yw^T x}) + 1/2\|w\|_2^2\). Let \(w^* = \min_{w \in F} R(w)\) denote the decision variable minimizing \(R(w)\). Then, \(R(w^*)\) denotes the minimum expected risk over \(F\). Given a sample \(S\) consisting of \(n\) training data points drawn from \(P\), the empirical risk function measuring the average loss of \(w\) over \(S\) is defined as:

\[R_S(w) = \frac{1}{n} \sum_{x \in S} f(x, w)\]

The best we can do is to find the Empirical Risk Minimizer (ERM), namely the weights \(w_S^* = \arg\min_{w \in F} R_S(w)\).

Statistical Efficiency. Suppose that an algorithm produces an approximate solution \(w_S\) over \(S\). \(\mathbb{E}[R_S(w_S^*) - R(w^*)]\) is called statistical error, where the expectation is over the randomness of \(n\)-sample \(S\). The statistical error is usually bounded by \(H(n) = cn^{-\alpha}\), where \(c\) is a constant and \(1/2 \leq \alpha \leq 1\). \(\mathbb{E}[R_S(w_S) - R_S(w_S^*)]\) is called optimization error. Let \(\epsilon(n)\) be an upper bound on the optimization error, then the total error is bounded by \(H(n) + \epsilon(n)\). Given a computational budget, \(\epsilon(n)\) is typically increasing with \(n\), whereas \(H(n)\) is always decreasing. To minimize the total error, the literature on learning theory suggests reducing \(\epsilon(n)\) to asymptotically balance with \(H(n)\) [11].

Streaming Data Scenario. This paper considers the maintenance of a model in the same streaming data scenario as mentioned in [10]: Let \(X_i\) denote the training data points (zero or more) arriving at time step \(i\), and each data point is drawn from some underlying probability distribution \(P\) (unknown to the algorithm). Let \(S_i\) denote the set of data points arriving in time steps \(1\) to \(i\) \((S_i = \cup_{j=1}^i X_j)\). The optimal solution over \(S_i\) is denoted as \(w_S^* = w_{S_i}^* = \arg\min_{w \in F} R_{S_i}(w)\). Our goal is to produce a solution \(w_i^*\) of which the empirical risk approximates \(R_{S_i}(w_i^*)\) to the best. We focus on using SGD-style algorithms to achieve this goal.

2.2 SGD and its Variance-Reduced Variants
SGD is an iterative algorithm widely used to optimize problems of the finite-sum form. Generally, SGD starts with a known initial vector of weights \(w^0\), and updates the vector repeatedly. In every iteration, SGD samples a data point
uniformly from the training dataset, and computes the gradient at this point based on the current state of decision variable. Then, it uses the gradient to determine the update direction of the decision variable. Given the value of $w$ at the end of iteration $t$ (referred to as $w^t$), the updates for iteration $t + 1$ in SGD are generally as follows:

1. Sample a data point $i$ uniformly from $S$ at random.
2. Compute the gradient $\nabla f_i(w^t)$ at data point $i$.
3. Update $w$: $w^{t+1} = w^t - \eta \nabla f_i(w^t)$, where $\eta$ is the step size.

Because $w$ is updated through a gradient computed at a single data point in the standard SGD, the variance of the update direction tends to be high. To ensure convergence, decreasing step sizes are generally chosen for SGD, resulting in a slower sub-linear convergence rate than using constant step sizes. The variance-reduced variants of SGD are proposed to solve this problem. Here, we focus on the update scheme of SAGA, which is a state-of-the-art variance-reduced method. SAGA starts with a known initial vector of weights $w^0$, and uses a table $a$ to store historical gradients. The length of $a$ is equal to the number of training data points. Each element $a(p)$ in $a$ is the last gradient computed for each data point $p$ (referred to as $a(p) \in \mathbb{R}^d$), initialized to $\nabla f_p(w^0)$. Given the value of $w$ and each $a(p)$ at the end of iteration $t$, the updates for iteration $t + 1$ in SAGA are as follows:

1. Sample a data point $i$ uniformly from $S$ at random.
2. Compute the gradient $\nabla f_i(w^t)$ at data point $i$.
3. Update $w$ using $\nabla f_i(w^t)$, $a(i)$ and the historical gradient average: $w^{t+1} = w^t - \eta \left( \nabla f_i(w^t) - a(i) + \frac{1}{n} \sum_{i=1}^{n} a(i) \right)$, where $n$ is the number of training data points.
4. Update $a(i)$, and other historical gradients remain unchanged: $a(i) = \nabla f_i(w)$

To control the computation cost, SAGA computes a gradient at a single data point like SGD. To approximate a full-data gradient, SAGA stores the last gradient computed at each data point and uses their average at each iteration of gradient descent. In this way, SAGA yields a more accurate model than SGD. Note that SAGA is an offline algorithm, which assumes the entire dataset is available beforehand. To the best of our knowledge, STRSAGA is the best performing VR SGD algorithm for handling streaming data, which is designed based on SAGA. As mentioned in Section 2.1, to guarantee the model accuracy, it is suggested to reduce the optimization error to asymptotically balance with the statistical error. Intuitively, it means that it is better to concentrate the computational budget on fewer data. Following this principle, STRSAGA increases the size of the sample set in a controlled manner. It does not add new data points into the effective sample set until a sufficient number of SAGA steps have been performed on the current sample set.

### 2.3 Challenges for Parallelizing Variance-Reduced SGD on Streaming Data

As parallelization is a crucial requirement for large-scale applications, this paper discusses the parallelization of VR SGD algorithm on streaming data with high performance and accuracy, and presents LFS-STRSAGA based on STRSAGA. To implement LFS-STRSAGA, we face the following challenges.

#### Challenge 1. How to sample from dynamically changing dataset in parallel?

In streaming data scenario, with the arrival of new data points, the training dataset changes over time. In parallel setting, concurrent threads may contend for changing the state of training dataset at the same time. Following STRSAGA, we add new training data points to the sample set in a controlled manner, and sampling threads only pick data points from the effective sample set for gradient computation. Similar to changing the training dataset, concurrent threads may contend for adding one training data point. In addition, changing the state of training dataset must be notified to sampling threads without preventing them from making progress. Therefore, we need to carefully design the lock-free synchronization scheme to coordinate threads with changing the state of training dataset and sample set in parallel.

#### Challenge 2. How to maintain the average of last gradients computed at different data points with low cost?

Note that computing the average of historical gradients from scratch at each update step incurs non-trivial computation cost. To solve this problem, STRSAGA updates the sum of last gradients computed at different data points incrementally and computes the average. To adopt this method on streaming data at an update step, our approach needs to get the size of the dynamic sample set. In parallel setting, suppose that a thread first reads the sum of historical gradients and then the size of sample set, and that some new data points are added into the sample set in the time interval between the two read operations. In this case, the sum of historical gradients and the size of sample set read by this thread does not correspond to the same state of the effective sample set. In addition, different threads may contend for updating the historical gradient associated with a data point at the same time. Therefore, in parallel setting, it is difficult to maintain the average of historical gradients with low cost in a lock-free manner.

### 3 LFS-STRSAGA: A Lock-Free Parallel Variant of Variance-Reduced SGD on Streaming Data

In this section, we present the details of LFS-STRSAGA. We begin with the high-level design of LFS-STRSAGA, followed by presenting a lock-free data structure for storing the streaming data. Next, we discuss how to process the arrival of new training data points based on the lock-free data structure. Finally, we introduce the details of update scheme used in LFS-STRSAGA.

#### 3.1 High-Level Design of LFS-STRSAGA

**Computation Model.** Like most concurrent algorithms, LFS-STRSAGA is designed for asynchronous shared memory systems [12], where an application runs with $n$ deterministic threads. In LFS-STRSAGA, threads are split between dedicated producers and consumers. Fig. 1 shows a diagram for this computation model. As shown in the diagram, between any two time steps $i$ and $i + 1$ ($i > 0$), producer and consumer threads work in parallel. To be specific, producer

1. Assume STRSAGA computes a gradient $g$ at a data point $p$ at an update step. The current sum of historical gradients is $s$, and the last gradient computed at $p$ is $g$. Then, STRSAGA updates the sum to $s + g - g$. 
threads continuously process the storage of new data points arriving in the time interval \([i, i + 1]\), and consumer threads update the model over a set of data points observed till the time step \(i\). The time step 1 is the initial time for LFS-STRSAGA, at which no data points have arrived. Because LFS-STRSAGA is time efficient, consumer threads will complete the model update steps over the data points observed till the time step \(i\) before the next time step \(i + 1\).

Following STRSAGA, LFS-STRSAGA adds the new data points into the effective sample set after performing a sufficient number of update steps on the current sample set. As mentioned in Section 2.3, we face two challenges to achieve this goal in parallel. For challenge 1, we carefully design an infinite array. Fig. 2 provides a diagram for the array.

As shown in Fig. 2, the infinite array is a 4-tuple \((Dataset, \tilde{T}, \tilde{S}, N)\). Dataset is used to store the observed data points. \(T, S, \) and \(N\) are three indexes dividing the data points of Dataset into three classes. Dataset\([0, \ldots, \min(T, S)]\) acts as the effective sample set in LFS-STRSAGA. \(T\) is only updated atomically by consumer threads for adding data points into the effective sample set. \(S\) denotes the number of data points that have been deposited into Dataset, which is only updated by producer threads. \(S\) marks the upper boundary for the effective sample set. In other words, consumer threads extend the effective sample set at most to Dataset\([0, \ldots, S]\). The storage of a new data point consists of two atomic operations performed by a producer thread that first gets an index and then deposits the data point into the corresponding location of Dataset. If \(S\) is used to distribute the indexes of new data points, consumer threads may sample a data point that only gets an index but has not been deposited into Dataset. Therefore, we set another global variable \(N\) for distributing the index of every new data point processed by producer threads. Like \(S\), \(N\) is only updated atomically by producer threads. At each time step \(i (i > 0)\), a producer thread (referred to as main producer) is responsible for updating \(S\) to let consumer threads see all the stored data points arriving till the time step \(i\). All the initial values of \(T, S, \) and \(N\) are 0.

Like the work on concurrent array-based data structures \([13], [14]\), the infinite array can be implemented by a singly-linked list of equal-sized array segments. \(S\): number of data points that have been deposited into Dataset, initialized to 0; \(N\): number of data points that have arrived now, initialized to 0.

Algorithm 1. Storing Training Data Points by Individual Producer Threads

**Global shared variables:** Dataset\([1, \ldots]\): infinite array for storing observed data points, which can be implemented by a singly-linked list of equal-sized array segments; \(S\): number of data points that have been deposited into Dataset, initialized to 0; \(N\): number of data points that have arrived now, initialized to 0.

1: for \(i = 1, 2, 3, \ldots\) do 2: for every time receive a data point \(P\) arriving between time step \(i\) and \(i + 1\) do 3: // store the observed data point 4: \(j \leftarrow \text{FAA}(\&N, 1)\) 5: Dataset\([j] \leftarrow P\) 6: end for 7: Wait for other producer threads 8: if \(i\) am the main producer then 9: // let consumer threads see the new data points 10: \(S \leftarrow N\) 11: end if 12: Wait for other producer threads 13: end for

3.2 Storing New Training Data Points

Algorithm 1 presents how to store training data points by individual producer threads. Once a new data point arrives, LFS-STRSAGA distributes it to a thread for storing it into the array named Dataset. As different threads may process the storage of new data points at the same time, we need to consider the problem that threads could overwrite each other’s progress by depositing different data points into the same location of the array. LFS-STRSAGA uses the Fetch-and-Add (FAA) \(^2\) atomic operation to solve this problem. A thread first obtains the index of the location for storing the data point by performing a FAA on the index \(N\) (line 4), and then stores the data point in the associated location of the array (line 5). The atomic feature of FAA ensures each data point gets a unique index. Among the producer threads, a thread acts as a main producer. At time step \(i + 1 (i > 0)\), before all the producer threads continue to store the training data points arriving between the time steps \(i + 1\) and \(i + 2\), the main producer updates \(S\) to the current value of \(N\) in order to let consumer threads see all the stored data.

2. FAA(addr, v): assuming the content stored in addr are \(a\), this operation returns \(a\) and stores \(a + v\) at addr.
points arriving till the time step \( i + 1 \) (lines 8-11). Then, consumer threads start to update the model over the stored data points, and producer threads continue to process data points arriving between the time steps \( i + 1 \) and \( i + 2 \).

**Algorithm 2. Model Updated Over a Set of Training Data Points \( S_i \), That Arrived in Time Steps 1 to \( i \) by Individual Consumer Threads, \( i > 0 \)**

Global shared variables: \( \omega \): decision variable; \( Dataset[s,1,\ldots] \): storing observed data points; \( \tilde{S} \): number of data points that have been deposited into \( Dataset \), initialized to 0; \( T \): index of next data point added to the effective sample set, initialized to 0; \( t \): iteration count, initialized to 0 at each time step.

Thread-local variables: \( \alpha \): a table for storing local historical gradients, initialized to 0; \( next.p \): recording the index of the next data point added by the current thread to the effective sample set, initialized to -1; \( sum \): the sum of all entries in \( \alpha \), initialized to 0 and maintained incrementally (\( d \) is the number of features in a gradient);

\[
\begin{align*}
\text{for} & (t \leftarrow \text{FAA}(\omega, t); t < \rho; t \leftarrow \text{FAA}(\omega, 1)) \text{ do} \\
\text{effective\_sample\_size} & \leftarrow 0 \\
\text{if} & t \text{ is even then} / / \text{try to extend the effective sample set in the even iteration number} \\
\text{if} & next.p \neq -1 \text{ and } next.p < \tilde{S} \text{ then} / / \text{add the data point} \\
\text{point} & \text{ reserved in the previous even iteration} \\
\text{p} & \leftarrow next.p, next.p \leftarrow -1 \\
\text{effective\_sample\_size} & \leftarrow p + 1 \\
\text{else} & / / \text{otherwise, reserve the index returned with FAA} \\
\text{next.p} & \leftarrow p \\
\text{end if}
\end{align*}
\]

3.3 Update Scheme

Algorithm 2 shows the details of updating the model over a set of training data points \( S_i \), that arrived in time steps 1 to \( i \) by individual consumer threads. The algorithm mainly repeats three procedures: 1) pick a data point for gradient computation (lines 3-21); 2) update the decision variable by using the picked data point and local historical gradients to approximate the full-data gradients (lines 22-26); 3) update the local historical gradients and their sum (lines 27-28). Table 1 describes the symbols used in Algorithm 2.

Before presenting our sample strategy, let us first recall the sample strategy adopted by the latest version of STRSAGA. At each time step, STRSAGA starts to perform \( \rho \) iterations of gradient descent. To control the expansion of the effective sample set, STRSAGA first adds the new data points to a buffer. Then, it chooses a new data point from the buffer every two iterations, and adds the data point to the effective sample set followed by computing a gradient at this data point. In other iterations, STRSAGA directly samples a data point from the effective sample set for gradient computation without expanding the effective sample set. In LFS-STRSAGA, \( Dataset[T,\ldots,S] \) and \( Dataset[0,\ldots,\min(T,S)] \) are equivalent to the buffer and effective sample set in STRSAGA, respectively.

Parallel Sample Strategy. LFS-STRSAGA distributes the \( \rho \) iterations of gradient descent starting at each time step to consumer threads dynamically, and coordinates these threads to expand the effective sample set in a controlled manner by performing FAA operations on \( \bar{T} \) every two iterations (line 10). Note that consumer threads extend the effective sample set at most to \( Dataset[0,\ldots,S] \). When \( \bar{T} \) is updated to a value larger than \( S \), \( Dataset[0,\ldots,S] \) denotes the effective sample set. Therefore, before performing a FAA in the even iterations, a consumer thread first judges whether \( \bar{T} \) is currently less than \( S \) (line 8). If yes, the thread first performs a FAA on \( \bar{T} \) (line 9), and then judges whether the value returned with FAA is less than \( S \) (line 10). If yes, it means that the value returned with FAA is equal to the index of a data point that has been stored into the array \( Dataset[s,1,\ldots] \), and the thread selects the data point for gradient computation afterwards. Otherwise, because \( \bar{T} \) is monotonically increased by FAA operations, LFS-STRSAGA may not ensure a sufficient number of gradient computation is performed on data points stored afterwards. To solve the

| \( |\alpha| \) | number of gradients contained in \( \alpha \) |
| \( e(p) \) | indexes of the coordinates on which the values of the data point indexed by \( p \) are non-zero |
| \( \eta \) | step size |
| \( \alpha(p) \) | if the local thread has not ever computed any gradient at the data point indexed by \( p \), return 0; otherwise, return the last gradient computed at the data point indexed by \( p \) |
| \( x_v \) | value of the vector \( x \) (e.g., \( \omega, g, \) and \( \alpha(p) \)) on the coordinate indexed by \( v \) |
| \( e_v \) | among the data points that have been picked by the local thread to update the decision variable, fraction of the data points that are non-zero on the \( v \)th coordinate |
problem, when the value returned with FAA is not less than 
$S$, the thread records the value in next $p$ (line 13) as the 
candidate index considered in the next even iteration (lines 4-
6). Therefore, at the start of every even iteration, a consumer 
thread first checks whether an index is reserved in next $p$ 
and the reserved index is less than $S$ (line 4). If yes 
(next $p \neq -1$ and next $p < S$), the thread directly selects 
the reserved index as the target index, and resets next $p$ to 
-1 for the next reservation (line 5). If the current iteration 
number is odd or the consumer thread fails to extend the 
effective sample set in the even iteration number, the thread 
directly samples a target index uniformly from $[0, \min(T, 
S))$ (lines 19-20). Once the target index is determined, the 
consumer thread computes a gradient at the indexed data 
point on the current state of $w$ (lines 22).

Asynchronous Strategy for Approximating Full-Data Gradients. 
Different from STRSAGA, each consumer thread maintains a 
local vector of historical gradients to approximate full-data 
gradients in LFS-STRSAGA, and incrementally updates the 
sum of the local historical gradients. Because the local vector 
of historical gradients is only updated by its owner thread, 
LFS-STRSAGA avoids the inconsistent read problem mentioned 
in Section 2.3. Under this scheme, each consumer thread can easily maintain the average of local historical 
gradients with low cost. As mentioned in Section 2.2, SAGA-style 
algorithm use the gradient computed at a random data point 
and the average of historical gradients to update $w$. To reduce 
the computation expenses, we borrow an idea from ASAGA 
[15]. When computing the average of the local historical 
gradients, each thread only considers the coordinates on which 
the values of the picked data point are non-zero in LFS-
STRSAGA (lines 24-26). To make the update unbiased, assuming 
the picked data point is indexed by $p$, we multiply the value 
of the sum of the local historical gradients on the $v$th coordinate 
($v \in c(p)$) by a coefficient $\frac{1}{n}$ (line 25) for approximating 
the full sum. The definition of $e_v$ is given in Table 1. 
Then, we get the average of local historical gradients on the 
vth coordinate by the equation $\overline{w} = \sum_{v \in c(p)} w_v$.

At the end of each iteration, we update $\alpha$ with the last 
gradient computed at the data point indexed by $p$ (line 28), 
and the sum of local historical gradients incrementally (line 
27). When all the consumer threads complete the dispatched 
iterations of gradient descent, they wait for the update 
phase at the next time step, and the current effective sample 
set is referred to as $T_i$.

4 THEORETICAL ANALYSIS

In this section, we analyze the accuracy of LFS-STRSAGA in 
terms of sub-optimality. According to the proof technique 
in previous work [10], the accuracy of a streaming data VR 
SGD algorithm depends on the convergence rate of its off-
line version. Therefore, we first present the offline version of 
LFS-STRSAGA, and then analyze its convergence rate in 
Section 4.1. Finally, we show that LFS-STRSAGA retains the 
linear convergence rate of STRSAGA to reach statistical 
accuracy in sparsity assumption in Section 4.2.

Following the assumptions in previous work [5, 9, 10, 
16], we assume that all $f_k$ are convex and their gradients 
are $L$-Lipschitz continuous. In addition, $R_S$ is $\mu$-strongly 
convex for the training dataset $S$.

4.1 Convergence Analysis for Offline Version of 
LFS-STRSAGA

The proof technique of STRSAGA relies on the convergence 
rate of SAGA, which can be seen as the offline version of 
STRSAGA. The convergence rate of SAGA is $1-\min(\frac{\lambda}{\mu}, \frac{\lambda}{\mu})$. 
Following this proof technique, we first define the offline 
version of LFS-STRSAGA as follows:

Offline Version of LFS-STRSAGA. For the offline version of 
LFS-STRSAGA, the entire training dataset $S$ is available 
beforehand. Thus, the effective sample set is fixed with size 
n. The offline version of LFS-STRSAGA starts with a known 
initial vector of weights $w^0$, and each processor core $c$ uses a 
table $\alpha(c)$ to store local historical gradients. The length of 
each $\alpha(c)$ is equal to the number of training data points. Each 
element $\alpha(c(i), \alpha(c)$ initialized to 0. Following Recht et al. 
[9], we define $w^t$ to be the state of $w$ after $t$ updates have been 
performed. Intuitively, $t$ refers to the global iteration 
counts. As $w^t$ is generally updated with a state gradient 
based on the state of $w$ read many clock cycles earlier, we use 
$w^{k(t)}$ to denote the state of $w$ when the update to $w^t$ was 
read. Assuming the update step is run on the $c$th core in the 
global $t$th iteration, the updates for iteration $t + 1$ in the 
offline version of LFS-STRSAGA are as follows:

1. Sample a point $i$ uniformly from $S$ at random.
2. Compute the gradient $\nabla f_i(w^{k(t)})$ at point $i$.
3. Update $w$ using $\nabla f_i(w^{k(t)})$, $\alpha(c, i)$ and the average of 
local historical gradients: $w^{k+1} = w^t - \eta g(w^{k(t)}, c, i)$, where 
$g(w^{k(t)}, c, i) = \nabla f_i(w^{k(t)}) - \nabla f_i(w^{k(t)}) - \alpha(c, i) + D_\bar{P}(c', \alpha(c, i))$ denotes the last local gradient computed locally on the 
$c$th core at the data point indexed by $i$, $D_\bar{P}(c', \alpha(c))$ denotes the average of historical gradients 
maintained on the $c$th core, and $D$ is a diagonal 
matrix equal to $\frac{1}{n}$ on the $c$th diagonal and zeros 
elsewhere ($c \in c(i)$). The definitions of $c_e$ and $c(i)$ 
are given in Table 1.
4. Update $\alpha(c, i)$, and other local historical gradients 
remain unchanged: $\alpha(c, i) = \nabla f_i(w^{k(t)})$.

There are three differences between the offline version of 
LFS-STRSAGA and SAGA: (1) the offline version of LFS-
STRSAGA uses the local historical gradients and their 
sparse average at each iteration of gradient descent, rather 
than the full average of historical gradients like SAGA; (2) 
co-running threads in the offline version of LFS-STRSAGA 
lead to that $w^t$ is generally updated with a stale gradient 
based on the state of $w$ read many clock cycles earlier; (3) 
threads could overwrite each other’s progress of updating 
the decision variable, which cannot occur in the serial 
SAGA. In the following, we will prove that the offline 
version of LFS-STRSAGA has the same convergence rate as 
SAGA in the big data regime under sparsity assumption.

Unbiased Condition for an Update Scheme. The unbiased 
condition is the heart of most convergence proofs for the 
asynchronous parallel SGD and its variants. To prove that 
the update schemes adopted in the offline version of LFS-
STRSAGA are unbiased, we need to show $Eg_\alpha(w^{k(t)}, c', i) = \nabla f_\alpha(w^{k(t)})$, where $\nabla f_\alpha(w^{k(t)})$ is a full-data gradient: $\nabla f_\alpha(w^{k(t)}) = \frac{1}{n} \sum_{k=1}^{n} \nabla f_k(w^{k(t)})$.

Lemma 1: $Eg_\alpha(w^{k(t)}, c', i) = \nabla f(w^{k(t)})$ is satisfied in LFS-
STRSAGA.
Suppose the total number of cores is $S$ and $\prod_{i} = E_{t \in d^{2R}} \left[ \sum_{k=1}^{n} \sum_{\rho \in \rho(k)} \frac{g_{i}(c')}{\rho} \right]$. Note that $d_{i}$ denotes the number of training data points (in $S$) equal to non-zero on the $t$th coordinate. $x_{i}$ is equal to 1 on the $t$th coordinate and zeros elsewhere. According to the definition given in Table 1, the values of $c_{1}, c_{2}, \ldots, c_{d}$ depend on the data points picked for gradient computation in the previous iterations. Because these data points are picked uniformly from the sample set, we have: $E_{v_{i}} = \frac{\rho}{n}$ ($v = 1, 2, \ldots, d$).

Thus, $E_{D}(c') = \mathbb{E}(c')$. For any $c', \mathbb{E}a(c', i) = \frac{1}{\sum_{k=1}^{n}} \sum_{\rho \in \rho(k)} \frac{g_{i}(c')}{\rho}$. Then, $E_{D}(\pi(v, c'), i) = E_{D}(f_{i}(w^{k}(i)) - \mathbb{E}a(c', i) = E_{D}(\frac{1}{\sum_{k=1}^{n}} \sum_{\rho \in \rho(k)} \frac{g_{i}(c')}{\rho})$. This completes the proof.

Following Recht et al. [9], for any $w^{k}$ and $w^{k'}$, we define the bound of $t - k$ as $\tau$. In addition, we define $\Delta$ as: $\Delta = \max_{i \in [m], c \in [d]} \left[ \sum_{t \leq n} \rho_{t} \right]$, i.e., the maximum frequency that any feature appears in a training data point. With Lemma 1 holding, we reach the following lemma.

**Lemma 2:** (Combining Theorem 2 and Corollary 3 in [15]) Suppose $\tau \leq O(n)$ and $\tau \leq O(\frac{1}{\sqrt{n}} \log(\frac{1}{\sqrt{n}})).$ Let $a^{*}(\tau) = \frac{1}{\sum_{t \leq n} \rho_{t}}$, where $\xi(\kappa, \tau) = \sqrt{1 + \frac{1}{\sqrt{\kappa}} \min(\frac{1}{\sqrt{\kappa}}, \tau)}$ and $\kappa = \frac{1}{\sqrt{n}}$. Then using the step size $\eta = \frac{\rho^{1/2}}{n}$, the offline version of LFS-STRSAGA has the same convergence rate as SAGA: $E_{D}(f_{i}(w^{k}(i)) - f_{i}(w^{*})) \leq \rho C_{0}$, where $\rho = 1 - \min(\frac{1}{\frac{1}{\sqrt{n}}}, \tau)$, and $C_{0}$ is a constant independent of $t$.

When a machine learning problem is very sparse, the precondition of Lemma 2 is easily satisfied because $\Delta$ is a very small value in this case. Therefore, the offline version of LFS-STRSAGA has the same convergence rate as SAGA in sparsity assumption, which is a common phenomenon in machine learning problems.

### 4.2 Sub-Optimality of LFS-STRSAGA

Now, with sparsity assumption, we analyze the accuracy of LFS-STRSAGA in terms of the expected sub-optimality.

**Metric for Sub-Optimality.** Following Jothimurugesan et al. [10], we define the sub-optimality of an algorithm $A$ over the training dataset $S$ as follows:

$$\text{SUBOPT}_{S}(A) = \mathcal{R}_{S}(w) - \mathcal{R}_{S}(w^{*}),$$

where $w$ is the solution produced by $A$ and $w^{*}$ is the optimal solution over $S$.

To prove the sub-optimality of LFS-STRSAGA, we use the upper bound function $U$ defined in [5]

$$U(t, n) = \min \left\{ \rho_{n} U(t - 1, n) \prod_{m < n} U(t, m) + \frac{ew}{n} \mathcal{H}(m) \right\}.$$  \hspace{1cm} (2)

For a given function $U(x, y)$, $x$ denotes the number of iterations that have been performed, and $y$ denotes the effective sample size at the end of iteration $x$. When using the $U$ function to analyze the sub-optimality of LFS-STRSAGA, $\rho_{n}$ is the convergence rate of the offline version of LFS-STRSAGA for the effective sample set with size $n$.

Let $T_{i}$ denote the effective sample set at the end of all update steps started at time step $i$ in LFS-STRSAGA, and let $t^{i}_{i}$ denote the size of $T_{i}$. We reach the following lemma.

**Lemma 3:** Suppose the condition number $L$ and the initial sub-optimality of LFS-STRSAGA are bounded by a constant. At the end of each time step $i$, the expected sub-optimality of LFS-STRSAGA over $T_{i}$ is: $E_{D}(\text{SUBOPT}_{T_{i}}(LFS-STRSAGA)) \leq 1 + O(1) \mathcal{H}(t^{i}_{i})$.

**Proof:** In LFS-STRSAGA, each thread adds a new data point into the effective sample set every two iterations by a FAU. Suppose the total number of iterations performed by all the consumer threads is $t_{i}$ at the end of each time step $i$. The atomic feature of FAU ensures that $t_{i} \geq 2t^{i}_{i}$. According to Equation (2), we have $E_{D}(\text{SUBOPT}_{T_{i}}(LFS-STRSAGA)) \leq U(t_{i}, t^{i}_{i}) \leq \rho_{i}^{t_{i}} \mathcal{H}(t^{i}_{i})$. According to Lemma 2, $0 < \rho < 1$. Therefore, we have $E_{D}(\text{SUBOPT}_{T_{i}}(LFS-STRSAGA)) \leq U(2t^{i}_{i}, t^{i}_{i}) \leq \mathcal{H}(t^{i}_{i})$. Similar to the proof of Lemma 3 in [10], we have $E_{D}(\text{SUBOPT}_{T_{i}}(LFS-STRSAGA)) \leq \mathcal{H}(t^{i}_{i}) + \frac{\rho_{i}^{t_{i}}}{(1 - \rho_{i})^{1/2}} = 1 + O(1) \mathcal{H}(t^{i}_{i})$, where $C_{0}$ denotes the initial sub-optimality of LFS-STRSAGA. This completes the proof.

**Lemma 3** bounds the expected sub-optimality of LFS-STRSAGA over its effective sample set. However, our goal is to bound the expected sub-optimality of LFS-STRSAGA over $S_{i}$, where $S_{i}$ denotes the data points that have been observed till the time step $i$. Let $T_{i}$ denote the effective sample set at the end of all update steps started at time step $i$ in STRSAGA, and let $t^{i}_{i}$ denote the size of $T_{i}$. Lemma 3 in [10] shows that $E_{D}(\text{SUBOPT}_{T_{i}}(SAGA)) \leq 1 + O(1) \mathcal{H}(t^{i}_{i})$. Both LFS-STRSAGA and STRSAGA produce solutions over the same $S_{i}$, $t^{i}_{i}$ is at most $t^{i}_{i}$. The proof used to bound the expected sub-optimality of STRSAGA over $S_{i}$ also applies to LFS-STRSAGA. Therefore, LFS-STRSAGA retains the linear convergence rate of STRSAGA to reach the statistical accuracy over $S_{i}$ in sparsity assumption.

### 5 Experimental Results

In this section, we evaluate the competitiveness of LFS-STRSAGA in terms of model update time and sub-optimality, and compare it to the state-of-the-art SGD algorithms and variance-reduced improvements. All the algorithms are implemented in C++, and the experiments are run on a machine consisting of four eight-core 2.0 GHz Intel Xeon E7-4809 v3 processors. Each core supports two hyper-threads.

#### 5.1 Experimental Methodology

We compare LFS-STRSAGA to the following algorithms:

- **STRSAGA**: A state-of-the-art serial VR SGD algorithm.
- **S-HOGWILD!**: A natural streaming data version of HOGWILD!. To make the comparison fair, S-HOGWILD! also performs $\rho$ gradient computations to update the model at each time step $i$, and adopts the same sample scheme as LFS-STRSAGA. Note that our sample scheme is better than the original sample scheme used in HOGWILD!.
LF-STRSAGA: An algorithm that can be seen as a middle ground between LFS-STRSAGA and S-HOGWILD! The only difference between LF-STRSAGA and LFS-STRSAGA is that a thread uses the full average of local historical gradients in LF-STRSAGA at an update step.

We take STRSAGA as the performance and accuracy benchmarking because it is the original building blocks of LFS-STRSAGA. S-HOGWILD! is selected as a competitor because HOGWILD! is the foundation for the most parallel implementations of SGD and its variants. To show the impact of the sparse processing for the average of local historical gradients, we implement LF-STRSAGA and compare it to LFS-STRSAGA.

Benchmarks. We evaluate all the algorithms on two widely used machine learning problems: Logistic Regression and Matrix Factorization. For a data point in the logistic regression problem, the L2-regularized logistic loss function is: $f_{i,j}(w) = \log(1 + \exp(-y_iw^T{x_j})) + \frac{\lambda}{2}\|w\|^2_F$. We consider the matrix factorization problem of finding two rank-10 matrices. For a data point $M_{i,j}$ in the matrix factorization problem, the regularized loss function is: $f_{i,j}(w) = ((L^T L)_{i,j} - M_{i,j})^2 + \frac{\lambda}{2}(\|L\|^2_F + \|R\|^2_F)$. For the logistic regression problem, we use 2 real-world datasets (RCV1 [17] and URL [18]) to evaluate all the algorithms, while using 1 real-world dataset (MovieLens1M [19]) for the matrix factorization problem. Tables 2 and 3 describe the details of the datasets and the step sizes used in experiments. We obtain the optimal step size using grid search on single-threaded SGD after a single pass of each dataset. In the following, we refer to the logistic regression problem using RCV1 and URL datasets as RCV1 and URL benchmarks, respectively. Similarly, the matrix factorization problem using MovieLens1M dataset is referred to as MovieLens1M benchmark. In all the benchmarks, threads are evenly divided into dedicated producers and consumers. If a thread acts as a dedicated producer, it only receives and stores new data points at different time steps. Otherwise, the thread only updates the model decision variables at different time steps.

Arrival Distributions. Note that the datasets given in Tables 2 and 3 are static. Following Jothimurugesan et al. [10], we convert these static training data points into streams. For each dataset in our experiments, all the training data points arrive over the course of 100 time steps, and the number of data points arriving in each time step $i$ is a random variable $x_i$. Our experiments are conducted under two arrival distributions: skewed arrivals and Poisson arrivals. We choose the skewed arrivals and Poisson arrivals because they represent two commonly arrival patterns. The skewed arrival models bursty arrivals, which combines a number of time steps without arrivals of new data points and occasional bursts of new data points, reflecting the worst case for streaming data algorithms to deal with at limited processing rates. Meanwhile, a large number of arrival events follow the Poisson distribution in real scenarios. In skewed arrivals, $x_i = M$ with a probability $\frac{\lambda}{M}$, and $x_i = 0$ with a probability $1 - \frac{\lambda}{M}$, where $M > 0$ is an integer. In Poisson arrivals, $x_i$ follows a Poisson distribution. In both arrivals, $\mathbb{E}[x_i] = \lambda$, where $\lambda = \frac{\text{Number of Training Data Points}}{\text{Number of Time Steps}}$.

5.2 Model Update Time
In this subsection, we present our experimental results for the model update time of different algorithms over the course of 100 time steps in all the benchmarks. For a given benchmark, the model update time of each algorithm running with different number of consumer threads is normalized to the model update time of STRSAGA. To avoid the scalability problem of parallel SGD-style algorithms on modern multicore machine with Non-Uniform Memory Access (NUMA) architecture [20], we allocate all the consumer threads to one processor and all the producer threads to another processor. In all the benchmarks, the number of consumer threads is up to 16, which is the maximum parallelism of one processor on our testbed like [9], [21]. Fig. 3 shows the normalized model update time of different algorithms for all the benchmarks under Poisson arrivals. The lower update time indicates the better performance.

As shown in Fig. 3, with the number of consumer threads increasing, the model update time of LF-STRSAGA is significantly reduced for all the benchmarks. Benefit from the sparse processing on the average of historical gradients, LFS-STRSAGA further reduces the model update time of LF-STRSAGA by over 98 percent. This is because the tested datasets are sparse and LFS-STRSAGA only considers the average of historical gradients on a small fraction of the coordinates when updating the decision variable, yielding less computation expenses than LF-STRSAGA. The time complexity of LFS-STRSAGA and LF-STRSAGA depends on the machine learning problem and the characteristics of datasets. In logistic regression problem, assuming $d$ and $density$ are the number of features and the average sparse density of a given dataset, the time complexity of updating the decision variables $w$ is $O(density)$ in LFS-STRSAGA and

### Table 2

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Number of Training Data Points</th>
<th>Number of Features</th>
<th>Density</th>
<th>Steps Size $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCV1</td>
<td>677399</td>
<td>47236</td>
<td>0.15%</td>
<td>0.5</td>
</tr>
<tr>
<td>URL</td>
<td>2396130</td>
<td>3231961</td>
<td>0.004%</td>
<td>0.005</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Users</th>
<th>Movies</th>
<th>Date Range</th>
<th>Rating Scale</th>
<th>Density</th>
<th>Steps Size $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MovieLens1M</td>
<td>6040</td>
<td>3952</td>
<td>4/2000-2/2003</td>
<td>1-5, stars</td>
<td>4.47%</td>
<td>0.005</td>
</tr>
</tbody>
</table>
In matrix factorization problem, assuming \(row, col, rank\) are the row, column, rank numbers of a given data matrix, the time complexity of updating the decision variables \(w\) is \(O(rank)\) in LFS-STRSAGA and \(O(row \times rank + col \times rank)\) in LF-STRSAGA. The experimental results under skewed arrivals have the similar figures.

5.3 Sub-Optimality

In this subsection, we evaluate the accuracy of different algorithms in terms of their sub-optimality over the data points that have been observed till each time step \(i\). The metric for sub-optimality is defined in Equation (1) (as shown in Section 4.2). Lower sub-optimality indicates higher accuracy.

Figs. 4 and 5 show the sub-optimality of different algorithms for all the benchmarks under skewed arrivals and Poisson arrivals, respectively. In both figures, the top row indicates the case in which all the algorithms (except STRSAGA) run with 1 consumer thread, and the bottom row indicates the case in which they run with 16 consumer threads. Because LF-STRSAGA with 1 consumer thread is equivalent to STRSAGA, we only take STRSAGA as the accuracy benchmarking in the case that other algorithms run with 16 consumer threads. Because LF-STRSAGA with 1 consumer thread is equivalent to STRSAGA, we only take STRSAGA as the accuracy benchmarking in the case that other algorithms run with 16 consumer threads. We can observe that both LFS-STRSAGA and LF-STRSAGA significantly outperform S-HOGWILD! in most cases because they benefit from the faster convergence rate by using variance-reduced method. At some time steps, the sub-optimality spikes because many new data points arrive now and the number of data points that have yet to be processed bursts. Because the sparse processing on the average of historical gradients loses some information from other coordinates, LFS-STRSAGA is slightly less accurate than LF-STRSAGA in some cases. When the number of consumer threads reaches to 16, the sub-optimality of LFS-STRSAGA almost matches the sub-optimality of STRSAGA for RCV1 and URL in most cases, but is relatively bigger for movielens1m. This is because the movielens1m dataset is more dense than another two datasets. More dense the dataset is, more update conflicts among threads will occur.

**Summary.** LFS-STRSAGA almost matches the accuracy of STRSAGA. Although LFS-STRSAGA is less accurate than LF-STRSAGA, it has a big advantage in the model update time. Because maintaining an accurate model on streaming data is limited in processing time, LFS-STRSAGA is a better alternative than LF-STRSAGA.

6 RELATED WORK

Our work is related to the research on gradient descent methods and their parallel implementations. In this section, we briefly discuss the most related work in turn.
Gradient Descent Methods

Gradient descent methods are widely used to pursue model parameters in the training phase of machine learning problems [22], [23]. According to the amount of data points used at each update step, gradient descent methods are generally divided into two classes: Batching Gradient Descent (BGD) and Stochastic Gradient Descent (SGD) [1]. BGD computes the full-data gradient to determine the update direction at each iteration of gradient descent. In contrast, SGD selects a single data point randomly from the training dataset to compute the gradient, requiring much less computation than BGD but missing the information from other data points. Thus, the variance of SGD can be high. To solve the problem, variance-reduced variants of SGD are proposed generally by incorporating a correction term that approximates a full-data gradient at each iteration of gradient descent [3], [4], [6], [7], [8], [24].

Roux et al. propose SAG [3], a representative variance-reduced method that stores the last gradient computed for each data point and uses their average at each iteration of gradient descent. Inspired from SAG, Defazio et al. propose SAGA [4], which optimizes SAG by eliminating a bias in the update step. Johnson et al. propose SVRG [6] that periodically computes a full-data gradient without the storage of historical gradients. Obviously, SVRG requires more computation than SAG and SAGA. To reduce the computation cost, Konečný et al. propose Semi-Stochastic Gradient Descent (S2GD) [7] based on SVRG. It uses a random-data-gradient and a full-data gradient at each iteration occasionally. Shah et al. propose another variant of SVRG, named CHEAPSVRG [8], which computes the gradient on a subset of training data points.

To improve the convergence rate of variance-reduced methods, Daneshmand et al. suggest to gradually increase the size of effective sample set [5]. They apply this idea to SAGA and propose DYNASAAGA [5]. As a result, DYNASAAGA achieves the statistical accuracy in $O(n)$ iterations, while all of the above methods require $O(n \log n)$ iterations. Jothimurugesan et al. extend DYNASAAGA to maintain a model over streaming data, and propose STRSAGA [10]. STRSAGA significantly outperforms another state-of-the-art streaming version of VR SGD named Streaming SVRG (SSVRG) [25]. Because fitting an accurate model over streaming data requires the optimization algorithm to be time-efficient, we present LFS-STRSAGA, which can be seen as a lock-free parallelization of STRSAGA. Compared to STRSAGA, LFS-STRSAGA almost matches the accuracy of STRSAGA under sparsity assumption, and LFS-STRSAGA reduces the model update time by over an order of magnitude.

Parallelization of SGD and its Variants

Parallelization is a crucial requirement for large-scale applications. In parallel setting, many asynchronous variants of SGD are proposed on both a single multicore machine and distributed systems. The seminal text presented by Bertsekas and Tsitsiklis [26] is the foundation for the most work on parallelizing SGD and its variants. The authors present ideas that using the stale gradient computed across many computers in a master-worker setting at each update step and different processors only access particular components of the decision variable. Inspired from these ideas, Niu et al. propose a lock-free approach to parallelizing stochastic gradient descent named HOGWILD! [9]. The update scheme of HOGWILD! allows processors to directly modify the decision variable without using any locking at each iterations of gradient descent. When most updates only modify small and disjoint parts of the decision variable, HOGWILD! achieves a nearly optimal rate of convergence. Nguyen et al. further proves the convergence of HOGWILD! without the bounded gradients assumption [27], and Alistarh et al. proves the convergence of parallel SGD in asynchronous shared memory [28]. The key idea of HOGWILD! is extended to the most work on lock-free parallelization of VR SGD including the asynchronous variants of coordinate descent [29], SVRG [21], and SAGA [15]. So far, all the parallel implementations of SGD-like methods focus on the offline algorithms, in which the entire training dataset is assumed available beforehand. In this paper, we focus on streaming
data arriving as an endless sequence of data points. In this setting, because the size of training dataset changes over time, it is more challenging to parallelize VR SGD with high performance and accuracy. The main challenge of parallelizing VR SGD on streaming data is how to store new data points and sample from dynamically changing dataset in parallel. The research work on non-blocking data structures [13], [14], [30] inspires the design of the data structures adopted in LFS-STRSAGA.

7 CONCLUSION AND FUTURE WORK

In this paper, we present a lock-free approach to parallelizing the VR SGD algorithm on streaming data. Specifically, we have focused on a modification of STRSAGA by presenting a lock-free data structure to process the arrival of streaming data in parallel, and asynchronously maintain the essential information to approximate the full-data gradients with low cost. Both our theoretical and empirical results show that LFS-STRSAGA matches the accuracy of the state-of-the-art VR SGD algorithm on streaming data in sparse setting that is common to machine learning problems, and that it reduces the model update time by over an order of magnitude.

This paper focuses on the design of LFS-STRSAGA for the settings where memory is not limited. We will explore how to restrict LFS-STRSAGA to use limited memory in our future work. For example, we can set a maximum size for the effective sample set. When the number of the data points added to the effective sample set reaches to the maximum size, some data points will be removed from the effective sample set. One problem is how to choose the data points for removal, and an alternative scheme is to remove most sampled data points because a relatively sufficient number of SAGA steps have been performed on these data points. Another problem is how to remove any data point from the effective sample set without using any locking. In addition, we also consider to enhance the scalability of LFS-STRSAGA on NUMA systems.

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