

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to convert an array, with priorities stored at subscripts 1 through  $n$ , to a minheap is in:

A.  $\Theta(n)$       B.  $\Theta(\max(m, n, p))$       C.  $\Theta(n^3)$       D.  $\Theta(mnp)$

2. The number of calls to `mergeAB` while performing `mergesort` on  $n$  items is:

A.  $\Theta(\log n)$       B.  $\Theta(m + n)$       C.  $\Theta(n)$       D.  $\Theta(n \log n)$

3. Which of the following is not true?

A.  $n^2 \in O(n^3)$

B.  $n \log n \in \Omega(n^2)$

C.  $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$

D.  $3^n \in \Omega(2^n)$

4. The cost function for the optimal matrix multiplication problem is:

A.  $C(i, j) = \min_{i \leq k < j} \{C(i, k) + C(k, j) + P_{i-1}P_kP_j\}$

B.  $C(i, j) = \min_{i \leq k < j} \{C(i, k) + C(k + 1, j) + P_iP_kP_j\}$

C.  $C(i, j) = \min_{i \leq k < j} \{C(i, k) + C(k + 1, j) + P_{i-1}P_kP_j\}$

D.  $C(i, j) = \max\{C(i, j - 1), C(i - 1, j)\}$  if  $x_i \neq y_j$

5. The function  $n + 3n \log n$  is in which set?

A.  $\Omega(n^2)$       B.  $\Theta(\log n)$       C.  $\Theta(n)$       D.  $\Theta(n \log n)$

6.  $\log(n!)$  is in all of the following sets, except

A.  $\Omega(\log n)$       B.  $\Theta(n \log n)$       C.  $O(n^2)$       D.  $\Omega(n^2)$

7. Which statement is correct regarding the unweighted and weighted activity scheduling problems?

A. Both require dynamic programming

B. Both are easily solved using a greedy technique

C. Unweighted is solved using a greedy technique, weighted is solved by dynamic programming

D. Weighted is solved using a greedy technique, unweighted is solved by dynamic programming

8. The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:

A. to assure that the final solution is free of duplicate values

B. to determine the longest possible increasing subsequence terminated by a particular input value

C. to search a table that will contain only the LIS elements at termination

D. to sort the original input

9. Suppose you have correctly determined some  $c$  and  $n_0$  to prove  $f(n) \in O(g(n))$ . Which of the following is not necessarily true?

A.  $c$  may be increased      B.  $n_0$  may be decreased      C.  $n_0$  may be increased      D.  $g(n) \in \Omega(f(n))$

10. Suppose you are using the substitution method to establish a  $\Theta$  bound on a recurrence  $T(n)$  and you already know

$T(n) \in \Omega(n \log n)$  and  $T(n) \in O(n^3)$ . Which of the following cannot be shown as an improvement?

A.  $T(n) \in O(n \log n)$       B.  $T(n) \in O(n)$       C.  $T(n) \in \Omega(n^2)$       D.  $T(n) \in \Omega(n^2 \log n)$

Short answer. 3 points each

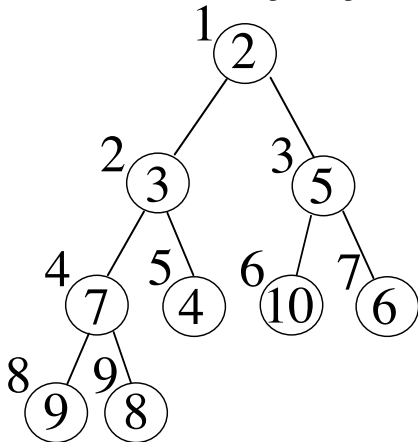
1. Give the value of  $H_4$ .
2. What is  $n$ , the number of elements, for the largest table that can be processed by binary search using no more than 6 probes?
3. Give the subscripts for the parent, left child, and right child for the maxheap element stored at subscript 451. (The heap is currently storing 1000 elements in a table with 2000 slots.)
4. Give the worst-case and expected number of (key) comparisons when insertion sort is used to sort tables with 100 keys.
5. What is the main issue when using a greedy algorithm?

Long Answer

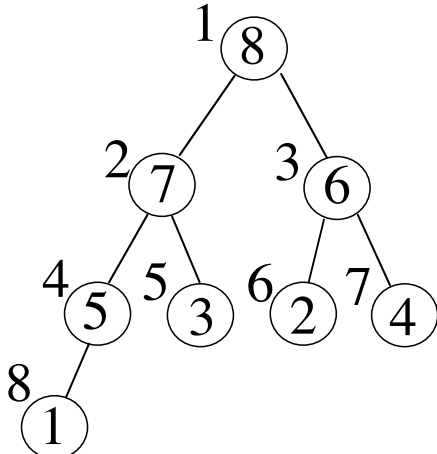
1. Prove that if  $f(n) \in O(g(n))$  then  $g(n) \in \Omega(f(n))$ . 5 points
2. Use dynamic programming to find a longest common subsequence of the following sequences. Be sure to provide the DP table and the backtrace for your LCS. 10 points  

0	1	2	0	1	2
0	0	1	1	2	2
3. Use the recursion-tree method to show that  $T(n) = 2T(\frac{n}{4}) + 1$  is in  $\Theta(\sqrt{n})$ . 10 points
4. Use the substitution method to show that  $T(n) = 2T(\frac{n}{4}) + 1$  is in  $\Theta(\sqrt{n})$ . 10 points
5. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points  

A	0.25
B	0.03
C	0.5
D	0.03
E	0.12
F	0.07
6. a. Show the minheap after performing `getmin`. 5 points



- b. Show the maxheap after changing the priority at subscript 6 to 9. 5 points



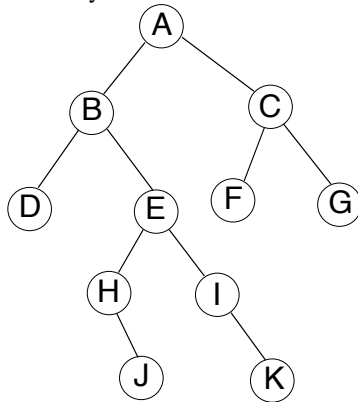
Fall 2007

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Why is it common for a circular queue implementation to waste one table element?

- A. To avoid confusing an empty queue with a full queue
- B. To have a place to store the tail and head values
- C. To make sure the queue always has at least one element in use
- D. To perform some loops faster

2. Suppose the tree below is a binary search tree whose keys are not shown. Which node will contain the key that is the predecessor of the key stored at K?

- A. A
- B. E
- C. F
- D. I

3. Which of the following sorts is not based on key comparisons?

- A. COUNTING-SORT
- B. INSERTION-SORT
- C. MERGESORT
- D. QUICKSORT

4. What is the worst-case time to perform MINIMUM(L) for a unsorted, doubly-linked list with  $n$  nodes?

- A.  $\Theta(1)$
- B.  $\Theta(\log n)$
- C.  $\Theta(n)$
- D.  $\Theta(n \log n)$

5. In the example of recycling the elements of a list in  $O(1)$  time, which situation holds?

- A. Both lists are not circular
- B. Both lists are circular
- C. The garbage list is circular, the list to be recycled is not
- D. The list to be recycled is circular, the garbage list is not

6. When evaluating a prefix expression, the stack contains

- A. Both operands and operators
- B. Both parentheses and operators
- C. Operands only
- D. Operators only

7. An array with 1500 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?

- A. If 1468 is returned from PARTITION, we must continue.
- B. If 1469 is returned from PARTITION, we are done.
- C. If 1470 is returned from PARTITION, we must continue.
- D. If 1481 is returned from PARTITION, we must continue.

8. Suppose a red-black tree has a sentinel. What is the first step to search for key  $x$ ?

- A. Compare  $x$  to the root's key
- B. Inorder traversal
- C. Copy the value of  $x$  into the sentinel
- D. Use rotations to bring the node of  $x$  to the root position

9. Which of the following may be performed in  $\Theta(1)$  worst-case time?

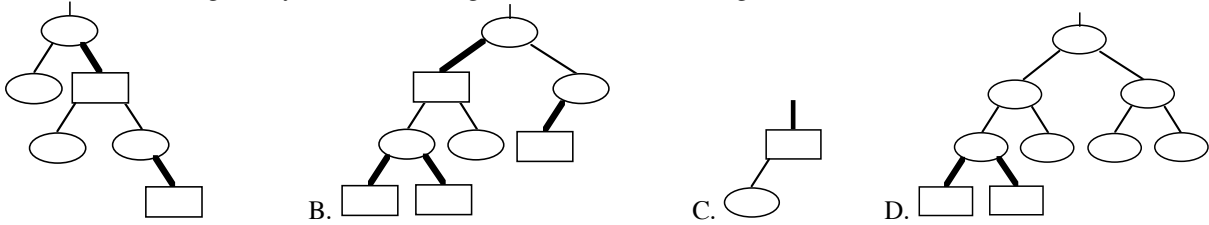
- A. SEARCH( $L, k$ ) on a sorted, singly linked list
- B. SEARCH( $L, k$ ) on an unsorted, singly linked list
- C. LOGICALPREDECESSOR( $L, x$ ) on a sorted, singly linked list
- D. LOGICALPREDECESSOR( $L, x$ ) on a sorted, doubly linked list

10. The stack for rat-in-a-maze stores

- A. all positions that have walls
- B. maze positions that must be in the final path

- C. the current path being explored
- D. the shortest known path that leads to the cheese

11. Which of the following binary trees has an illegal red-black tree coloring?



12. Which of the following will not be true regarding the decision tree for MERGESORT for sorting  $n$  input values?

- A. Every path from the root to a leaf will have  $O(n \log n)$  decisions.
- B. The height of the tree is  $\Omega(n \log n)$ .
- C. There will be a path from the root to a leaf with  $\Omega(n^2)$  decisions.
- D. There will be  $n!$  leaves.

13. In the example of concatenating two strings in  $O(1)$  time, which situation holds?

- A. Both lists are not circular
- B. Both lists are circular
- C. The first list is circular, the list to be attached is not
- D. The first list is not circular, the list to be attached is circular

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
  if (x==null)
    return;
  traverse(x.left);
  traverse(x.right);
  // process x here
}
```

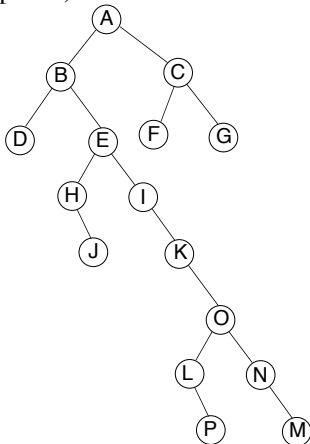
- A. inorder
- B. postorder
- C. preorder
- D. search for key  $x$

15. Suppose that only numbers in  $1 \dots 1000$  appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?

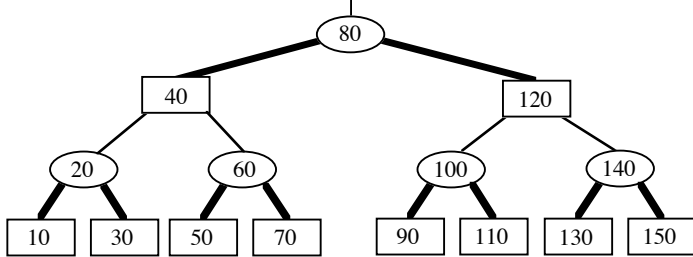
- A. 10, 200, 300, 100, 500
- B. 100, 1000, 200, 900, 300, 800, 400, 700, 500
- C. 200, 300, 400, 700, 600, 500
- D. 600, 100, 550, 540, 500

Long Answer

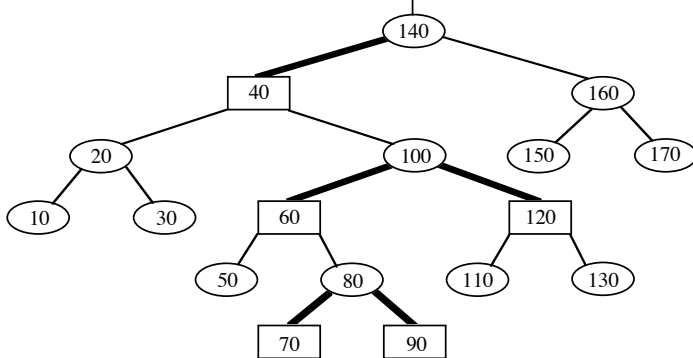
1. Give the unbalanced binary search tree that results when the keys 50, 70, 60, 90, 100, 80, 120 are inserted, in the given order, into an initially empty tree. (5 points)
2. Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (10 points)



- Show the result after PARTITION manipulates the following subarray. Be sure to indicate which version of PARTITION you applied. (10 points)  
 1            8            2            6            3            7            4            9            0            5
- A billion integers in the range  $0 \dots 999,999$  are to be sorted by LSD radix sort. How much faster will this be done if radix  $0 \dots 999$  is used rather than decimal ( $0 \dots 9$ ) radix? Show your work. (10 points)
- Insert 115 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



- Insert 95 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



CSE 2320

Name \_\_\_\_\_

Test 3

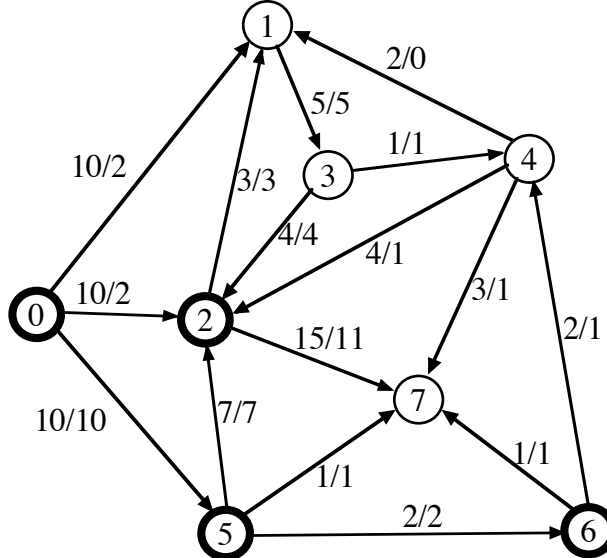
Fall 2007

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Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

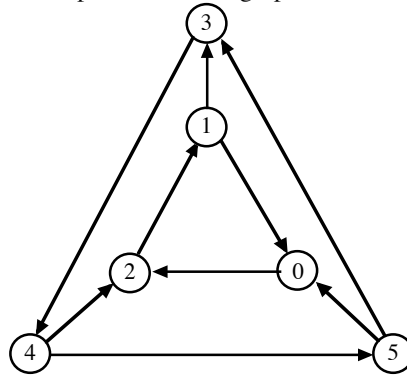
- The worst-case time for Prim's algorithm implemented with a T-heap is:  
 A.  $\theta(V + E)$             B.  $\theta(E \lg V)$             C.  $\theta(V \lg V)$             D.  $\theta(V \lg E)$

Problems 2, 3, and 4 refer to the following network. 0 is the source. 7 is the sink. Each edge is labeled with capacity/flow.

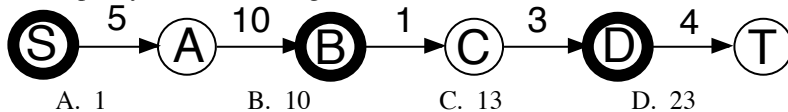


- The capacity of the indicated cut (S vertices are bold) is:  
 A. 20            B. 30            C. 31            D. 32
- The net flow across the given cut is:  
 A. 14            B. 16            C. 18            D. 20
- Suppose the flow is increased as much as possible using the augmenting path  $0 \rightarrow 2 \rightarrow 4 \rightarrow 7$ . Which is the critical edge?  
 A.  $0 \rightarrow 2$             B.  $2 \rightarrow 4$             C.  $4 \rightarrow 7$             D. Insufficient information

5. Which statement is incorrect regarding depth-first search on a directed graph?
- Exploring an edge whose head is colored black will cause the edge to be a back edge.
  - Exploring an edge whose head is colored gray will cause the edge to be a back edge.
  - Exploring an edge whose head is colored white will cause the edge to be a tree edge.
  - The run time is  $\Theta(m + n)$ , where  $m$  is the number of edges and  $n$  is the number of vertices.
6. What is the number of strongly connected components in this graph?



- A. 1                      B. 2                      C. 3                      D. 4
7. Which of the following uses a min-heap?
- Breadth-first search
  - Depth-first search
  - Dijkstra's algorithm
  - Maximum capacity path
8. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers  $0, 1, \dots, V - 1$ .)
- Adjacency lists (ordered):  $\Theta(\log V)$
  - Adjacency lists (unordered):  $\Theta(V)$
  - Adjacency matrix:  $\Theta(1)$
  - Compressed adjacency lists (ordered):  $\Theta(\log V)$
9. The capacity of the following cut is \_\_\_\_\_. (S vertices are bold.)

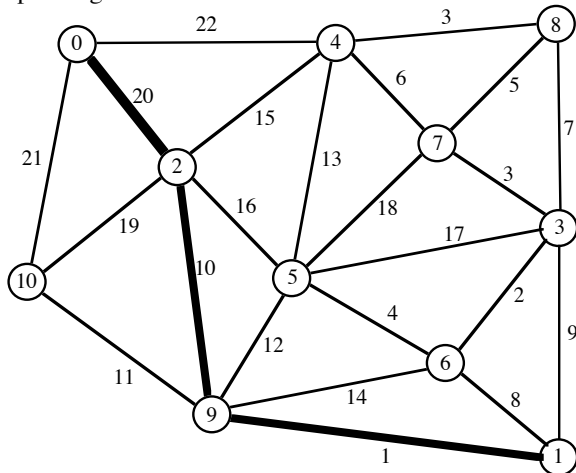


- A. 1                      B. 10                      C. 13                      D. 23
10. When a graph is sparse, the best way to find a minimum spanning tree is:
- Floyd-Warshall algorithm
  - Prim's algorithm using heap
  - Prim's algorithm using T-table
  - Warshall's algorithm
11. Which of the following is not true regarding the Edmonds-Karp variant.
- An augmenting path is found using breadth-first search.
  - An augmenting path may be used several times.
  - An edge may go back-and-forth between being saturated and unsaturated.
  - It solves the network flow problem in polynomial time.
12. When finding the strongly connected components, the number of components is indicated by:
- The number of back edges found during the first depth-first search.
  - The number of cross edges found during the second depth-first search.
  - The number of restarts for the first depth-first search.
  - The number of restarts for the second depth-first search.
13. The fastest method for finding the diameter of a directed graph (where distance is measured in "hops") is to:
- Use breadth-first search.
  - Use Dijkstra's algorithm.
  - Use the Floyd-Warshall algorithm.
  - Use the Ford-Fulkerson algorithm.
14. Suppose a depth-first search is performed on an undirected graph. What is the situation regarding edge types?
- no edge can be a cross edge or a forward edge

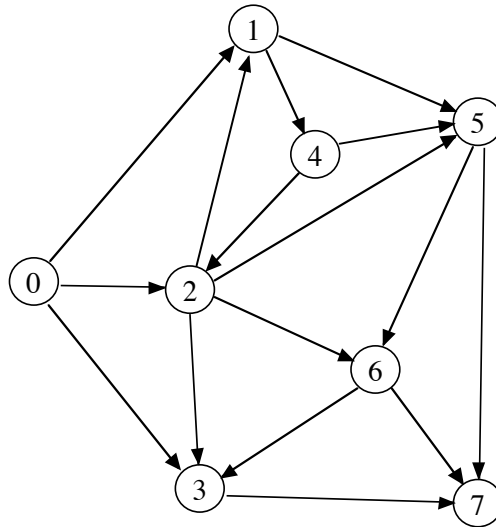
- B. both C and D  
 C. every edge is a tree edge  
 D. there cannot be a back edge
15. The expected number of probes for a successful search in hashing by chaining with  $\alpha$  as the load factor is:  
 A.  $\frac{\alpha}{2}$  B.  $\frac{2}{3}\alpha$  C.  $\alpha$  D.  $2\alpha$
16. The main disadvantage of compressed adjacency lists is  
 A. Directed graphs may not be represented  
 B. It is difficult to change the graph  
 C. They waste space  
 D. Undirected graphs may not be represented
17. Suppose that an instance of bipartite matching has 15 vertices in the left column, 15 vertices in the right column, and 25 edges. The number of edges in the corresponding instance of network flow is:  
 A. 25  
 B. 30  
 C. 55  
 D. 150
18. The number of potential probe sequences when using linear probing with a table with  $m$  entries ( $m$  is prime) is:  
 A.  $O(\log m)$  B.  $m$  C.  $m(m-1)$  D.  $m!$
19. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 0 to 1?  
 -1 3 3 3 3  
 -1 3 3 3 4  
 -1 1 1 1 4  
 -1 2 2 2 2  
 -1 -1 -1 -1 -1  
 A. 0 B. 1 C. 2 D. 3
20. Which of the following is not true about probe sequences for an implementation of double hashing?  
 A. All slots in the hash table appear in each probe sequence  
 B. Every key has a probe sequence different from the probe sequences for other keys  
 C. The elements of a probe sequence are subscripts in the hash table  
 D. The probe sequence for a key cannot change

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.

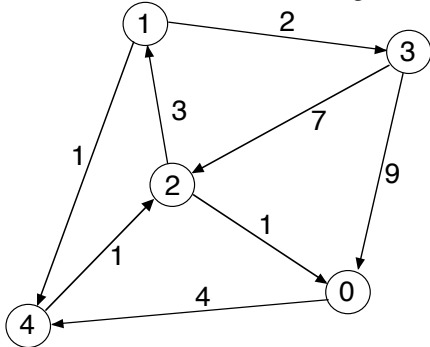


2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ordered. Write your answer in the tables below. 10 points

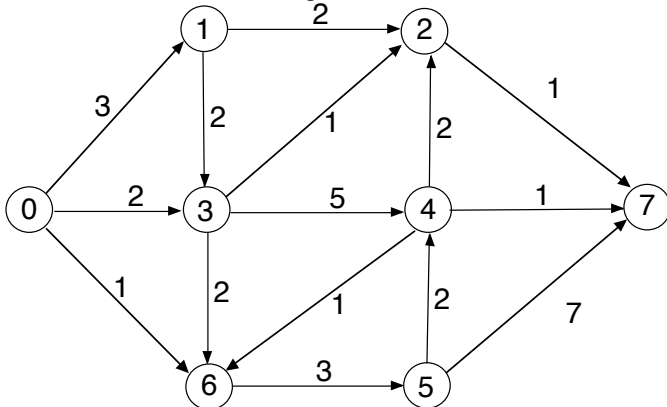


Vertex	Start	Finish	Edge	Type	Edge	Type
0	<u>1</u>	---	0 1	---	2 6	---
1	---	---	0 2	---	3 7	---
2	---	---	0 3	---	4 2	---
3	---	---	1 4	---	4 5	---
4	---	---	1 5	---	5 6	---
5	---	---	2 1	---	5 7	---
6	---	---	2 3	---	6 3	---
7	---	---	2 5	---	6 7	---

3. Demonstrate the Floyd-Warshall algorithm, with successors, for the following graph. The paths indicated in the final matrix must have at least one edge. You are not required to show the intermediate matrices. 10 points.



4. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.



Minimum Cut:

S vertices: 0

T vertices: 7

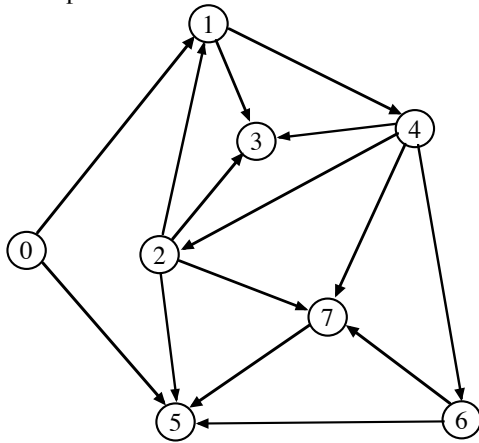
Augmenting Paths and Contribution to Flow:



5. Consider the following hash table whose keys were stored by double hashing using  $h_1(\text{key}) = \text{key} \% 11$  and  $h_2(\text{key}) = 1 + (\text{key} \% 10)$ . Show your work.

0 22  
 1  
 2  
 3 17  
 4 4  
 5 15  
 6 28  
 7  
 8  
 9  
 10 10

- a. Suppose 142 is to be inserted (using double hashing). Which slot will be used? (5 points)
  - b. Suppose 130 is to be inserted (using double hashing) *after* 142 has been stored. Which slot will be used? (5 points)
6. Perform a breadth-first search on the following graph listing the BFS number, shortest path distance (hops) from the source (0), and the predecessor for each vertex. Assume that the adjacency lists are ordered. 10 points



Vertex	BFS Number	Distance	Predecessor
0	_____	_____	_____
1	_____	_____	_____
2	_____	_____	_____
3	_____	_____	_____
4	_____	_____	_____
5	_____	_____	_____
6	_____	_____	_____
7	_____	_____	_____