CSE 2320 Name ____________________

Test 1

Fall 2008 Last 4 Digits of Mav ID # ______________

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to compute the sum of the n elements of an integer array is in:

   A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

2. When solving the fractional knapsack problem, the items are processed in the following order.
   A. Ascending order of weight
   B. Ascending order of $$$/lb
   C. Descending order of weight
   D. Descending order of $$$/lb

3. Suppose the input to HEAPSORT is always a table of identical integers. The worst-case time will be

   A. $\Theta(1)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

4. What is the definition of H_n?

   A. $\Theta(\sqrt{n})$ B. $\sum_{k=1}^{n} k$ C. $\ln n$ D. $\sum_{k=1}^{n} \frac{1}{k}$

5. The function $n + \log n$ is in which set?

   A. $\Omega(n \log n)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

6. Which of the following is not true?

   A. $n^2 \in \Omega(n^3)$ B. $n \log n \in O(n^2)$

   C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$ D. $\log n \in \Omega(\log \log n)$

7. Suppose a binary search is to be performed on a table with 20 elements. The maximum number of elements that could be examined (probes) is:

   A. 4 B. 5 C. 6 D. 7

8. Which of the following functions is not in $O(n^2)$?

   A. n B. $n \lg n$ C. n^2 D. n^3

9. Which statement is correct regarding the unweighted and weighted activity scheduling problems?
   A. Both require dynamic programming
   B. Both are easily solved using a greedy technique
   C. Unweighted is solved using a greedy technique, weighted is solved by dynamic programming
   D. Weighted is solved using a greedy technique, unweighted is solved by dynamic programming

10. What is required when calling `union(i,j)` for maintaining disjoint subsets?
    A. `i` and `j` are in the same subset
    B. `i` and `j` are leaders for different subsets
    C. `i` and `j` are leaders for the same subset
    D. `i` is the ancestor of `j` in one of the trees

11. $4^{\lg 7}$ evaluates to which of the following? (Recall that $\lg x = \log_2 x$.)

    A. $\sqrt{7}$ B. 7 C. 30 D. 49

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^3)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in O(1)$ B. $T(n) \in O(\log n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

13. What is the value of $\sum_{k=0}^{t} 2^k$?

    A. 2^k B. 2^t C. $2^{t+1} - 1$ D. $2^{t+1} + 1$

14. Suppose that you have correctly determined some c and n_o to prove $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

A. c may be decreased B. c may be increased C. n_o may be increased D. $f(n) \in O(g(n))$

15. Suppose there is a large table with n integers, possibly with repeated values, in ascending order. How much time is needed to determine the number of occurences of a particular value?

A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

Long Answer

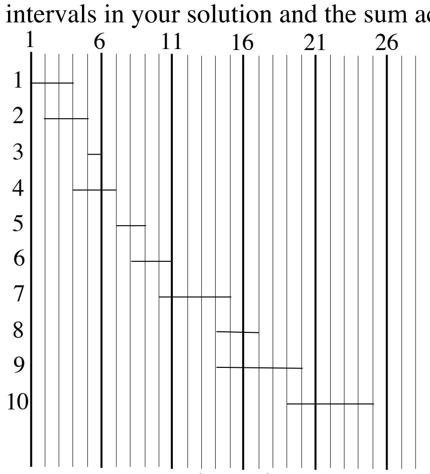
1. Two `int` arrays, `A` and `B`, contain `m` and `n` `int`s each, respectively. The elements within each of these arrays appear in ascending order without duplication (i.e. each table represents a set). Give Java code for a $\Theta(m+n)$ algorithm to find the **<u>set intersection</u>** by producing a third array `C` with the values that are common to both `A` and `B` (in ascending order) **<u>and</u>** sets the variable `p` to the final number of elements copied to `C`. (Details of input/output, allocation, declarations, error checking, comments and style **<u>are unnecessary</u>**.) 10 points2.Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

A	0.25
B	0.02
C	0.5
D	0.04
E	0.12
F	0.07

3. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + n$ is in $\Theta(n)$. 10 points

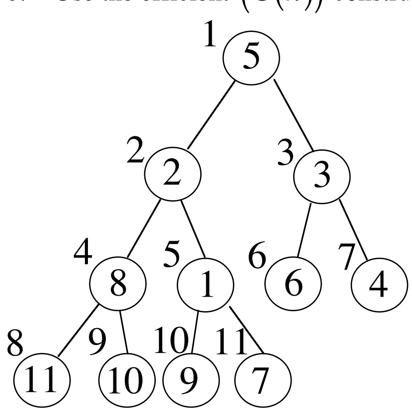
4. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + n$ is in $\Theta(n)$. 10 points

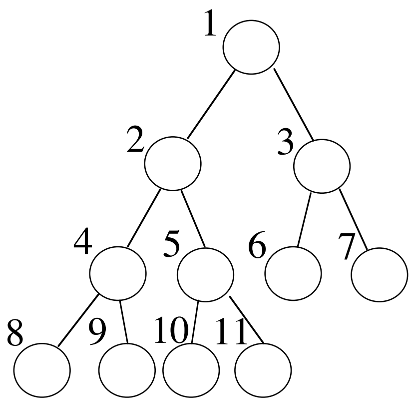
5. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



i	v_i	p_i	$m(i)$
1	1	0	
2	5	0	
3	3	2	
4	8	1	
5	1	4	
6	2	4	
7	3	5	
8	1	6	
9	4	6	
10	1	8	

6. Use the efficient $(\Theta(n))$ construction to convert into a minHeap. 5 points

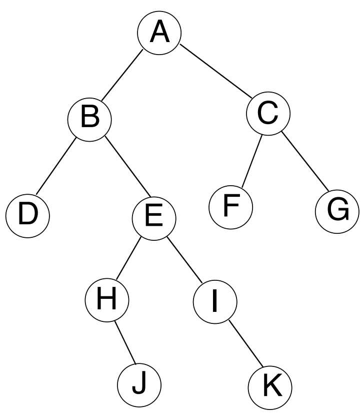




CSE 2320 Name ________________________
Test 2
Fall 2008 Last 4 Digits of Student ID # ____________

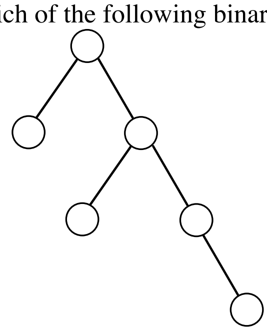
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

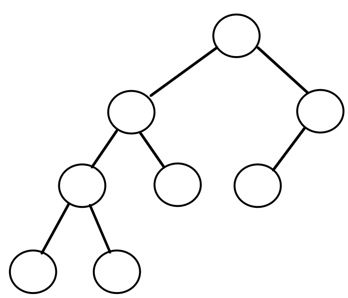
1. The queue for breadth-first rat-in-a-maze stores
   A. all maze positions that have walls
   B. maze positions that must be in the final path
   C. maze positions that have been reached
   D. the current path being explored
2. Suppose the tree below is a binary search tree whose keys <u>are not shown</u>. Which node will contain the key that is the predecessor of the key stored at F?



   A. A
   B. B
   C. C
   D. G

3. Which of the following is a longest common subsequence for 0 1 2 0 1 2 and 0 0 1 1 2 2?
   A. 0 0 1 1 B. 0 0 1 1 2 C. 0 0 1 2 D. 0 1 2 0
4. What is the worst-case time to perform MINIMUM(L) for an unordered, doubly-linked list with n nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
5. Given a pointer to a node, the worst-case time to delete the node from a doubly-linked list with n nodes in ascending order is:
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$
6. What is the worst-case time to find the predecessor of a key in an unbalanced binary search tree storing n keys? Assume that parent pointers are available.
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
7. The two mandatory pointers in a node for a rooted tree with linked siblings are:
   A. First child and right sibling
   B. Left child and right child
   C. Left child and parent
   D. Left sibling and right sibling
8. In which situation will a sentinel be inappropriate?
   A. Binary search for a key in an ordered table, to simplify and speed-up code
   B. Search for a key in an unordered table, to simplify and speed-up code
   C. Search for a key in an unordered linked list, to simplify and speed-up code
   D. Red-black tree, to simplify code
9. In a red-black tree holding n keys, what is the total number of left and right pointers that will be set to `nil` (the sentinel)?
   A. $n - 1$
   B. n
   C. $n + 1$
   D. None of the above
10. How should the successor of a node without a right child in an unbalanced binary search tree be found?
   A. Examine the ancestors of the node
   B. Go left, then proceed to the right
   C. Go right, then proceed to the left
   D. Preorder traversal
11. Which of the following binary trees has *multiple* legal colorings as a red-black tree?


A.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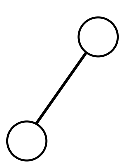
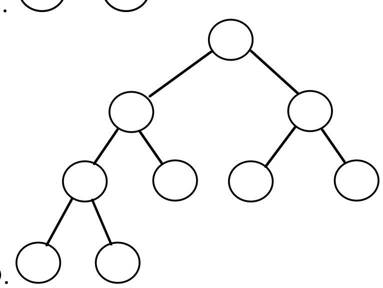

C.


D.

12. Recursion is often an alternative to using which data structure?
   A. Linked list B. Queue C. Stack D. 2-d array
13. The expected number of comparisons for finding the kth largest of n keys using PARTITION is in which asymptotic set?
   A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$
14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
```

```
if (x==null)
  return;
traverse(x.left);
// process x here
traverse(x.right);
}
```

A. inorder B. postorder C. preorder D. search for key `x`

15. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
   A. 700, 200, 600, 550, 500
   B. 200, 700, 600, 300, 400, 500
   C. 100, 1000, 200, 800, 300, 900, 500
   D. 300, 400, 900, 800, 500

Long Answer

1. List the four phases in a counting sort and give the asymptotic time needed for each phase. (5 points)
2. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

```
p[0]=6
p[1]=2
p[2]=4
p[3]=3
p[4]=2
```

	1		2		3		4	
1	0	0	48	1	60	1	???	?
2	-------		0	0	24	2	36	3
3	-------		-------		0	0	24	3
4	-------		-------		-------		0	0

3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

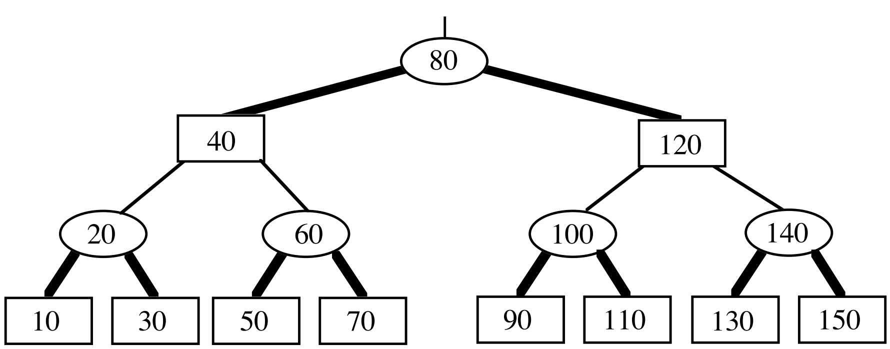
```
8    2    5    3    4    1    9    0    7    6
```

   Version: 1 2/Sedgewick

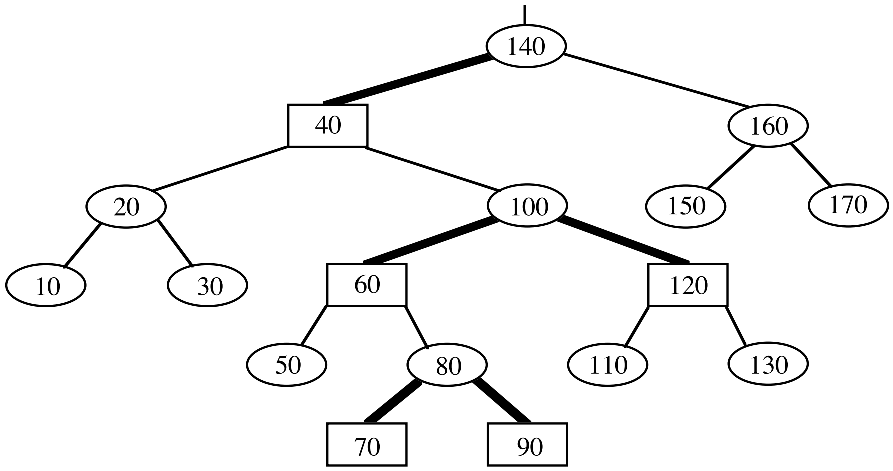
4. Use dynamic programming to solve the following instance of the strictly longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. 10 points

```
5   10   15   20   25   7   10   15   22   25   5   10   22   26   27
```

5. Insert 55 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



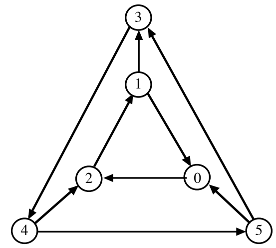
6. Insert 95 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



CSE 2320 Name ________________________________
Test 3
Fall 2008 Last 4 Digits of Student ID # ______________________

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. Which edge is chosen in a phase of Prim's algorithm?
   A. A minimum-weight edge connecting T to S.
   B. A minimum-weight edge that keeps the result free of cycles
   C. An edge of maximum-weight in a cycle (to be excluded)
   D. An edge that is on a shortest path from the source
2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from X to Y, its type will be:
   A. Back B. Cross C. Forward D. Tree
3. Which statement is incorrect regarding depth-first search on a directed graph?
   A. Exploring an edge whose head is colored black will cause the edge to be a back edge.
   B. Exploring an edge whose head is colored gray will cause the edge to be a back edge.
   C. Exploring an edge whose head is colored white will cause the edge to be a tree edge.
   D. The run time is $\Theta(m+n)$, where m is the number of edges and n is the number of vertices.
4. Suppose a depth-first search is performed on an undirected graph. What is the situation regarding edge types?
   A. no edge can be a cross edge or a forward edge
   B. both C and D
   C. every edge is a tree edge
   D. there cannot be a back edge
5. Suppose a depth-first search is performed on a directed graph. There are no cycles if:
   A. no edge is a cross edge or forward edge
   B. both C and D
   C. there are no restarts
   D. there are no back edges
6. A topological ordering of a directed graph may be computed by:
   A. Ordering the vertices by ascending discovery time after DFS
   B. Ordering the vertices by ascending finish time after DFS
   C. Ordering the vertices by descending discovery time after DFS
   D. Ordering the vertices by descending finish time after DFS
7. The worst case for double hashing is:
   A. All stored keys have the same h_1.
   B. All stored keys have the same h_2.
   C. All stored keys have the same h_1 and h_2.
   D. Inserting each key requires probing the slots for all previously inserted keys
8. What is the number of strongly connected components in this graph?



A. 1 B. 2 C. 3 D. 4

9. During a breadth-first search, the status of a gray vertex is:
   A. It has been completely processed.
   B. It is in the FIFO queue.
   C. It is in the priority queue.
   D. It is undiscovered.
10. When a graph is dense, the best way to find a minimum spanning tree is:
    A. Floyd-Warshall algorithm
    B. Prim's algorithm using heap
    C. Prim's algorithm using T-table
    D. Warshall's algorithm
11. Suppose a depth-first search is performed on an undirected graph. The graph is a free (i.e. unrooted) tree if:

    A. all edges are tree edges
    B. both C and D
    C. there are no restarts
    D. there are no back edges
12. Which of the following cannot occur when additional edges are included in a directed graph?
    A. The number of strong components may remain the same.
    B. The number of strong components may decrease.
    C. The number of strong components may increase.
    D. The graph acquires a cycle.
13. What is the purpose of the first depth-first search when finding strongly connected components?
    A. To assure that two vertices that are in the same cycle will be output in the same component
    B. To assure that two vertices with no paths between them are not output in the same component
    C. To assure that two vertices, X and Y, with paths from X to Y but not from Y to X, are output in different components.
    D. To make sure that the input graph has no cycles.
14. The number of potential probe sequences when using double hashing with a table with *m* entries (*m* is prime) is:
    A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$
15. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers 0, 1, . . ., V - 1.)
    A. Adjacency lists (ordered): $\Theta(V)$
    B. Adjacency lists (unordered): $\Theta(V)$
    C. Adjacency matrix: $\Theta(1)$
    D. Compressed adjacency lists (ordered): $\Theta(V)$
16. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 3 to 1?

-1	3	3	3	3
-1	3	3	3	4
-1	1	1	1	4
-1	2	2	2	2
-1	-1	-1	-1	-1

    A. 0 B. 1 C. 2 D. 3
17. The fastest method for finding the diameter of a tree (where distance is measured in "hops") is to:
    A. Use breadth-first search.
    B. Use Dijkstra's algorithm.
    C. Use the Floyd-Warshall algorithm.
    D. Use the Ford-Fulkerson algorithm.

18. Suppose the compressed adjacency list representation is used for a directed graph with *n* vertices and *m* edges. The number of entries in the two tables are:
    A. *n* for both
    B. *m* for both
    C. *n* and *m*
    D. *n* + 1 and *m*
19. Suppose Warshall's algorithm is used on a directed graph with vertices 0 . . . 25, but is stopped after column 10 in the matrix is processed. Which paths will be represented in the matrix?
    A. All paths that start at some vertex in 0 . . . 10 and stop at some vertex in 0 . . . 10.
    B. All paths that start at some vertex in 0 . . . 10, stop at some vertex in 0 . . . 10, and have vertices in 0 . . . 10 in between.
    C. All paths that start at some vertex in 0 . . . 25, stop at some vertex in 0 . . . 25, and have vertices in 0 . . . 10 in between.
    D. All paths with no more than 12 edges.
20. Which of the following is not true about probe sequences for an implementation of double hashing?
    A. All slots in the hash table appear in each probe sequence
    B. The elements of a probe sequence are possible keys for the hash table
    C. Two keys could have the same probe sequence
    D. The probe sequence for a key cannot change
21. Suppose a double hash table has $\alpha = 0.8$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:
    A. 1.2 B. 2 C. 5 D. 10
22. The main disadvantage of compressed adjacency lists is:
    A. Directed graphs may not be represented
    B. It is difficult to change the graph
    C. They waste space
    D. Undirected graphs may not be represented

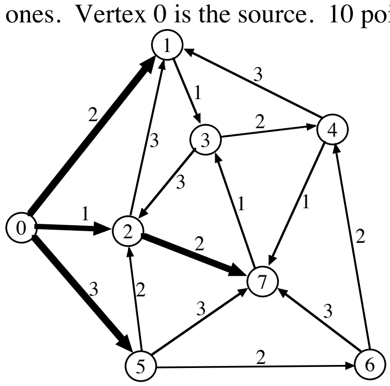
Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using h(key) = key % 13.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

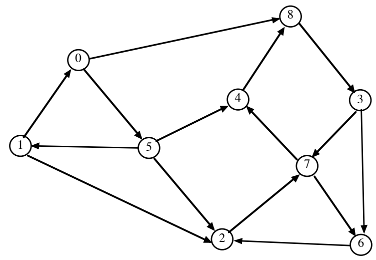
23. 143 would be inserted into which slot of the given table?
    A. 0 B. 1 C. 2 D. 11
24. 136 would be inserted into which slot of the given table? (without previously inserting 143)
    A. 0 B. 4 C. 6 D. 11
25. Which edge is chosen in a phase of Kruskal's algorithm?
    A. A minimum-weight edge connecting T to S.
    B. A minimum-weight edge that keeps the result free of cycles
    C. An edge of maximum-weight in a cycle (to be excluded)
    D. An edge that is on a shortest path from the source

Long Answer

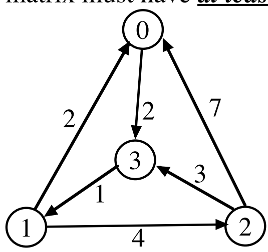
1. What are the entries in the heap (for ***Dijkstra's*** algorithm) ***before and after*** moving the next vertex and edge into the shortest path tree? DO NOT COMPLETE THE ENTIRE TREE!!! Edges already in the shortest path tree are the thick ones. Vertex 0 is the source. 10 points.



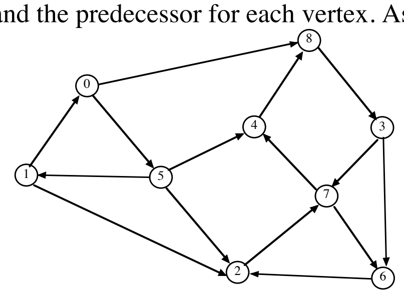
2. Demonstrate, for the graph below, the algorithm that uses ***two*** depth-first searches to determine the strongly-connected components. 10 points



3. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following graph. The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points.

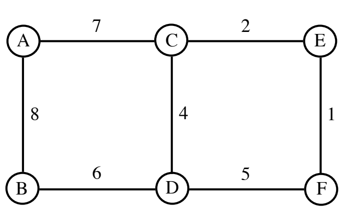


4. Perform a breadth-first search on the following graph listing the BFS number, shortest path distance (hops) from the source (0), and the predecessor for each vertex. Assume the adjacency lists are **ordered**. 10 points

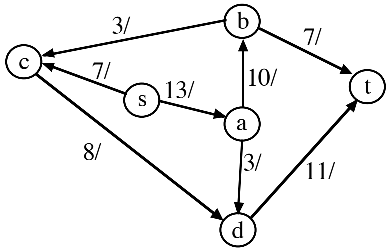


Vertex	BFS Number	Distance	Predecessor
0	_______	_______	_______
1	_______	_______	_______
2	_______	_______	_______
3	_______	_______	_______
4	_______	_______	_______
5	_______	_______	_______
6	_______	_______	_______
7	_______	_______	_______
8	_______	_______	_______

5. Demonstrate Kruskal's algorithm on this graph. 10 points



Extra Credit (5 points): Find a maximum flow for this network using augmenting paths.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