CSE 2320 Name ______________________________

Test 1

Fall 2012 Last 4 Digits of Mav ID # ________________

Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. The time to find the maximum of the n elements of an integer array is in:

   A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

2. The time to run the code below is in:

```
for (i=n; i>=0; i--)
  for (j=0; j<n; j+=2)
    sum+=i+j;
```

   A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

3. Suppose you have correctly determined some c and n_o to prove $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?

   A. c may be decreased B. n_o may be decreased C. n_o may be increased D. $f(n) \in O(g(n))$

4. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

   A. $T(n) \in \Omega(n^3)$ B. $T(n) \in O(\log n)$ C. $T(n) \in O(n)$ D. $T(n) \in \Omega(n^2)$

5. The number of calls to `mergeAB` while performing `mergesort` on n items is in:

   A. $\Theta(\log n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

6. Which of the following is not true?

   A. $n^3 \in \Omega(n^2)$ B. $n^2 \in \Omega(n \log n)$

   C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$ D. $\log\log n \in \Omega(\log n)$

7. Suppose a binary search is to be performed on a table with 50 elements. The maximum number of elements that could be examined (probes) is:

   A. 4 B. 5 C. 6 D. 7

8. The time to run the code below is in:

```
sum=1;
for (i=1; i<n; i=3*i)
  sum++;
```

   A. $\Theta(\log n)$ B. $\Theta(\sqrt{n})$ C. $\Theta(n)$ D. $\Theta(n \log n)$

9. What is the definition of H_n?

   A. $\Theta(\sqrt{n})$ B. $\sum_{k=1}^{n} k$ C. $\ln n$ D. $\sum_{k=1}^{n} \frac{1}{k}$

10. The expected time for insertion sort for n keys is in which set? (All n! input permutations are equally likely.)

   A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

Long Answer.

1. Suppose an `int` array `a` contains `m` zeroes followed by `n` ones, where `m` and `n` are unknown. The size of the array is given to you as `p`, i.e. `p==m+n`. Give C code for a binary search to determine `m` in $O(\log p)$ time. (Only the code for this task is needed. I/O, declarations, etc. are unnecessary)
   20 points

2. Use the recursion-tree method to show that $T(n) = 8T\left(\frac{n}{2}\right) + n^2$ is in $\Theta(n^3)$. 20 points

3. Use the substitution method to show that $T(n) = 8T\left(\frac{n}{2}\right) + n^2$ is in $O\left(n^3\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(n^3\right)$.) 20 points

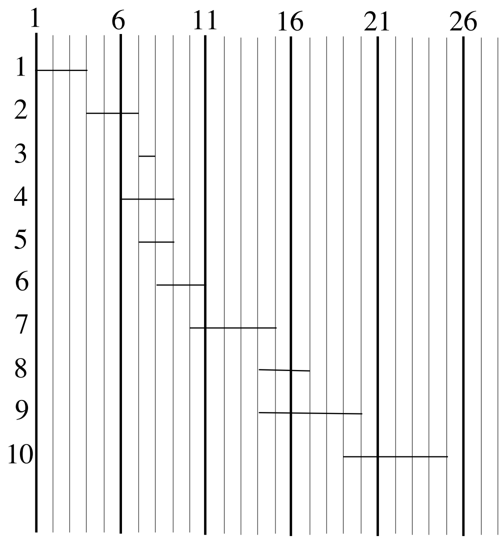
CSE 2320 Name ____________________
Test 2
Fall 2012 Last 4 Digits of Student ID # ____________

Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. The disadvantage of using LCS to solve an instance of LIS is:
   A. it only works for the monotone case
   B. it takes $\Theta\left(n^2\right)$ time in the worst case
   C. the input must be free of duplicate values
   D. it requires sorting
2. The goal of the optimal matrix multiplication problem is to:
   A. Minimize the number of C(i,j) instances evaluated.
   B. Minimize the number of matrix multiplications.
   C. Minimize the number of scalar multiplications.
   D. Minimize the number of scalar additions.
3. The time to optimally convert an array, with priorities stored at subscripts 1 through n, to a minheap is in:
   A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta\left(n^2\right)$ D. $\Theta(\log n)$
4. Which of the following is not a property of HEAPSORT for sorting keys into ascending order?
   A. A maxheap is used. B. It uses many `fixUp`s.
   C. The heap loses one element after each round. D. It is not stable.
5. The subset sum problem takes n input values and attempts to find a combination of those values whose sum is m. The worst-case time to extract the solution from the dynamic programming table is:
   A. $\Theta(\log m)$ B. $\Theta(n)$ C. $\Theta(m)$ D. $\Theta(mn)$
6. Memoization is associated with which technique?
   A. bottom-up dynamic programming B. `fixDown`
   C. greedy methods D. top-down dynamic programming
7. Which situation for operating on a maxheap uses a `fixUp`?
   A. `decreaseKey` B. `PQdelmax` C. `PQinsert` D. heapsort
8. The fractional knapsack problem may be solved optimally by a greedy method by taking a fraction of no more than this number of items.
   A. 1 B. 2 C. 3 D. 4
9. Which of the following is not true for the activity scheduling problem?
   A. The activities may have various durations.
   B. The greedy solution is optimal.
   C. There may be several optimal solutions.
   D. The goal is to minimize the number of activities chosen.
10. The time to extract the LCS (for sequences of lengths m and n) after filling in the dynamic programming matrix is in:
    A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

Long Answer

1. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 15 points



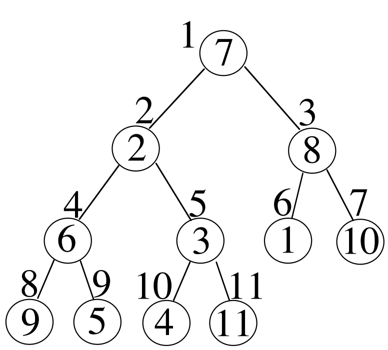
	v_i	p_i	$m(i)$
1	1	0	
2	5	1	
3	3	2	
4	8	1	
5	1	2	
6	2	3	
7	3	5	
8	1	6	
9	4	6	
10	1	8	

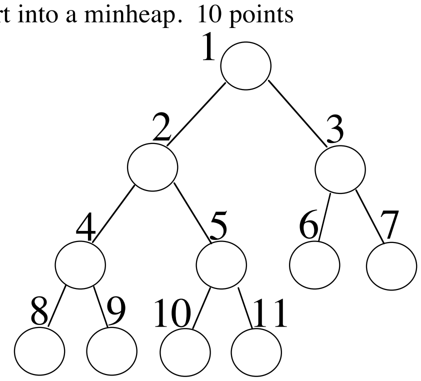
2. Use the greedy method for unweighted interval scheduling for the set of intervals in the previous problem. You may give your solution as the numbers of the chosen intervals. 5 points
2. Complete the following example of the efficient dynamic programming technique for finding a Longest Increasing Subsequence (monotone). Circle the inputs that are in the final LIS. (15 points)

	1	2	3	4	5	6	7	8	9	10	11	12
	40	10	30	15	35	25	20	37	47	27	45	17
C	1	1	2	2								
j	0	0	2	2								

```
1  10/2
2  15/4
3
4
5
6
7
8
```

3. Use the efficient construction to convert into a minheap. 10 points





4. Provide an LCS and indicate the backtrace using arrows in the matrix below. 15 points
a b c d d c b a a b c d
a a b b c c d d a b a c

		a	a	b	b	c	c	d	d	a	b	a	c
	0	0	0	0	0	0	0	0	0	0	0	0	0
a	0	1	1	1	1	1	1	1	1	1	1	1	1

b	0	1	1	2	2	2	2	2	2	2	2	2	2
c	0	1	1	2	2	3	3	3	3	3	3	3	3
d	0	1	1	2	2	3	3	4	4	4	4	4	4
d	0	1	1	2	2	3	3	4	5	5	5	5	5
c	0	1	1	2	2	3	4	4	5	5	5	5	6
b	0	1	1	2	3	3	4	4	5	5	6	6	6
a	0	1	2	2	3	3	4	4	5	6	6	7	7
a	0	1	2	2	3	3	4	4	5	6	6	7	7
b	0	1	2	3	3	3	4	4	5	6	7	7	7
c	0	1	2	3	3	4	4	4	5	6	7	7	8
d	0	1	2	3	3	4	4	5	5	6	7	7	8

CSE 2320 Name ______________________________
Test 3
Fall 2012 Last 4 Digits of Student ID # ________________

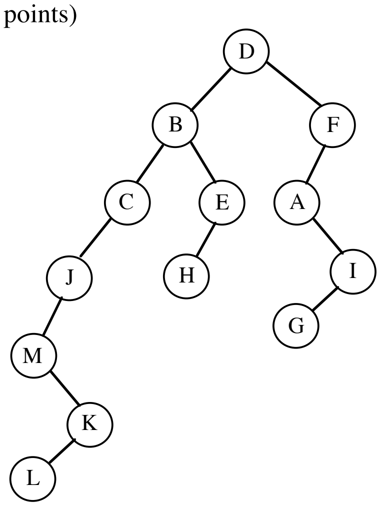
Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. Which sort treats keys as several digits and uses a counting sort for each position?
   A. counting
   B. insertion
   C. merge
   D. radix
2. Which binary tree traversal corresponds to the following recursive code?
```
void traverse(noderef x)
{
if (x==null)
  return;
// process x here
traverse(x.left);
traverse(x.right);
}
```
   A. preorder B. postorder C. inorder D. search for key `x`
3. What is the worst-case time to perform MINIMUM(L) for an unordered, doubly-linked list with n nodes?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
4. Given a pointer to a node, the worst-case time to delete the node from an unordered doubly-linked list with n nodes is:
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$
5. Which of the following sorts is stable?
   A. heapsort B. insertion C. quick D. shell
6. How should the successor of a node without a right child in an unbalanced binary search tree be found?
   A. Examine the ancestors of the node B. Go left, then proceed to the right
   C. Go right, then proceed to the left D. Preorder traversal
7. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?
   A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
   B. The height of the tree is $\Omega(n \log n)$.
   C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
   D. There will be $n!$ leaves.
8. The two mandatory pointers in a node for a rooted tree with linked siblings are:
   A. First child and right sibling B. Left child and right child
   C. Left child and parent D. Left sibling and right sibling

9. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with n keys and height h is:
   A. $O(1)$ B. $O(\log n)$ C. $O(h)$ D. $O(n)$
10. The worst-case number of comparisons for finding the kth largest of n keys using PARTITION is in which asymptotic set?
   A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

Long Answer

1. Give the unbalanced binary search tree that results when the keys 100, 50, 70, 60, 90, 80, 120 are inserted, in the given order, into an initially empty tree. (10 points)
2. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (15 points)
   7 8 2 5 3 1 4 9 0 6
   Version: 1 2/Sedgewick
3. Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (15 points)



4. List the four phases in a counting sort and give the asymptotic time needed for each phase. (10 points)
5. Twenty million positive integers in the range 0 . . . 99,999,999 are to be sorted by LSD radix sort. Compare the performance for using radix 0 . . . 9999 and radix 0 . . . 9. Show your work. (10 points)

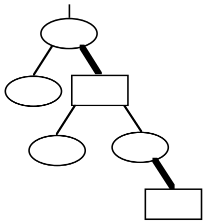
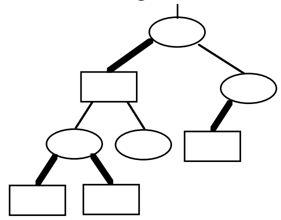
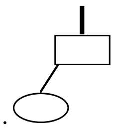
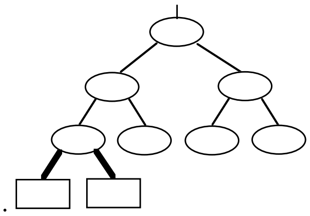
CSE 2320 Name ________________________________
Test 4
Fall 2012 Last 4 Digits of Student ID # ______________________

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

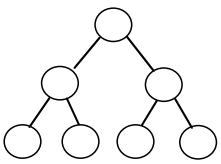
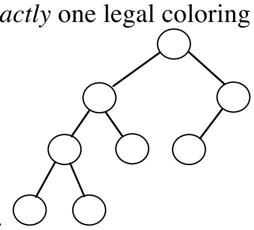
1. The worst-case time for Prim's algorithm implemented with a T-table is:
   A. θ(E lg V) B. $\theta(V^2 + E)$ C. θ(V lg E) D. θ(V lg V)
2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Y to Z, then its type will be:
   A. Tree B. Back C. Cross D. Forward
3. The worst-case time for depth-first search is:
   A. θ(E lg V) B. θ(V lg V) C. θ(V + E) D. θ(V lg E)
4. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from Y to X, then its type will be:
   A. Cross B. Forward C. Tree D. Back
5. During a breadth-first search, the status of a gray vertex is:
   A. It has been completely processed. B. It is undiscovered.
   C. It is in the priority queue. D. It is in the FIFO queue.
6. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times is:
   A. finish(X) > finish(Y) B. finish(X) < finish(Y)
   C. finish(X) = finish(Y) D. could be either A. or B.
7. Which of the following cannot occur when additional edges are included in a directed graph?
   A. The number of strong components may remain the same.
   B. The number of strong components may decrease.
   C. The number of strong components may increase.

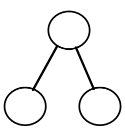
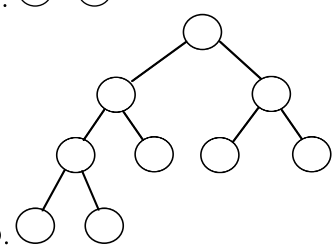
D. The graph acquires a cycle.

8. Suppose a depth-first search is applied to either an undirected or a directed graph. Suppose for two vertices discovery(X) < discovery(Y). Which of the following is not possible?
   A. finish(Y) < finish(X) B. finish(X) < discovery(Y)
   C. discovery(X) < finish(X) D. finish(X) < finish(Y)
9. Which of the following binary trees has an illegal red-black tree coloring?

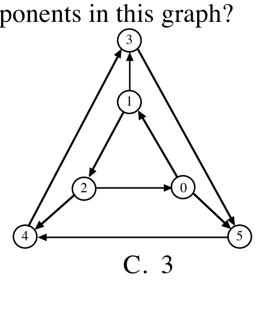
A.  B.  C.  D. 

10. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?

A.  B. 

C.  D. 

11. When a graph is dense, the best way to find a minimum spanning tree is:
    A. Floyd-Warshall algorithm B. Prim's algorithm using heap
    C. Prim's algorithm using T-table D. Warshall's algorithm
12. Which edge is chosen in a phase of Kruskal's algorithm?
    A. An edge of maximum-weight in a cycle (to be excluded)
    B. An edge that is on a shortest path from the source
    C. The unprocessed edge (x, y) of smallest weight such that `find(x)!=find(y)`
    D. The unprocessed edge (x, y) of smallest weight such that `find(x)==find(y)`
13. When finding the strongly connected components, the number of components is indicated by:
    A. The number of cross edges found during the second depth-first search.
    B. The number of back edges found during the first depth-first search.
    C. The number of restarts for the second depth-first search.
    D. The number of restarts for the first depth-first search.
14. The fastest method for finding the diameter of a tree (where distance is measured in "hops") is to:
    A. Use breadth-first search. B. Use Dijkstra's algorithm.
    C. Use the Floyd-Warshall algorithm. D. Use Prim's algorithm.
15. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:
    A. $\frac{\alpha}{2}$ B. $\frac{2}{3}\alpha$ C. α D. $\frac{1}{1-\alpha}$
16. Which algorithm maintains multiple subtrees?
    A. Dijkstra's. B. Kruskal's C. Prim's D. Warshall's
17. What is the number of strongly connected components in this graph?



A. 1 B. 2 C. 3 D. 4

18. For a double hash table with $\alpha = 0.5$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:
    A. 1.2 B. 2 C. 5 D. 10
19. Suppose that there is only one path from vertex 5 to vertex 10 in a directed graph: $5 \rightarrow 7 \rightarrow 8 \rightarrow 3 \rightarrow 2 \rightarrow 10$. During the scan of which column will Warshall's algorithm record the presence of this path?
    A. 2 B. 5 C. 8 D. 10
20. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?
    A. hashing with chaining B. open addressing
    C. ordered linked list D. unbalanced binary search tree

Problems 21 and 22 refer to the following hash table whose keys are stored by double hashing using h_1(key) = key % 13 and h_2(key) = 1 + (key % 12).

0	1	2	3	4	5	6	7	8	9	10	11	12
			120	186	187	162		122	110			194

21. 263 would be inserted into which slot of the given table?
    A. 0 B. 2 C. 3 D. 7
22. 303 would be inserted into which slot of the given table?
    A. 0 B. 2 C. 3 D. 7

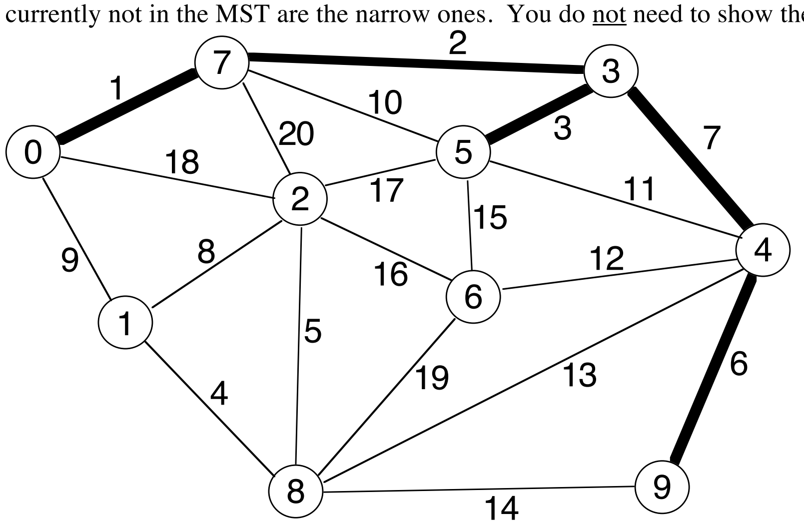
Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using h(key) = key % 13.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

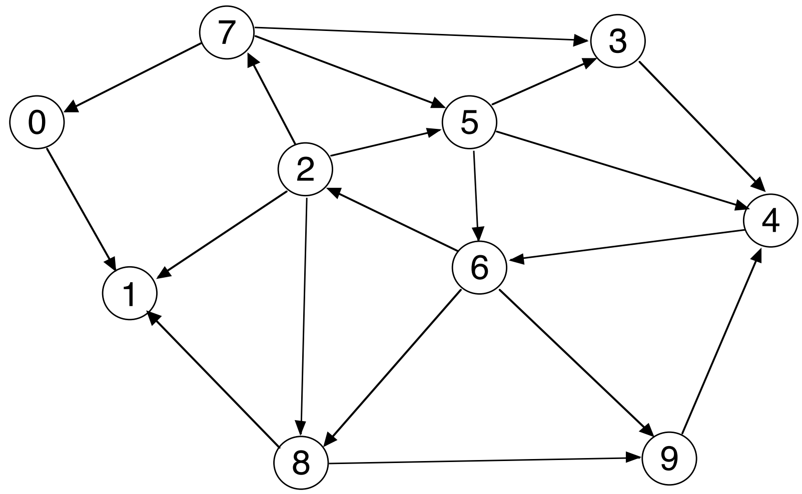
23. 143 would be inserted into which slot of the given table?
    A. 0 B. 1 C. 2 D. 11
24. 136 would be inserted into which slot of the given table?
    A. 0 B. 4 C. 6 D. 11
25. A topological ordering of a directed graph may be computed by:
    A. Ordering the vertices by descending finish time after DFS
    B. Ordering the vertices by ascending discovery time after DFS
    C. Ordering the vertices by ascending finish time after DFS
    D. Ordering the vertices by descending discovery time after DFS

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before <u>and</u> after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do <u>not</u> need to show the binary tree for the heap ordering. 10 points.

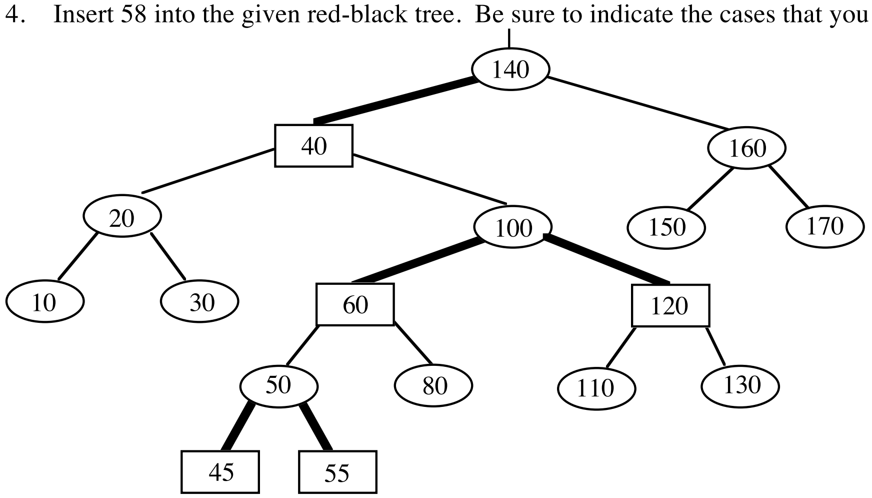


2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ***<u>ordered</u>***. Write your answer in the tables below. 10 points



Vertex	Start	Finish	Edge	Type	Edge	Type
0	1		0 1		6 2	
1			2 1		6 8	
2			2 5		6 9	
3			2 7		7 0	
4			2 8		7 3	
5			3 4		7 5	
6			4 6		8 1	
7			5 3		8 9	
8			5 4		9 4	
9			5 6			

3. Suppose a conventional implementation (i.e. columns scanned left-to-right) of Warshall's algorithm with successors is applied to a directed graph. From vertex X to vertex Y there can be many different paths, but only one path gets recorded. Give a mathematically precise statement that identifies which path gets recorded. (Don't say the "first" one . . .) 10 points.
4. Insert 58 into the given red-black tree. Be sure to indicate the cases that you used. 10 points.



5. Demonstrate, for the graph below, the algorithm that uses ***two*** depth-first searches to determine the strongly-connected components. 10 points