CSE 2320
Test 1
Fall 2018

Multiple Choice:

1. Write the letter of your answer on the line ( _____ ) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. The time for the following code is in which set?
   for (i=0; i<n-5; i++)
   for (j=2; j<n; j=j+2)
   {
       c[i][j] = 0;
       for (k=0; k<n; k++)
       c[i][j] += a[i][k]*b[k][j];
   }
   A. \( \Theta(n) \) B. \( \Theta(n^2) \) C. \( \Theta(n^2 \log n) \) D. \( \Theta(n^3) \)

2. The recursion tree for mergesort has which property?
   A. it leads to a definite geometric sum
   B. it leads to a harmonic sum
   C. each level has the same contribution
   D. it leads to an indefinite geometric sum

3. Which of the following is false?
   A. Subscript 1 will store the maximum priority.
   B. The parent for the node with subscript 500 is stored at subscript 250.
   C. The left child for the node with subscript 200 is stored at subscript 400.
   D. The right child for the node with subscript 405 is stored at subscript 911.

4. Which of the following is not true regarding a maxheap with 1000 elements?
   A. Subscript 1 will store the maximum priority.
   B. The parent for the node with subscript 500 is stored at subscript 250.
   C. The left child for the node with subscript 200 is stored at subscript 400.
   D. The right child for the node with subscript 405 is stored at subscript 911.

5. The function \( 2n \log n + \log n \) is in which set?
   A. \( \Theta(n) \) B. \( \Theta(n \log n) \) C. \( \Omega(n^2) \) D. \( \Theta(\log n) \)

6. \( f(n) = n \log n \) is in all of the following sets, except
   A. \( O(\log n) \) B. \( \Theta(\log(n!)) \) C. \( \Omega(n) \) D. \( O(n^2) \)

7. Suppose the input to heapsort is always a table of \( n \) zeroes and ones. The worst-case time will be:
   A. \( O(n \log n) \) B. \( \Theta(n \log n) \) C. \( \Omega(n \log n) \) D. \( \Theta(\log n) \)

8. The time to run the code below is in:
   for (i=5; i<=n-5; i++)
   for (j=2; j<n; j=2*j+1)
   sum+=i+j;
   A. \( \Theta(n \log n) \) B. \( \Theta(n^2) \) C. \( \Theta(n^3) \) D. \( \Theta(n) \)

9. Suppose that you have correctly determined some \( c \) and \( n_c \) to prove that \( g(n) \in \Omega(f(n)) \). Which of the following is not necessarily true?
   A. \( c \) may be decreased
   B. \( n_c \) may be increased
   C. \( n_c \) may be decreased
   D. \( f(n) \in O(g(n)) \)

10. Suppose you are using the substitution method to establish a \( \Theta \) bound on a recurrence \( T(n) \) and that you already know that \( T(n) \in \Omega(\log n) \)
    and \( T(n) \in O(n^2) \). Which of the following cannot be shown as an improvement?
    A. \( T(n) \in \Omega(n^3) \) B. \( T(n) \in O(n \log n) \) C. \( T(n) \in O(n) \) D. \( T(n) \in \Omega(n^2) \)

11. What is \( n \), the number of elements, for the largest table that can be processed by binary search using no more than 10 probes?
    A. 511 B. 1023 C. 2047 D. 4095

12. Heapsort may be viewed as being a faster version of which sort?
    A. Insertion B. Mergesort C. Selection D. qsort

13. Which of the following functions is not in \( \Omega(n^2) \)?
14. \(4^{\log 7}\) evaluates to which of the following? (Recall that \(\log x = \log_2 x\).)

- A. \(\sqrt{7}\)
- B. 7
- C. 25
- D. 49

15. Which of the following is solved heuristically by a greedy method?

- A. Fractional knapsack
- B. This semester’s first lab assignment
- C. Unweighted interval scheduling
- D. 0/1 knapsack

16. What is the value of \(\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k\)?

- A. \(\frac{1}{3}\)
- B. \(\frac{2}{3}\)
- C. \(\frac{3}{2}\)
- D. 3

17. Suppose there is a large table with \(n\) integers, possibly with repeated values, in ascending order. How much time is needed to determine the number of occurrences of a particular value?

- A. \(\Theta(1)\)
- B. \(\Theta(\log n)\)
- C. \(\Theta(n)\)
- D. \(\Theta(n \log n)\)

18. The number of calls to \(\text{merge}\) while performing \(\text{mergesort}\) on \(n\) items is in:

- A. \(\Theta(\log n)\)
- B. \(\Theta(1)\)
- C. \(\Theta(n)\)
- D. \(\Theta(n \log n)\)

19. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.

- A. Descending order of start time
- B. Ascending order of interval length
- C. Descending order of finish time
- D. Ascending order of finish time

20. The goal of the Huffman coding method is:

- A. Construct a tree that is order preserving
- B. Minimize the expected bits per symbol.
- C. Find the symbols with high probability of occurring.
- D. Maximize the compression for every string.

Long Answer

1. Use the efficient construction from Notes 05 to convert into a maxheap. 10 points

2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points
3. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.12</td>
</tr>
<tr>
<td>B</td>
<td>0.13</td>
</tr>
<tr>
<td>C</td>
<td>0.15</td>
</tr>
<tr>
<td>D</td>
<td>0.15</td>
</tr>
<tr>
<td>E</td>
<td>0.2</td>
</tr>
<tr>
<td>F</td>
<td>0.25</td>
</tr>
</tbody>
</table>

4. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

\[
\begin{array}{cccccc}
1 & 6 & 11 & 16 & 21 & 26 \\
\hline
1 & 0 & 0 & 120 & 1 & 88 & 138 \\
2 & 48 & 2 & 88 & 3 & 156 & 3 \\
3 & 0 & 0 & 60 & 3 & 132 & 3 \\
4 & 0 & 0 & 60 & 4 & \ \\
5 & 0 & 0 & \ \\
\end{array}
\]

5. Use the recursion-tree method to show that \( T(n) = 2T\left(\frac{n}{4}\right) + n \) is in \( \Theta(n) \). 10 points

6. Use the substitution method to show that \( T(n) = 2T\left(\frac{n}{4}\right) + n \) is in \( O(n) \). (You do not need to show that \( T(n) \) is in \( \Omega(n) \).) 10 points

CSE 2320
Test 2
Fall 2018

Multiple Choice:
1. Based on dictionary search performance alone, the best justification for ordering a linked list is:
   A. Many more misses than hits are expected
   B. Sentinels are more effective in speeding up search
   C. Many more hits than misses are expected
   D. Less storage will be needed

2. What is the worst-case time to perform \textsc{minimum}(L) for a sorted, doubly-linked list with \( n \) nodes?
   A. \( \Theta(1) \)  B. \( \Theta(\log n) \)  C. \( \Theta(n) \)  D. \( \Theta(n \log n) \)

3. Suppose a postfix evaluator has already processed \( 3 2 1 + \ast 4 5 + \) (with more to follow). What will be the contents of the stack (shown bottom-to-top going left-to-right)?
   A. 3 2 1 4 5  B. 9 9  C. 3 3 4 5  D. 18

4. Suppose that only numbers in 1 . . . 100 appear as keys in a binary search tree. While searching for 50, which of the following sequences of keys could not be examined?
   A. 10, 40, 70, 30, 50  B. 10, 30, 70, 60, 50  C. 1, 100, 20, 70, 50  D. 100, 20, 80, 30, 50

5. Which phase of counting sort actually “counts”?
   A. first  B. second  C. third  D. fourth
6. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with \( n \) keys and height \( h \) is:
   
   ______
   A. \( \Theta(1) \)           B. \( \Theta(\log n) \)           C. \( \Theta(h) \)           D. \( \Theta(n) \)
   
7. For which of the following sorts does the decision tree model not apply?
   
   ______
   A. Insertion       B. LSD Radix Sort       C. MERGE-SORT       D. QUICKSORT
   
8. If \( \text{POP} \) is implemented as return \( \text{stack}[-\text{SP}] \), then \( \text{PUSH} \) of element \( X \) is implemented as:
   
   ______
   A. \( \text{stack}[++\text{SP}] = X \)       B. \( \text{return stack[SP++]} \)       C. \( \text{stack[SP++]} = X \)       D. \( \text{stack}[SP] = X \)
   
9. Suppose the tree below is a binary search tree whose keys and subtree sizes are not shown. Which node will contain the key with rank 8?
   
   ______
   A. the header points at itself       B. the header points at \((-1)\)       C. \( \text{next}[n] = 0 \)       D. the recycling list is empty
   
10. Which of the following will not be true regarding the decision tree for QUICKSORT for sorting \( n \) input values?

   ______
   A. There will be \( n! \) leaves.
   B. Every path from the root to a leaf will have \( \Omega(n \log n) \) decisions.
   C. There will be a path from the root to a leaf with \( \Omega(n^2) \) decisions.
   D. The height of the tree is \( \Omega(n \log n) \).

11. The expected number of comparisons for finding the \( k \)th largest of \( n \) keys using \( \text{PARTITION} \) is in which asymptotic set?

   ______
   A. \( \Theta(\log n) \)       B. \( \Theta(n) \)       C. \( \Theta(n \log n) \)       D. \( \Theta(n^2) \)

12. Recently, we considered an abstraction supporting the operations \( \text{allocate, allocateAny, and freeup} \) in constant time. How does the \( \text{allocateAny} \) operation detect that all items have already been allocated?

   ______
   A. The header points at itself       B. The header points at \((-1)\)       C. \( \text{next}[n] = 0 \)       D. The recycling list is empty

13. Which of the following is a longest common subsequence for 0 1 2 0 1 2 and 0 0 1 2 1 2?

   ______
   A. 0 0 1 1       B. 0 1 2 1 2       C. 0 0 1 2       D. 0 1 2 0

14. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?

   ______
   A. Like a circular queue, the maximum number of items is determined at initialization.
   B. One node is always wasted.
   C. The head points to the first element and the tail points to the last element.
   D. The tail points to the first element and the head points to the last element.

15. Suppose a value \( k \) appears for \( p \) entries in the cost function table \( C \) for an instance of the longest monotonically increasing subsequence problem. Going left-to-right across the corresponding input sequence values \( (y_j) \), which statement is true?

   (Stated formally: For \( i_1 < i_2 < \cdots < i_p \), suppose \( C_{i_1} = C_{i_2} = \cdots = C_{i_p} = k \). Which statement is true regarding \( y_{i_1}, y_{i_2}, \ldots, y_{i_p} \)?)

   ______
   A. They are monotonically decreasing        B. They are strictly increasing
   C. They are monotonically increasing        D. They are strictly decreasing

   Long Answer
   1. Give the unbalanced binary search tree that results when the keys 60, 50, 40, 20, 80, 70, 90, 30 are inserted, \( \text{in the given order} \), into an initially empty tree. (5 points)
   2. A billion integers in the range 0 . . . \( 2^{30} - 1 \) will be sorted by LSD radix sort. How much faster is this done using radix 0 . . . \( 2^3 \) - 1 rather than 0 . . . \( 2^1 \) - 1? Show your work. (10 points)
   3. Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (10 points)

     ______

   4. Show the result after \( \text{PARTITION} \) (Version 1) manipulates the following subarray. Recall that both pointers start at the left end of the subarray. (10 points)
5. Use the dynamic programming solution for subset sums to determine a subset that sums to 14. Be sure to give the complete table that would be produced. (10 points, no points for solving by inspection)

\[
\begin{array}{ccccccc}
 i & 0 & 1 & 2 & 3 & 4 & 5 \\
 S_i & 0 & 2 & 3 & 5 & 7 & 11 \\
\end{array}
\]

6. Use dynamic programming to solve the following instance of the strictly longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. (10 points, no points for solving by inspection)

<table>
<thead>
<tr>
<th>i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_i</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>22</td>
<td>25</td>
<td>5</td>
<td>10</td>
<td>22</td>
<td>26</td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

Test 3
Fall 2018
CSE 2320
Name ________________________________
Your name as it appears on your UTA ID Card
11. Suppose that there is only one path from vertex 5 to vertex 10 in a directed graph:
\[ 5 \rightarrow 7 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 10. \] During the scan of which column will Warshall’s algorithm record the presence of this path?

12. A topological ordering of a directed graph may be computed by:
   A. Ordering the vertices by descending finish time after DFS
   B. Ordering the vertices by ascending discovery time after DFS
   C. Ordering the vertices by ascending finish time after DFS
   D. Ordering the vertices by descending discovery time after DFS

13. The number of potential probe sequences when using double hashing with a table with \( m \) entries (\( m \) is prime) is:
   A. \( \Theta(\log m) \)
   B. \( m \)
   C. \( m(m - 1) \)
   D. \( m! \)

14. What is the number of strongly connected components in this graph?

15. The worst-case time for Prim’s algorithm implemented with a minheap is:
   A. \( \Theta(V + E) \)
   B. \( \Theta(E \log V) \)
   C. \( \Theta(V \log V) \)
   D. \( \Theta(V \log E) \)

16. Which of the following cannot occur when additional edges are included in a directed graph?
   A. The graph acquires a cycle.
   B. The number of strong components may remain the same.
   C. The number of strong components may decrease.
   D. The number of strong components may increase.

17. What is required when calling \( \text{union}(i, j) \) for maintaining disjoint subsets?
   A. \( i \) and \( j \) are leaders for the same subset
   B. \( i \) and \( j \) are in the same subset
   C. \( i \) and \( j \) are leaders for different subsets
   D. \( i \) is the ancestor of \( j \) in one of the trees

18. The maximum number of rotations while inserting a key into a red-black tree is:
   A. 1
   B. 2
   C. 3
   D. the black-height

19. When finding the strongly connected components, the number of components is indicated by:
   A. The number of cross edges found during the second depth-first search.
   B. The number of back edges found during the first depth-first search.
   C. The number of restarts for the second depth-first search.
   D. The number of restarts for the first depth-first search.

20. In Dijkstra’s algorithm, the final shortest path distance from the source \( s \) to a vertex \( x \) is known when:
   A. \( x \) is placed on the heap.
   B. \( x \) has its entry extracted from the heap.
   C. \( x \) is read from the input file.
   D. some vertex \( y \) moves from \( T \) to \( S \) and there is an edge from \( y \) to \( x \).

Long Answer

1. What are the entries in the heap (for Prim’s algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.

2. Consider the following hash table whose keys were stored by double hashing using
\( h_1(key) = \text{key} \mod 11 \) and \( h_2(key) = 1 + (\text{key} \mod 10) \). \textbf{Show your work.}

1. 22
2. 17
3. 4
4. 15
5. 28
6. 10
7. 10
8. 10
a. Suppose 142 is to be inserted (using double hashing). Which slot will be used? (5 points)
b. Suppose 130 is to be inserted (using double hashing) \textit{after} 142 has been stored. Which slot will be used? (5 points)

3. Show the \textit{compressed} adjacency list representation this weighted graph. (Answers using conventional adjacency lists will receive no credit.) 10 points.

4. Demonstrate the Floyd-Warshall algorithm, \textit{with successors}, for the following input adjacency matrix. (\( \infty \) represents infinity) The paths indicated in the final matrix must have \textit{at least one} edge. You are \textbf{not} required to show the intermediate matrices. 10 points.

5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are \textit{ordered}. Write your answer in the tables below. 10 points.

6. Insert 42 into the given red-black tree. Be sure to indicate the cases that you used. 10 points.