CSE 2320 Lab Assignment 3

Due April 27, 2010

Goals:

1. Understanding of graph representations.

2. Understanding of reachability.

Requirements:

1. Write a Java program that uses the "pebble game" technique to test if an input graph is *minimally rigid*. The first line of the input will be n, the number of vertices. n will not exceed 100. Each of the remaining $2n - 3$ lines will contain two values in the range $0 \ldots n - 1$ to designate an undirected edge. (Self-loops and parallel edges will not be included.) Each input line should be echoed to the output before being processed. The last line of your output should be either 1) an indication that the graph is minimally rigid ("Laman") or 2) an indication that the last edge processed is "redundant" along with the set of vertices giving the rigid subgraph for the redundant edge.

2. Email your program (as an attachment) to `fawaz.bokhari@mavs.uta.edu` by 10:45 a.m. on April 27, 2010.

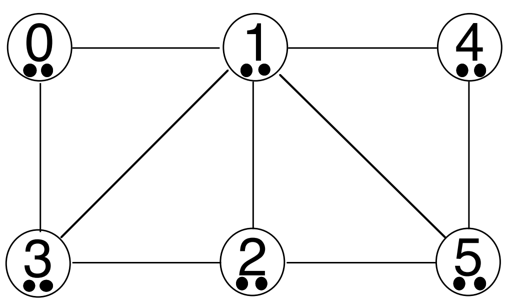
Getting Started:

1. The input is easily read by importing `java.util.Scanner` and using code like:

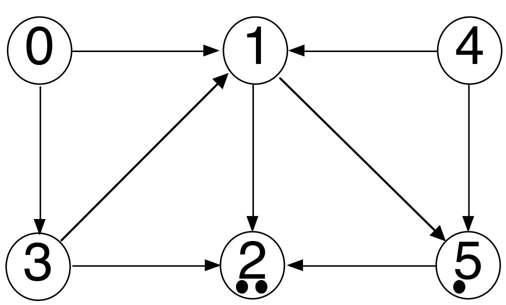
```
Scanner sc=new Scanner(System.in);
int n=sc.nextInt();
```

Do not prompt for a file name!

2. The following bar-and-joint system has $n = 6$ vertices and $2n - 3 = 9$ edges:

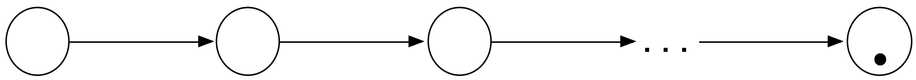


For analyzing rigidity, a pebble on a vertex may be moved to an incident (undirected) edge, which then directs the edge away from the vertex. For this system, the nine edges may be directed as:

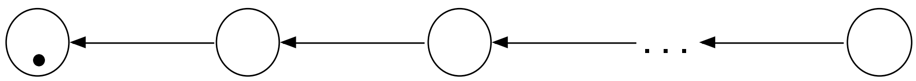


Observe that each vertex has no more than two outgoing edges.

3. Coordinating the edge directions is facilitated by the observation that a pebble may be moved against the direction of a path. i.e. the path



may be converted to:



4. The following characterizations of Laman give an algorithmically expensive way of testing rigidity:

   The edges of a graph $G = (V, E)$ are independent (non-redundant) in two dimensions if and only if no subgraph $G' = (V', E')$ (with n' vertices) has more than $2n' - 3$ edges.

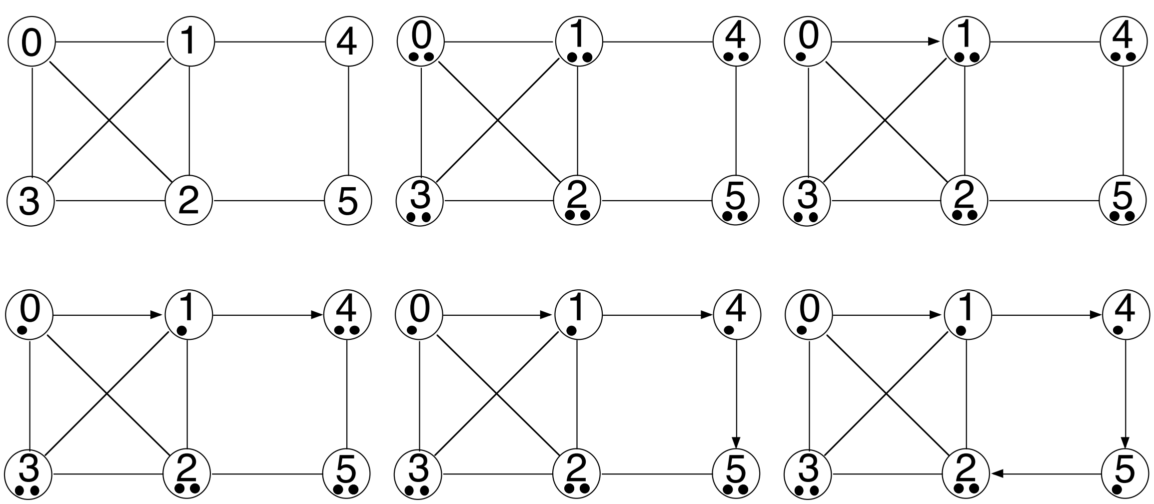
   (Corollary) A graph with $2n - 3$ edges is rigid in two dimensions if and only if no subgraph G' (with n' vertices) has more than $2n' - 3$ edges.

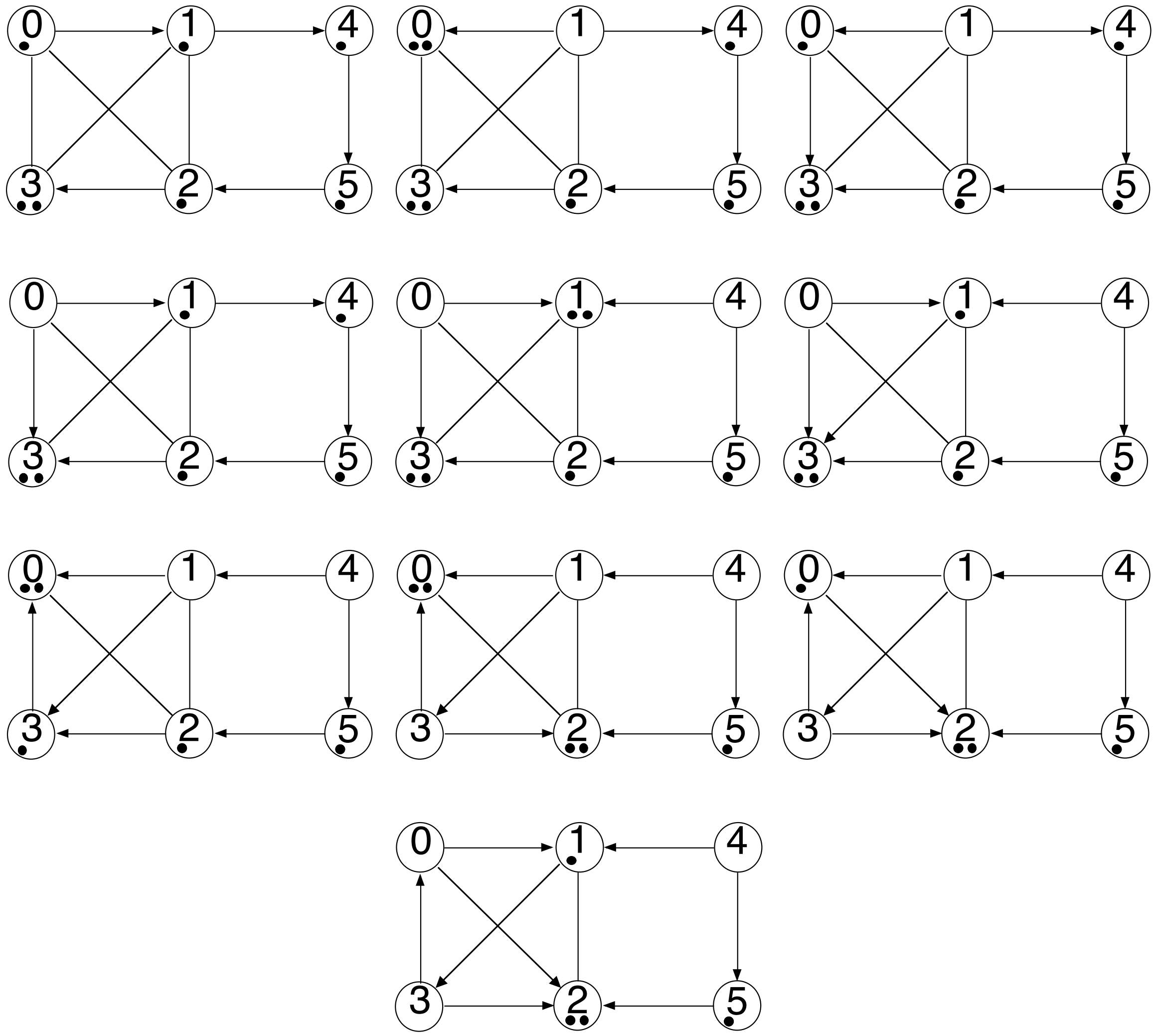
5. Jacobs and Hendrickson developed the following "pebble game" technique for testing rigidity in $O(n^2)$ time:

```
Place two pebbles at each vertex.
for each input edge {x y}
  Attempt to get two pebbles (four total) at vertices x and y.
    This may be done using BFS (or DFS) to find a directed path
    to a vertex (other than x or y) with a free pebble.  After
    the vertex is found, the path must be reversed.
  if four pebbles are achieved
    Use one pebble on x to record the directed edge x → y.
  else
    // Only three pebbles were achieved for {x y}.
    {x y} is a redundant edge.
    The vertices reached during the last search induce a rigid subgraph.

// All edges have been directed . . .  Graph is minimally rigid.
```

6. The following bar-and-joint system has $n = 6$ vertices and $2n - 3 = 9$ edges, but it has issues with rigidity:





7. Due to the limited number of edges (2*n* - 3) and the characteristics of the pebble game, only a simple table is needed to store the pebbling status. Other data structures will be needed to support the graph search technique that you choose.

8. `http://minerva.cs.mtholyoke.edu/demos/pebbleGames/2dbarjoint.php` has the applet used during lecture.