

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

- The time to compute the sum of the n elements of an integer array is:
 - $\Theta(n)$
 - $\Theta(n \log n)$
 - $\Theta(n^2)$
 - $\Theta(n^3)$
- The number of calls to `getmin` to build a Huffman code tree for n symbols is:
 - $\Theta(\log n)$
 - $n - 1$
 - n
 - $2n - 2$
- Which of the following is not true?
 - $n^2 \in \Omega(n^3)$
 - $n \log n \in O(n^2)$
 - $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$
 - $3^n \in \Omega(2^n)$
- For an instance of the subset sum problem, n positive integer values and another positive integer m are given. The time for the dynamic programming solution is:
 - $\Theta(m + n)$
 - $\Theta(m)$
 - $\Theta(n)$
 - $\Theta(mn)$
- The function $2n + \log n$ is in which set?
 - $\Omega(n \log n)$
 - $\Theta(\log n)$
 - $\Theta(n)$
 - $\Theta(n \log n)$
- $f(n) = n \lg n$ is in all of the following sets, except
 - $O(\log n)$
 - $\Theta(\log(n!))$
 - $\Omega(n)$
 - $O(n^2)$
- Which statement is correct regarding the unweighted and weighted interval scheduling problems?
 - Weighted is solved using a greedy technique, unweighted is solved by dynamic programming
 - Unweighted is solved using a greedy technique, weighted is solved by dynamic programming
 - Both are easily solved using a greedy technique
 - Both require dynamic programming
- The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:
 - to assure that the final solution is free of duplicate values
 - to determine the longest possible increasing subsequence terminated by a particular input value
 - to search a table that will contain only the LIS elements at termination
 - to sort the original input
- Suppose that you have correctly determined some c and n_0 to prove that $g(n) \in \Omega(f(n))$. Which of the following is not necessarily true?
 - c may be decreased
 - c may be increased
 - n_0 may be increased
 - $f(n) \in O(g(n))$
- Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^3)$. Which of the following cannot be shown as an improvement?
 - $T(n) \in O(1)$
 - $T(n) \in O(\log n)$
 - $T(n) \in \Omega(n^2)$
 - $T(n) \in \Omega(n^3)$
- What is n , the number of elements, for the largest table that can be processed by binary search using no more than 7 probes?
 - 31
 - 63
 - 64
 - 127
- $4^{\lg 7}$ evaluates to which of the following? (Recall that $\lg x = \log_2 x$.)
 - $\sqrt{7}$
 - 7
 - 30
 - 49

13. Which of the following functions is not in $\Omega(n^2)$?
- A. n B. n^2 C. $n^2 \lg n$ D. n^3
14. Which of the following best approximates $H_m - H_n$? ($m > n$)
- A. H_{m-n} B. $1/(m-n)$ C. $\ln(m/n)$ D. $\ln(m-n)$
15. Which of the following is solved heuristically by a greedy method?
- A. Fractional knapsack
 B. Huffman code
 C. Unweighted interval scheduling
 D. 0/1 knapsack

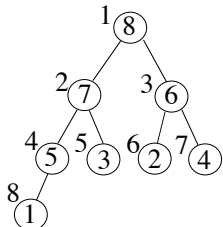
Long Answer

1. Prove that if $f(n) \in O(g(n))$ then $\frac{1}{f(n)} \in \Omega\left(\frac{1}{g(n)}\right)$. 5 points
2. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

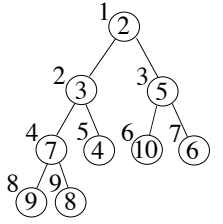
p[0]=5
 p[1]=2
 p[2]=2
 p[3]=4
 p[4]=6

	1	2	3	4	
1	0	0	20	1	56
2	-----	0	0	16	2
3	-----	-----	0	0	48
4	-----	-----	-----	0	0

3. Use dynamic programming to solve the following instance of (monotone) longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. 10 points
4. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{2}\right) + n^2$ is in $\Theta(n^2)$. 10 points
5. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{2}\right) + n^2$ is in $\Theta(n^2)$. 10 points
6. a. Show the maxheap after performing getmax. 5 points



- b. Show the minheap after changing the priority at subscript 8 to 1. 5 points



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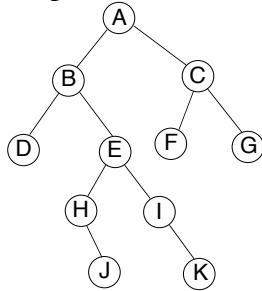
Test 2

Spring 2008

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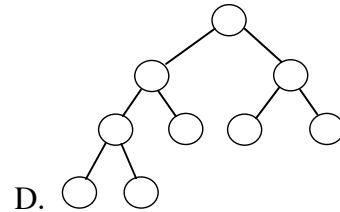
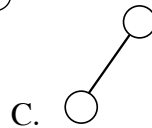
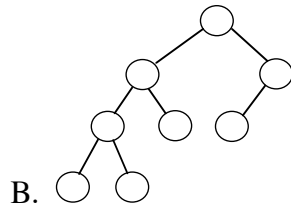
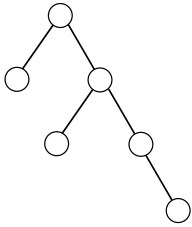
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

- Which version of rat-in-a-maze finds the shortest path?
A. queue B. recursive C. red-black tree D. stack
- Suppose the tree below is a binary search tree whose keys are not shown. Which node will contain the key that is the predecessor of the key stored at F?



- A
 - B
 - C
 - G
- Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?
A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
B. The height of the tree is $\Omega(n \log n)$.
C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
D. There will be $n!$ leaves.
 - What is the worst-case time to perform $\text{MINIMUM}(L)$ for an unsorted, doubly-linked list with n nodes?
A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
 - Which type of linked list will be the most convenient if $\text{LOGICALPREDECESSOR}$ and LOGICALSUCCESSOR operations will be frequent?
A. ordered, doubly-linked
B. ordered, singly-linked
C. unordered, doubly-linked
D. unordered, singly-linked
 - Suppose a postfix evaluator has already processed $3 \ 2 \ 1 \ + \ * \ 4 \ 5 \ +$ (with more to follow). What will be the contents of the stack (shown bottom-to-top going left-to-right)?
A. 3 2 1 4 5
B. 3 3 4 5
C. 18
D. 9 9
 - The two mandatory pointers in a node for a rooted tree with linked siblings are:
A. First child and right sibling
B. Left child and right child
C. Left child and parent
D. Left sibling and right sibling
 - In which situation will a sentinel be inappropriate?
A. Binary search for a key in an ordered table, to simplify and speed-up code
B. Search for a key in an unordered table, to simplify and speed-up code

- C. Search for a key in an unordered linked list, to simplify and speed-up code
 D. Red-black tree, to simplify code
9. What is the worst-case time to perform $\text{PREDECESSOR}(L, x)$ for an unsorted, singly-linked list with n nodes?
 A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
10. How should the successor of a node with a right child in an unbalanced binary search tree be found?
 A. Examine the ancestors of the node
 B. Go left, then proceed to the right
 C. Go right, then proceed to the left
 D. Preorder traversal
11. Which of the following binary trees can be legally colored as a red-black tree with its root colored red?



12. For which of the following sorts does the decision tree model not apply?
 A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT
13. What does counting sort count?
 A. the maximum length among all the strings being sorted
 B. the number of bytes in the input array
 C. the number of different input values that have occurred
 D. the number of occurrences for each possible key value
14. Which binary tree traversal corresponds to the following recursive code?

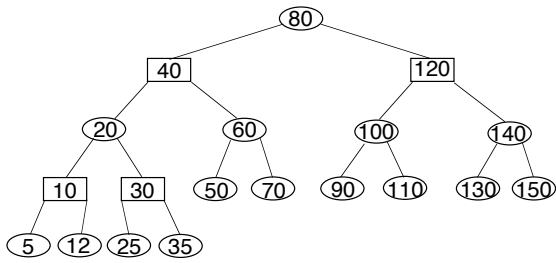
```
void traverse(noderef x)
{
  if (x==null)
    return;
  // process x here
  traverse(x.left);
  traverse(x.right);
}
```

 A. inorder B. postorder C. preorder D. search for key x

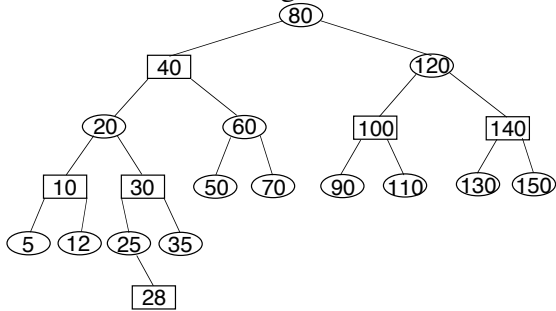
15. In a binary search tree, queries involving the ranks of keys are facilitated by:
 A. parent pointers B. red-black coloring C. storing ranks D. subtree sizes
- Long Answer

- Give the unbalanced binary search tree that results when the keys 50, 100, 80, 70, 60, 90, 120 are inserted, in the given order, into an initially empty tree. (5 points)
- Describe the three situations that can occur for deletion from an unbalanced binary search tree. An example for each situation will be fine. (10 points)
- Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

8	2	5	3	7	4	1	9	0	6
Version:		1			2/Sedgewick				
- Twenty (20) positive integers in the range $0 \dots 99,999,999$ are to be sorted by LSD radix sort. Compare the performance for using radix $0 \dots 9999$ and radix $0 \dots 9$. Show your work. (10 points)
- Insert 15 into the given red-black tree. Be sure to indicate the cases you used. (10 points)



6. Insert 26 into the given red-black tree. Be sure to indicate the cases you used. (10 points)



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Test 3

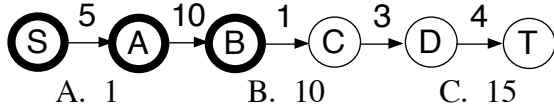
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Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

- The worst-case time for Prim's algorithm implemented with a T-table is:
 - $\theta(V^2 + E)$
 - $\theta(E \lg V)$
 - $\theta(V \lg V)$
 - $\theta(V \lg E)$
- Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Y to Z, then its type will be:
 - Back
 - Cross
 - Forward
 - Tree
- The worst-case time for depth-first search is:
 - $\theta(V \lg E)$
 - $\theta(E \lg V)$
 - $\theta(V \lg V)$
 - $\theta(V + E)$
- Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from Y to X, then its type will be:
 - Back
 - Cross
 - Forward
 - Tree
- During a breadth-first search, the status of a gray vertex is:
 - It has been completely processed.
 - It is in the FIFO queue.
 - It is in the priority queue.
 - It is undiscovered.
- Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times is:
 - $\text{finish}(X) < \text{finish}(Y)$
 - $\text{finish}(X) > \text{finish}(Y)$
 - $\text{finish}(X) = \text{finish}(Y)$
 - could be either A. or B.
- The capacity of any cut is:
 - A lower bound on the maximum flow.
 - An upper bound on the maximum flow.
 - The same as the capacity of all other cuts.
 - The same as the maximum attainable flow.
- An adjacency matrix is the most useful representation for which problem?
 - Breadth-first search
 - Finding strongly-connected components
 - Maximum network flow
 - Warshall's algorithm

9. The capacity of the following cut is _____. (S vertices are bold.)



- A. 1 B. 10 C. 15 D. 23
10. Prim's algorithm, when implemented with a heap, is most suitable for:
- Finding the minimum spanning tree of a dense graph.
 - Finding the minimum spanning tree of a sparse graph.
 - Finding the shortest paths from a designated source vertex in a dense graph.
 - Finding the shortest paths from a designated source vertex in a sparse graph.
11. Before searching for a minimum cut in a network, it is useful to do the following:
- Determine the type of each edge using depth-first search.
 - Find and record augmenting paths until none remains.
 - Find one augmenting path.
 - Perform a breadth-first search on the input network.
12. When finding the strongly connected components, the number of components is indicated by:
- The number of back edges found during the first depth-first search.
 - The number of cross edges found during the second depth-first search.
 - The number of restarts for the first depth-first search.
 - The number of restarts for the second depth-first search.
13. When using two breadth-first searches to find the diameter of a tree, the purpose of the first search is to find:
- all vertices that could be an end of a diameter.
 - both ends of a diameter.
 - one end of a diameter.
 - the number of edges in the diameter.
14. During depth-first search on a directed graph, a cycle is indicated by which edge type?
- Back
 - Cross
 - Forward
 - Tree
15. The expected number of probes for an unsuccessful search in hashing by chaining with α as the load factor is:
- $\frac{\alpha}{2}$
 - $\frac{2}{3}\alpha$
 - α
 - $\frac{1}{1-\alpha}$
16. Compressed adjacency lists have the following disadvantage:
- Testing whether an edge from X to Y is present will take $\theta(V + E)$ worst-case time.
 - They are static.
 - They can only be used for graphs without weights.
 - They require $\theta(V + E)$ space to store.
17. Suppose a depth-first search is performed on an undirected graph. The graph is a free (i.e. unrooted) tree if:
- all edges are tree edges
 - both C and D
 - there are no restarts
 - there are no back edges
18. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:
- $O(\log m)$
 - m
 - $m(m-1)$
 - $m!$
19. Suppose that there is only one path from vertex 5 to vertex 10 in a directed graph:

5 → 7 → 8 → 3 → 2 → 10. During the scan of which column will Warshall’s algorithm record the presence of this path?

- A. 2 B. 5 C. 8 D. 10

20. Which of the following is not true about probe sequences for an implementation of linear probing?

- A. All slots in the hash table appear in each probe sequence
 B. Every key has a probe sequence different from the probe sequences for other keys
 C. The elements of a probe sequence are subscripts in the hash table
 D. The probe sequence for a key cannot change

Problems 21 and 22 refer to the following hash table whose keys are stored by double hashing using $h_1(\text{key}) = \text{key} \% 13$ and $h_2(\text{key}) = 1 + (\text{key} \% 12)$.

0	1	2	3	4	5	6	7	8	9	10	11	12
			120	186	187	162		122	110			194

21. 263 would be inserted into which slot of the given table?

- A. 0 B. 2 C. 3 D. 7

22. 303 would be inserted into which slot of the given table?

- A. 0 B. 2 C. 3 D. 7

Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using $h(\text{key}) = \text{key} \% 13$.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

23. 143 would be inserted into which slot of the given table?

- A. 0 B. 1 C. 2 D. 11

24. 136 would be inserted into which slot of the given table?

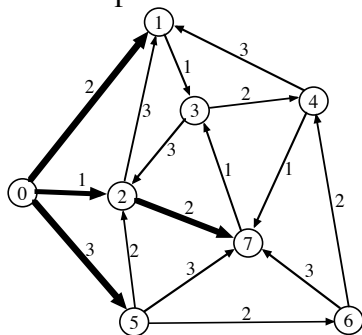
- A. 0 B. 4 C. 6 D. 11

25. The cycle property for minimum spanning trees may be used to find an MST by:

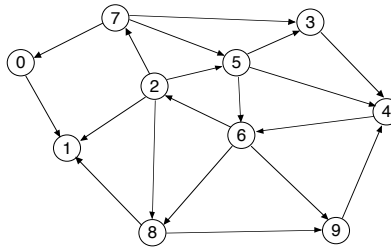
- A. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
 B. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
 C. Remove the maximum weight in any cycle until only a tree of edges remains.
 D. Remove the minimum weight in any cycle until only a tree of edges remains.

Long Answer

1. What are the entries in the heap (for Dijkstra’s algorithm) before and after moving the next vertex and edge into the shortest path tree? **DO NOT COMPLETE THE ENTIRE TREE!!!** Edges already in the shortest path tree are the thick ones. Vertex 0 is the source. 10 points.

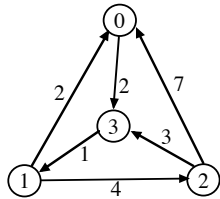


2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points

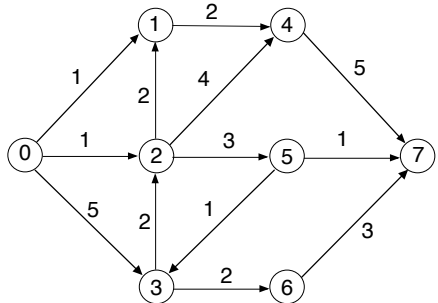


Vertex	Start	Finish	Edge	Type	Edge	Type
0	<u>1</u>	___	0 1	___	6 2	___
1	___	___	2 1	___	6 8	___
2	___	___	2 5	___	6 9	___
3	___	___	2 7	___	7 0	___
4	___	___	2 8	___	7 3	___
5	___	___	3 4	___	7 5	___
6	___	___	4 6	___	8 1	___
7	___	___	5 3	___	8 9	___
8	___	___	5 4	___	9 4	___
9	___	___	5 6	___		

3. Demonstrate the Floyd-Warshall algorithm, **with successors**, for the following graph. The paths indicated in the final matrix must have **at least one** edge. You **are not** required to show the intermediate matrices. 10 points.



4. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.



S vertices: 0

T vertices: 7

Augmenting Paths and Contribution to Flow:

5. Demonstrate, for the graph below, the algorithm that uses **two** depth-first searches to determine the strongly-connected components. 10 points

