CSE 2320 Name _____
Test 1
Spring 2013 Last 4 Digits of Mav ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to multiply an $m \times n$ matrix and a $n \times p$ matrix is in:

   A. $\Theta(n)$ B. $\Theta(\max(m,n,p))$ C. $\Theta(n^3)$ D. $\Theta(mnp)$

2. Which of the following is the best approximation for $H_{mn}$? (*m* and *n* are positive integers)

   A. $(\ln m)(\ln n)$ B. $\ln m + \ln n$ C. $\ln n$ D. $mn$

3. Which sort has both its worst-case and expected times in $\Theta(n^2)$?

   A. heap B. insertion C. merge D. selection

4. Suppose $H_n = \frac{11}{6}$. What is the value of *n*?

   A. 4 B. 3 C. 5 D. 2

5. The function $3\log n + 2n$ is in which set?

   A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Omega(n \log n)$ D. $\Theta(n \log n)$

6. Which of the following is not true?

   A. $n^2 \in \Omega(n \log n)$ B. $g(n) \in \Omega(f(n)) \Leftrightarrow f(n) \in O(g(n))$

   C. $2^n \in \Omega(3^n)$ D. $n^2 \in O(n^3)$

7. What is *n*, the number of elements, for the largest table that can be processed by binary search using no more than 7 probes?

   A. 31 B. 63 C. 64 D. 127

8. $f(n) = n \lg n$ is in all of the following sets, except

   A. $O(n^2)$ B. $\Omega(\log n)$ C. $\Theta(\log(n!))$ D. $O(n)$

9. Suppose there is a large table with *n* integers in descending order, possibly with repeated values. How much time is needed to determine the minimum value?

   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

10. Which of the following is true regarding mergesort?

    A. It is difficult to code without recursion
    B. It is difficult to code to ensure stability
    C. It may be coded to operate in a bottom-up fashion
    D. The input must be preprocessed to exploit ordered subarrays or sublists in the input.

11. The worst-case time to initially construct a minheap for *n* keys is in which set?

    A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know $T(n) \in \Omega(n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

    A. $T(n) \in O(\lg n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n \lg n)$

13. The time for the following code is in which set?

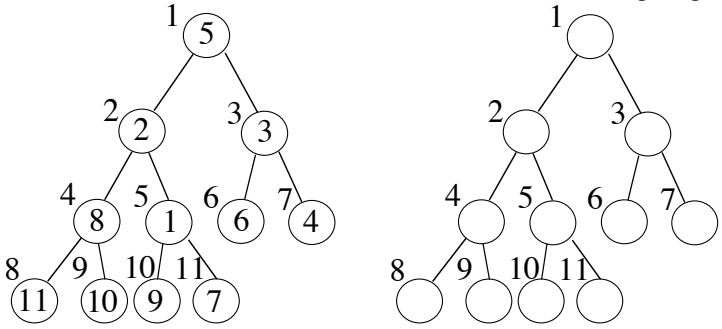
```
for (i=0; i<2; i++)
  for (j=2; j<n; j++)
  {
    c[i][j] = 0;
    for (k=0; k<n; k++)
      c[i][j] += a[i][k]*b[k][j];
  }
```

A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

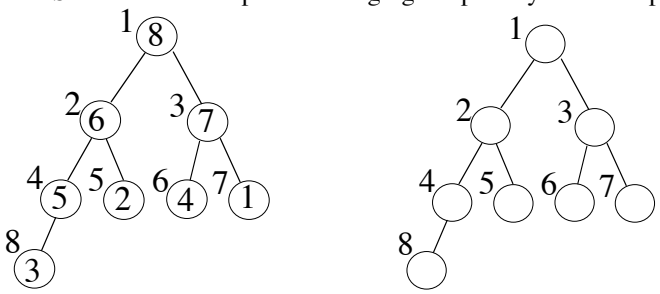
14. Which of the following is not true regarding a maxheap with 1000 elements?
    A. Subscript 1 will store the maximum priority.
    B. The parent for the node with subscript 500 is stored at subscript 250.
    C. The left child for the node with subscript 200 is stored at subscript 400.
    D. The right child for the node with subscript 405 is stored at subscript 911.
15. When solving the fractional knapsack problem, the items are processed in the following order.
    A. Ascending order of weight B. Ascending order of $$$/lb
    C. Descending order of weight D. Descending order of $$$/lb

Long Answer

1. Use the efficient construction to convert into a minHeap. 5 points



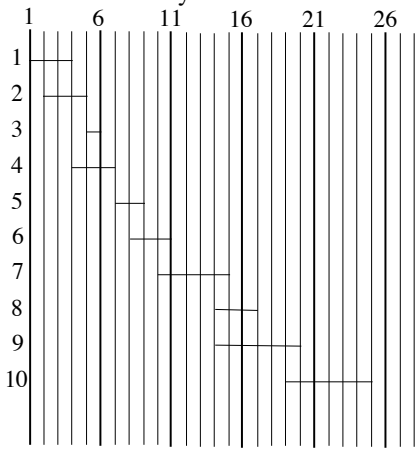

2. Show the maxheap after changing the priority at subscript 6 to 9. 5 points




3. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta\left(\sqrt{n} \log n\right)$. 10 points

4. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta\left(\sqrt{n} \log n\right)$. 10 points

5. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points




| | $v_i$ | $p_i$ | $m(i)$ |
|---|---|---|---|
| 1 | 1 | 0 | |
| 2 | 5 | 0 | |
| 3 | 3 | 2 | |
| 4 | 8 | 1 | |
| 5 | 1 | 4 | |
| 6 | 2 | 4 | |
| 7 | 3 | 5 | |
| 8 | 1 | 6 | |
| 9 | 4 | 6 | |
| 10 | 1 | 8 | |

6. Give the greedy solution for the unweighted interval scheduling problem using the set of intervals for problem 6. You may simply give the indices for the intervals in the solution. 5 points
7. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

```
p[0]=5
p[1]=2
p[2]=2
p[3]=4
p[4]=6
```

| | 1 | | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 20 | 1 | 56 | 1 | ??? | ? |
| 2 | ------- | | 0 | 0 | 16 | 2 | 64 | 3 |
| 3 | ------- | ------- | | 0 | 0 | 48 | 3 | |
| 4 | ------- | ------- | ------- | | 0 | 0 | | |

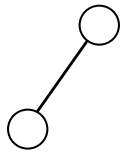
CSE 2320 Name _____
Test 2
Spring 2013 Last 4 Digits of Student ID # _____

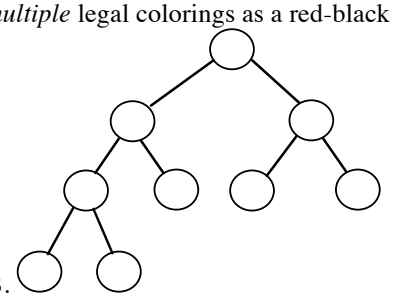
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

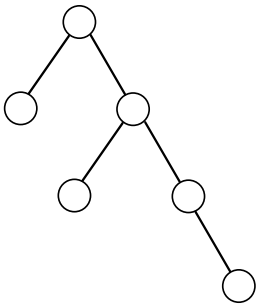
1. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?
   A. Like a circular queue, the maximum number of items is determined at initialization.
   B. One node is always wasted.
   C. The head points to the first element and the tail points to the last element.
   D. The tail points to the first element and the head points to the last element.
2. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with $n$ keys and height $h$ is:
   A. $O(1)$ B. $O(\log n)$ C. $O(h)$ D. $O(n)$
3. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting $n$ input values?
   A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
   B. The height of the tree is $\Omega(n \log n)$.
   C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
   D. There will be $n!$ leaves.
4. In the example of recycling the elements of a list in O(1) time, which situation holds?
   A. The list to be recycled is circular, the garbage list is not
   B. The garbage list is circular, the list to be recycled is not
   C. Both lists are circular
   D. Both lists are not circular
5. Given a pointer to a node, the worst-case time to delete the node from a singly-linked list with $n$ nodes in ascending order is:
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$
6. What is the worst-case time to find the predecessor of a key in an unbalanced binary search tree storing $n$ keys? Assume that parent pointers are available.
   A. $\Theta(n)$ B. A. $\Theta(1)$ C. $\Theta(\log n)$ D. $\Theta(n \log n)$
7. The two mandatory pointers in a node for a rooted tree with linked siblings are:
   A. First child and right sibling B. Left sibling and right sibling
   C. Left child and right child D. Left child and parent
8. In which situation will a sentinel be inappropriate?
   A. Search for a key in an unordered linked list, to simplify and speed-up code
   B. Red-black tree, to simplify code
   C. Search for a key in an unordered table, to simplify and speed-up code
   D. Binary search for a key in an ordered table, to simplify and speed-up code
9. In a red-black tree holding $n$ keys, what is the total number of left and right pointers that will be set to `nil` (the sentinel)?
   A. $n + 1$ B. $n$ C. $n - 1$ D. None of the above
10. Suppose a sequence of $n$ keys will be inserted into an initially-empty instance of the following data structures. Which of the following will not take $\Theta(n^2)$ worst-case time for the entire sequence?
    A. ordered table

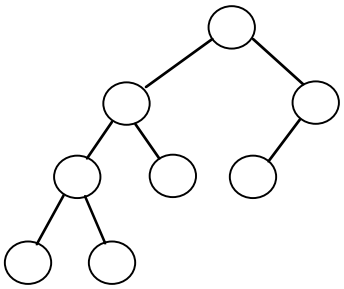
B. ordered linked list
C. red-black tree
D. unbalanced binary search tree

11. Which of the following binary trees has *multiple* legal colorings as a red-black tree?

A. 

B. 

C. 

D. 

12. Which of the following sorts is not based on key comparisons?
    A. COUNTING-SORT B. INSERTION-SORT C. MERGESORT D. QUICKSORT
13. An array with 150 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?
    A. If 120 is returned from PARTITION, we must continue.
    B. If 131 is returned from PARTITION, we must continue.
    C. If 118 is returned from PARTITION, we must continue.
    D. If 119 is returned from PARTITION, we are done.

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
if (x==null)
  return;
traverse(x.left);
traverse(x.right);
// process x here
}
```

    A. inorder B. postorder C. preorder D. search for key `x`

15. The worst-case number of comparisons for finding the *k*th largest of *n* keys using PARTITION is in which asymptotic set?

    A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

Long Answer

1. Give the unbalanced binary search tree that results when the keys 60, 90, 70, 100, 80, 50, 40, 30 are inserted, in the given order, into an initially empty tree. (5 points)
2. A billion integers in the range $0 \ldots 2^{20} - 1$ will be sorted by LSD radix sort. How much faster is this done using radix $0 \ldots 2^{10} - 1$ rather than $0 \ldots 2^5 - 1$? Show your work. (10 points)
3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

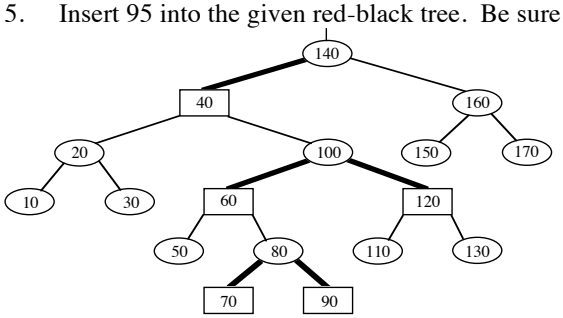
| 8 | 2 | 6 | 3 | 4 | 1 | 9 | 0 | 7 | 5 |
|---|---|---|---|---|---|---|---|---|---|

Version: 1 2/Sedgewick

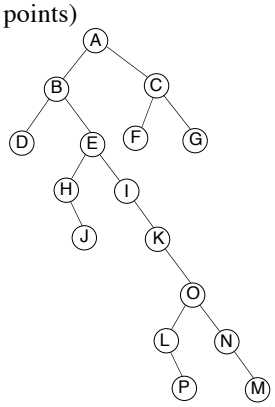
4. Use the dynamic programming solution for subset sums to determine a subset that sums to 15. (10 points)

| $i$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $S_i$ | 0 | 2 | 3 | 5 | 7 | 11 |

5. Insert 95 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



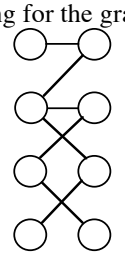

6. Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (10 points)




CSE 2320 Name _____
Test 3
Spring 2013 Last 4 Digits of Student ID # _____

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. The number of edges in a maximum bipartite matching for the graph below is:

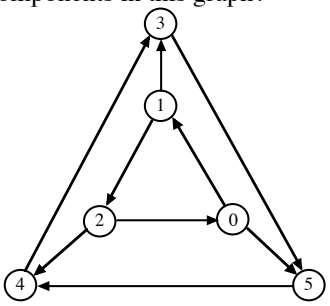


A. 2 B. 3 C. 4 D. 5

2. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?
   A. hashing with chaining B. open addressing
   C. ordered linked list D. unbalanced binary search tree
3. Which edge is chosen in a phase of Kruskal's algorithm?
   A. An edge that is on a shortest path from the source
   B. An edge of maximum-weight in a cycle (to be excluded)
   C. A minimum-weight edge that keeps the result free of cycles
   D. A minimum-weight edge connecting T to S.
4. The fastest method for finding the diameter of a tree is to:
   A. Use the Floyd-Warshall algorithm. B. Use the Ford-Fulkerson algorithm.
   C. Use breadth-first search. D. Use Dijkstra's algorithm.
5. The capacity of any cut is:
   A. The same as the maximum attainable flow. B. A lower bound on the maximum flow.
   C. An upper bound on the maximum flow. D. The same as the capacity of all other cuts.
6. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:
   A. finish(X) = finish(Y) B. could be either A. or B.
   C. finish(X) < finish(Y) D. finish(X) > finish(Y)
7. The relationship of the net flow across a cut and the amount of flow from the source to the sink is:
   A. The amount of flow does not exceed the net flow.
   B. The net flow does not exceed the amount of flow.

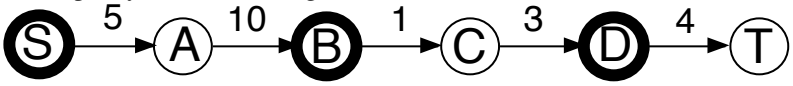
C. They are equal.
D. There is no relationship.

8. What is the number of strongly connected components in this graph?




A. 1 B. 2 C. 3 D. 4

9. The capacity of the following cut is _____. (S vertices are bold.)




A. 1 B. 10 C. 13 D. 23

10. When a graph is dense, the best way to find a minimum spanning tree is:
A. Prim's algorithm using T-table B. Floyd-Warshall algorithm
C. Prim's algorithm using heap D. Warshall's algorithm

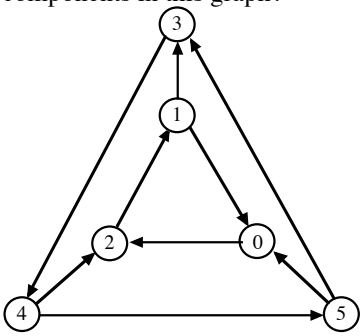
11. The Edmonds-Karp variant is important because:
A. It solves the bipartite matching problem.
B. It solves the network flow problem in polynomial time.
C. It solves the network flow problem using critical edges.
D. It solves the network flow problem without using augmenting paths.

12. Which of the following cannot occur when additional edges are included in a directed graph?
A. The number of strong components may decrease.
B. The number of strong components may increase.
C. The graph acquires a cycle.
D. The number of strong components may remain the same.

13. The number of potential probe sequences when using linear probing with a table with *m* entries is:

A. $O(\log m)$ B. $m$ C. $m(m-1)$ D. $m!$

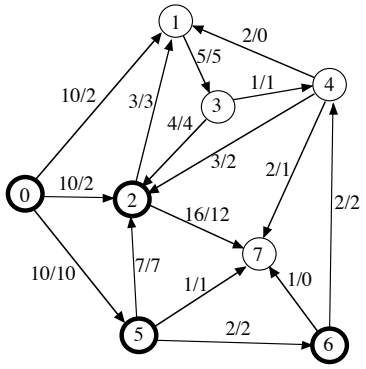
14. What is the number of strongly connected components in this graph?




A. 1 B. 2 C. 3 D. 4

15. The worst-case time for Prim's algorithm implemented with a minheap is:
A. θ(V + E) B. θ(E lg V) C. θ(V lg V) D. θ(V lg E)

Problems 16, 17, and 18 refer to the following network. 0 is the source. 7 is the sink. Each edge is labeled with capacity/flow.




16. The capacity of the indicated cut (S vertices are bold) is:
    A. 31 B. 32 C. 33 D. 34
17. The net flow across the given cut is:
    A. 14 B. 16 C. 18 D. 20
18. Suppose the flow is increased as much as possible using the augmenting path 0 → 2 → 4 → 7. Which is the critical edge?
    A. 0 → 2 B. 2 → 4 C. 4 → 7 D. Insufficient information
19. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers 0, 1, . . ., *V* - 1.)
    A. Adjacency lists (ordered): $\Theta(V)$ B. Adjacency lists (unordered): $\Theta(V)$
    C. Adjacency matrix: $\Theta(1)$ D. Compressed adjacency lists (ordered): $\Theta(V)$
20. The expected number of probes for a successful search in hashing by chaining with $\alpha$ as the load factor is:
    A. $\alpha$ B. $2\alpha$ C. $\frac{\alpha}{2}$ D. $\frac{2}{3}\alpha$

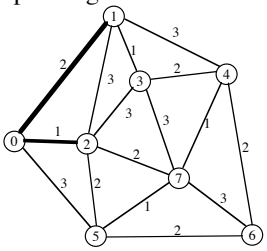
Long Answer

1. Consider the following hash table whose keys were stored by double hashing using $h_1(key) = key \% 17$ and $h_2(key) = 1 + (key \% 16)$.

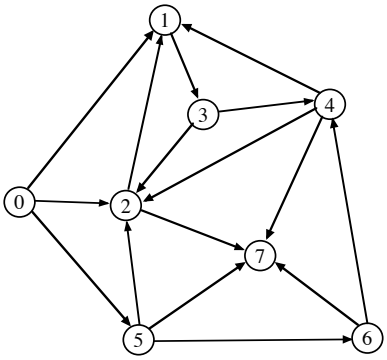
```
0  -1
1  800
2  -1
3  -1
4  701
5  -1
6  601
7  -1
8  501
9  -1
10 401
11 -1
12 301
13 -1
14 201
15 -1
16 101
```

   a. Suppose 1000 is to be inserted (using double hashing). Which slot will be used? (5 points)
   b. Suppose 1001 is to be inserted (using double hashing) *after* 1000 has been stored. Which slot will be used? (5 points)

2. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.

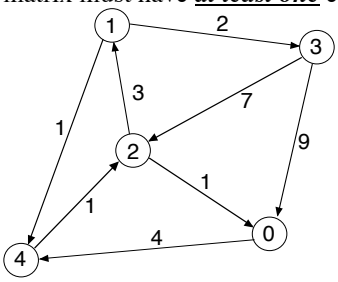



3. Perform a breadth-first search on the following graph listing the BFS number, shortest path distance (hops) from the source (0), and the predecessor for each vertex. Assume that the adjacency lists are **ordered**. 10 points

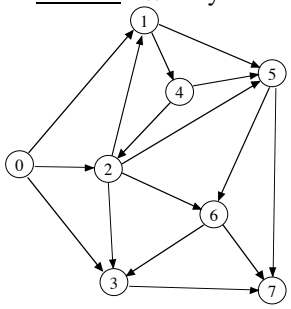



| Vertex | BFS Number | Distance | Predecessor |
|---|---|---|---|
| 0 | _____ | _____ | _____ |
| 1 | _____ | _____ | _____ |
| 2 | _____ | _____ | _____ |
| 3 | _____ | _____ | _____ |
| 4 | _____ | _____ | _____ |
| 5 | _____ | _____ | _____ |
| 6 | _____ | _____ | _____ |
| 7 | _____ | _____ | _____ |

4. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following graph. The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points.

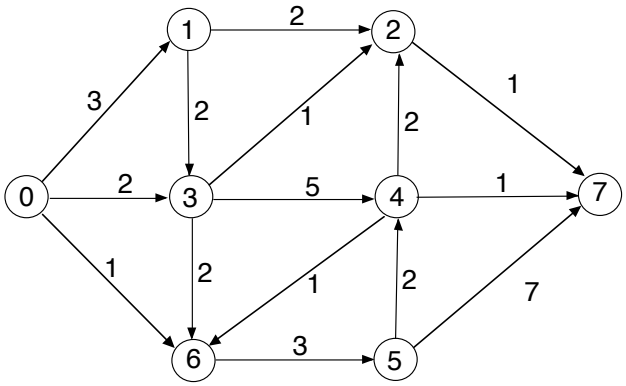



5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ***ordered***. Write your answer in the tables below. 10 points




| Vertex | Start | Finish | Edge | Type | Edge | Type |
|---|---|---|---|---|---|---|
| 0 | 1 | ____ | 0 1 | ____ | 2 6 | ____ |
| 1 | ____ | ____ | 0 2 | ____ | 3 7 | ____ |
| 2 | ____ | ____ | 0 3 | ____ | 4 2 | ____ |
| 3 | ____ | ____ | 1 4 | ____ | 4 5 | ____ |
| 4 | ____ | ____ | 1 5 | ____ | 5 6 | ____ |
| 5 | ____ | ____ | 2 1 | ____ | 5 7 | ____ |
| 6 | ____ | ____ | 2 3 | ____ | 6 3 | ____ |
| 7 | ____ | ____ | 2 5 | ____ | 6 7 | ____ |

6. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.




S vertices: 0

T vertices: 7

Augmenting Paths and Contribution to Flow:

Extra Credit: Fill in the KMP failure links. 10 points

| 1 | pattern | 2 |
|---|---|---|
| 0 | a | |
| 1 | b | |
| 2 | c | |
| 3 | d | |
| 4 | a | |
| 5 | b | |
| 6 | c | |
| 7 | a | |
| 8 | b | |
| 9 | a | |