CSE 2320 Name ________________________________

Test 1

Spring 2015

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to run the code below is in:

```
for (i=n; i>=1; i--)
  for (j=1; j<n; j=2*j+2)
    sum+=i+j;
```

A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

2. Which of the following is the best approximation for H_{mn}? (m and n are positive integers)

A. $\frac{1}{mn}$ B. $\ln m + \ln n$ C. $\ln n$ D. 1

3. The expected time for insertion sort for n keys is in which set? (All $n!$ input permutations are equally likely.)

A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

4. Which technique allows interfacing a priority queue with a dictionary?

A. Binary search tree B. `PQchange` C. Array of handles D. Binary search

5. The time to run the code below is in:

```
for (i=n; i>=0; i--)
  for (j=5; j<n; j+=2)
    sum+=i*j;
```

A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

6. Which of the following is not true?

A. $2^n \in \Omega(3^n)$ B. $n^2 \in O(n^3)$

C. $n^2 \in \Omega(n \log n)$ D. $g(n) \in \Omega(f(n)) \Leftrightarrow f(n) \in O(g(n))$

7. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 7 probes?

A. 31 B. 63 C. 64 D. 127

8. $f(n) = n \lg n$ is in all of the following sets, except

A. $\Omega(\log n)$ B. $\Theta(\log(n!))$ C. $O(n)$ D. $O(n^2)$

9. Suppose there is a large, unordered table with n integers, possibly with repeated values. How much time is needed to determine the minimum value?

A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

10. When solving the fractional knapsack problem, the items are processed in the following order.

A. Ascending order of weight B. Ascending order of $$$/lb

C. Descending order of weight D. Descending order of $$$/lb

11. Which of the following is not true regarding dynamic programming?

A. It is a form of exhaustive search B. It is a form of divide-and-conquer

C. A cost function must be defined D. The backtrace may be based on recomputing the cost function

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know $T(n) \in \Omega(n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A. $T(n) \in O(\lg n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n \lg n)$

13. What is required when calling `union(find(i),find(j))` for maintaining disjoint subsets?

A. `i` and `j` are leaders for different subsets

B. `i` and `j` are in the same subset

C. `i` and `j` are in different subsets

D. `i` is the ancestor of `j` in one of the trees

14. Which of the following is not true regarding a maxheap with 1000 elements?
    A. Subscript 1 will store the maximum priority.
    B. The parent for the node with subscript 500 is stored at subscript 250.
    C. The left child for the node with subscript 200 is stored at subscript 400.
    D. The right child for the node with subscript 405 is stored at subscript 911.
15. $\log(n!)$ is in all of the following sets, except

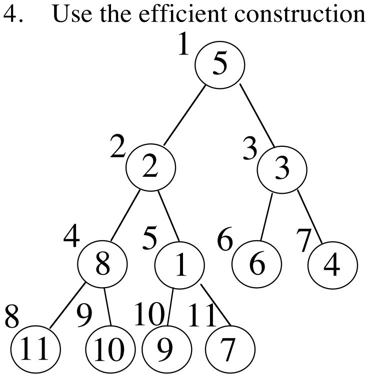
    A. $\Omega(\log n)$ B. $\Theta(n \log n)$ C. $O(n^2)$ D. $\Omega(n^2)$

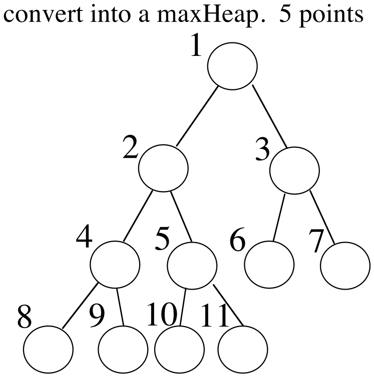
Long Answer

1. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 15 points

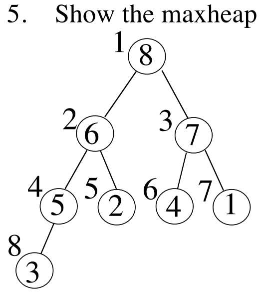
A	0.15
B	0.03
C	0.5
D	0.03
E	0.22
F	0.07

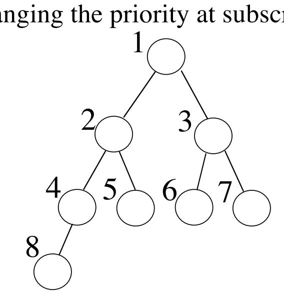
2. Use the substitution method to show that $T(n) = 3T\left(\frac{n}{2}\right) + n^2$ is in $O(n^2)$. (You do not need to show that $T(n)$ is in $\Omega(n^2)$.) 15 points
3. Use the recursion-tree method to show that $T(n) = 3T\left(\frac{n}{2}\right) + n^2$ is in $\Theta(n^2)$. 15 points
4. Use the efficient construction to convert into a maxHeap. 5 points





5. Show the maxheap after changing the priority at subscript 8 to 7. 5 points





CSE 2320 Name ________________________________

Test 2

Spring 2015

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. Recently, we considered an abstraction supporting the operations *allocate*, *allocateAny*, and *freeup* in constant time. Which of the following was not a feature of the implementation?

_____ A. a recycling list B. a circular, doubly-linked list C. a header D. arrays

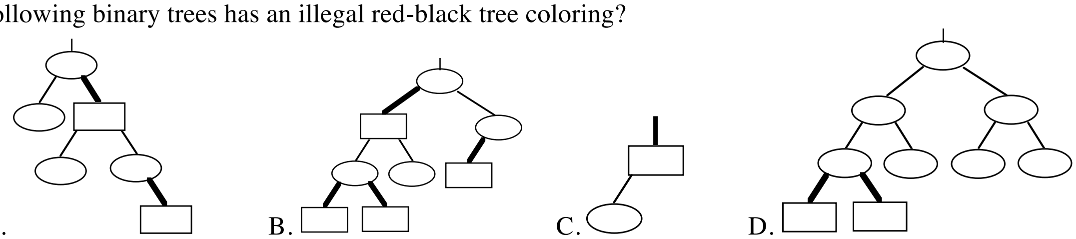
2. What is the worst-case time to perform MINIMUM(L) for an unsorted, doubly-linked list with n nodes?

_____ A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

3. The time to extract the LCS (for sequences of lengths m and n) after filling in the dynamic programming matrix is in:
_____ A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

4. Circular linked lists are occasionally useful because
_____ A. some operations may be done in constant time.
B. they are an alternative to red-black trees.
C. they are useful for implementing circular queues.
D. they avoid mallocs.

5. Which of the following binary trees has an illegal red-black tree coloring?



_____ A. B. C. D.

6. Which of the following will not be true regarding the decision tree for MERGESORT for sorting n input values?
_____ A. There will be n! leaves.
B. Every path from the root to a leaf will have $O(n^2)$ decisions.
C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
D. The height of the tree is $\Omega(n \log n)$.

7. Which phase of counting sort actually "counts"?
_____ A. first B. second C. third D. fourth

8. What is the worst-case time to find the predecessor of a key in an unbalanced binary search tree storing n keys? Assume that parent pointers are available.
_____ A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

9. For which of the following sorts does the decision tree model not apply?
_____ A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT

10. The expected number of comparisons for finding the kth largest of n keys using PARTITION is in which asymptotic set?
_____ A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

11. The subset sum problem takes n input values and attempts to find a combination of those values whose sum is m. The worst-case time to extract the solution from the dynamic programming table is:
_____ A. $\Theta(\log m)$ B. $\Theta(n)$ C. $\Theta(m)$ D. $\Theta(mn)$g

12. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:
_____ A. return stack[SP++] B. stack[SP++] = X C. stack[--SP] = X D. stack[++SP] = X

13. In a red-black tree holding n keys, what is the total number of left and right pointers that will be set to `nil` (the sentinel)?
_____ A. n - 1 B. n C. n + 1 D. None of the previous answers

14. The disadvantage of using LCS to solve an instance of LIS is:
_____ A. it only works for the monotone case
B. it takes $\Theta(n^2)$ time in the worst case
C. the input must be free of duplicate values
D. it requires sorting

15. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
_____ A. 300, 400, 900, 800, 500
B. 700, 200, 600, 550, 500
C. 200, 700, 600, 300, 400, 500
D. 100, 1000, 200, 800, 300, 900, 500

Long Answer

1. Give the unbalanced binary search tree that results when the keys 60, 90, 70, 100, 80, 50, 40, 30 are inserted, ***<u>in the given order</u>***, into an initially empty tree. (5 points)

2. Complete the following instance of the optimal matrix multiplication ordering problem, ***including the tree*** showing the optimal ordering. 10 points

```
p[0]=5     p[1]=4     p[2]=6     p[3]=2     p[4]=5     p[5]=6
```

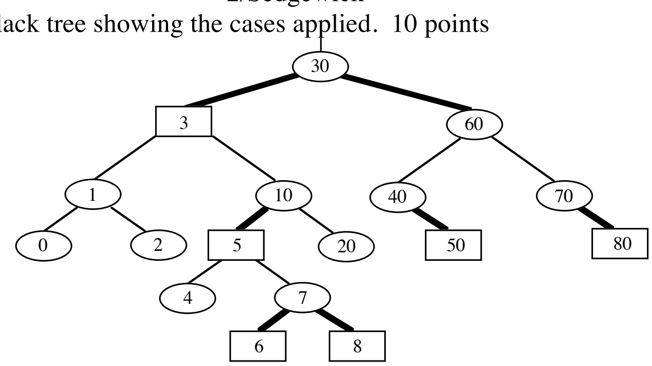
	1		2		3		4		5	
1	0	0	120	1	88	1	138	3	???	?
2	-------		0	0	48	2	88	3	156	3
3	-------		-------		0	0	60	3	132	3
4	-------		-------		-------		0	0	60	4
5	-------		-------		-------		-------		0	0

3. Show the result after PARTITION manipulates the following subarray. ***Be sure to circle*** which version of PARTITION you applied. (10 points)

```
8   2   6   3   4   1   9   0   7   5
```

Version: 1 2/Sedgewick

4. Insert 9 into the given red-black tree showing the cases applied. 10 points



5. Use the dynamic programming solution for subset sums to determine a subset that sums to 15.
Be sure to give the complete table that would be produced. (10 points)

i	0	1	2	3	4	5
S_i	0	2	3	5	7	11

6. Twenty million positive integers in the range 0 . . . 999,999,999 are to be sorted by LSD radix sort. Compare the performance for using radix 0 . . . 999 and radix 0 . . . 9. ***Show your work***. (10 points)

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Test 3

Spring 2015

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. Which edge is chosen in a phase of Kruskal's algorithm?

_____ A. An edge that is on a shortest path from the source

B. The unprocessed edge (x, y) of smallest weight such that `find(x)!=find(y)`

C. The unprocessed edge (x, y) of smallest weight such that `find(x)==find(y)`

D. An edge of maximum-weight in a cycle (to be excluded)

2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from X to Y, its type will be:

_____ A. Back B. Cross C. Forward D. Tree

3. Which statement is incorrect regarding depth-first search on a directed graph?

_____ A. Exploring an edge whose head is colored white will cause the edge to be a tree edge.

B. The run time is $\Theta(m+n)$, where m is the number of edges and n is the number of vertices.

C. Exploring an edge whose head is colored black will cause the edge to be a back edge.

D. Exploring an edge whose head is colored gray will cause the edge to be a back edge.

4. An adjacency matrix is the most useful representation for which problem?

_____ A. Breadth-first search B. Finding strongly-connected components

C. Topological Ordering D. Warshall's algorithm

5. Suppose an adjacency matrix represents a directed graph with V vertices (numbered 0 .. V-1) and an adjacency list representation (with unordered lists) represents a directed graph with the same vertices. How fast can you verify that the two representations are storing the same graph? You are allowed to use additional memory.

_____ A. $\Theta(V)$ B. $\Theta(V \log V)$ C. $\Theta(V^2)$ D. $\Theta(V^3)$

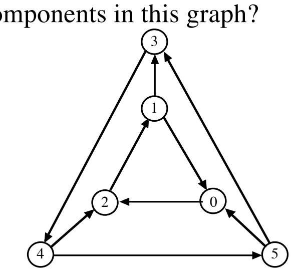
6. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?

_____ A. hashing with chaining B. open addressing
C. ordered linked list D. unbalanced binary search tree

7. The cycle property for minimum spanning trees may be used to find an MST by:

_____ A. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
B. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
C. Remove the maximum weight in any cycle until only a tree of edges remains.
D. Remove the minimum weight in any cycle until only a tree of edges remains.

8. What is the number of strongly connected components in this graph?



_____ A. 1 B. 2 C. 3 D. 4

9. During a breadth-first search, the status of a gray vertex is:

_____ A. It has been completely processed. B. It is in the priority queue.
C. It is in the FIFO queue. D. It is undiscovered.

10. When a graph is dense, the best way to find a minimum spanning tree is:

_____ A. Floyd-Warshall algorithm B. Prim's algorithm using heap
C. Prim's algorithm using T-table D. Warshall's algorithm

11. Suppose Warshall's algorithm is used on a directed graph with vertices 0 . . . 25, but is stopped after column 10 in the matrix is processed. Which paths will be represented in the matrix?

_____ A. All paths that start at some vertex in 0 . . . 10 and stop at some vertex in 0 . . . 10.
B. All paths that start at some vertex in 0 . . . 10, stop at some vertex in 0 . . . 10, and have only vertices in 0 . . . 10 in between.
C. All paths that start at some vertex in 0 . . . 25, stop at some vertex in 0 . . . 25, and have only vertices in 0 . . . 10 in between.
D. All paths with no more than 12 edges.

12. Which of the following cannot occur when additional edges are included in a directed graph?

_____ A. The graph acquires a cycle.
B. The number of strong components may remain the same.
C. The number of strong components may decrease.
D. The number of strong components may increase.

13. What is the purpose of the first depth-first search when finding strongly connected components?

_____ A. To assure that two vertices, X and Y, with paths from X to Y but not from Y to X, are output in different components.
B. To assure that two vertices that are in the same cycle will be output in the same component
C. To assure that two vertices with no paths between them are not output in the same component
D. To make sure that the input graph has no cycles.

14. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:

_____ A. $m(m-1)$ B. $m!$ C. $O(\log m)$ D. m

15. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **<u>incorrect</u>**? (Vertices are conveniently labeled by numbers 0, 1, . . ., V - 1.)

_____ A. Adjacency lists (ordered): $\Theta(V)$ B. Adjacency lists (unordered): $\Theta(V)$
C. Adjacency matrix: $\Theta(1)$ D. Compressed adjacency lists (ordered): $\Theta(V)$

16. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 3 to 1?

```
-1   3   3   3   3
-1   3   3   3   4
-1   1   1   1   4
-1   2   2   2   2
-1  -1  -1  -1  -1
```

_____ A. 0 B. 1 C. 2 D. 3

17. The fastest method for finding the diameter of a tree (where distance is measured in "hops") is to:

_____ A. Use breadth-first search. B. Use Dijkstra's algorithm. C. Use the Floyd-Warshall algorithm. D. Use Prim's algorithm.

18. The main disadvantage of compressed adjacency lists is:

_____ A. Undirected graphs may not be represented B. Directed graphs may not be represented C. It is difficult to change the graph D. They waste space

19. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

_____ A. α B. 2α C. $\frac{\alpha}{2}$ D. $\frac{2}{3}\alpha$

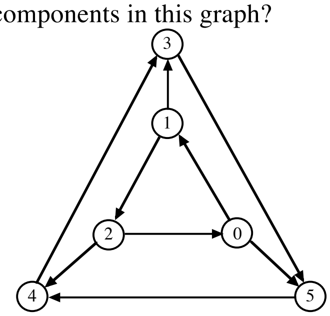
20. Which of the following is not true about probe sequences for an implementation of double hashing?

_____ A. Two keys could have the same probe sequence
B. The probe sequence for a key cannot change
C. All slots in the hash table appear in each probe sequence
D. The elements of a probe sequence are possible keys for the hash table

21. Suppose a double hash table has $\alpha = 0.8$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

_____ A. 1.2 B. 2 C. 5 D. 10

22. What is the number of strongly connected components in this graph?



_____ A. 1 B. 2 C. 3 D. 4

Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using h(key) = key % 13.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

23. 143 would be inserted into which slot of the given table?

_____ A. 0 B. 1 C. 2 D. 11

24. 136 would be inserted into which slot of the given table? (without previously inserting 143)

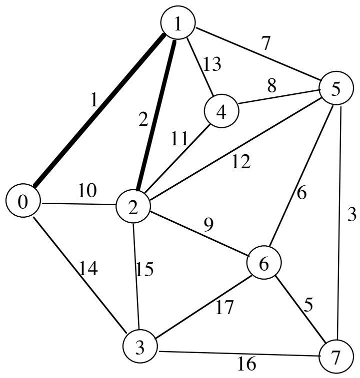
_____ A. 0 B. 4 C. 6 D. 11

25. Which algorithm maintains multiple subtrees?

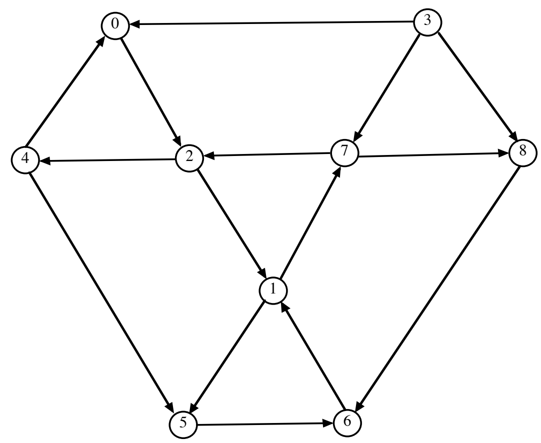
_____ A. Dijkstra's. B. Kruskal's C. Prim's D. Warshall's

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.



2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points



Vertex	Start	Finish
0	1	____
1	____	____
2	____	____
3	____	____
4	____	____
5	____	____
6	____	____
7	____	____
8	____	____

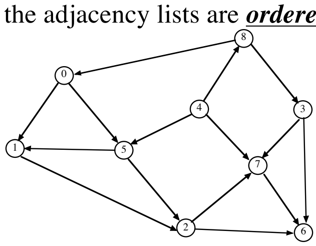
Edge	Type
0 2	____
1 5	____
1 7	____
2 1	____
2 4	____
3 0	____
3 7	____
3 8	____
4 0	____
4 5	____

Edge	Type
5 6	____
6 1	____
7 2	____
7 8	____
8 6	____

3. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following input adjacency matrix. (`999` represents infinity) The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points.

	0	1	2	3	4
0	999	999	3	999	4
1	999	999	11	12	4
2	8	6	999	5	999
3	15	999	20	999	999
4	999	999	5	999	999

4. Demonstrate, for the graph below, the algorithm that uses ***depth-first search*** to determine a topological ordering. Assume that the adjacency lists are ***ordered***. Show your work. 10 points



5. Consider the following hash table whose keys were stored by double hashing using $h_1(key) = key \% 11$ and $h_2(key) = 1 + (key \% 10)$. ***<u>Show your work</u>***.

```
0 22
1
2
3 17
4 4
5 15
6 28
7
8
9
10 10
```

a. Suppose 142 is to be inserted (using double hashing). Which slot will be used? (5 points)

b. Suppose 130 is to be inserted (using double hashing) ***<u>after</u>*** 142 has been stored. Which slot will be used? (5 points)