

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time for the following code is in which set?

```
for (i=0; i<n; i++)
  for (j=0; j<n; j++)
  {
    c[i][j] = 0;
    for (k=0; k<n; k++)
      c[i][j] += a[i][k]*b[k][j];
  }
```

A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

2. The fractional knapsack problem may be solved optimally by a greedy method by taking a fraction of no more than this number of items.

A. 0 B. 1 C. 2 D. 3

3. A sort is said to be stable when:

A. Duplicate copies of a key will appear in the same order in the output as in the input.
 B. It removes duplicate copies of any key in the final output.
 C. It runs in $O(n \log n)$ time.
 D. The average time and the worst-case time are the same.

4. What is the value of H_3 ?

A. $\lg 3$ B. $\frac{1}{3}$ C. $\frac{11}{6}$ D. 3

5. $f(n) = n \lg n$ is in all of the following sets, except

A. $O(\log n)$ B. $\Theta(\log(n!))$ C. $\Omega\left(\frac{1}{n}\right)$ D. $O(n^2)$

6. Which of the following is not true?

A. $n^3 \in \Omega(n^2)$
 B. $n \log n \in O(n^2)$
 C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$
 D. $\log \log n \in \Omega(\log n)$

7. Suppose a binary search is to be performed on a table with 50 elements. The maximum number of elements that could be examined (probes) is:

A. 4 B. 5 C. 6 D. 7

8. Which of the following functions is not in $\Omega(n^2)$?

A. n B. n^2 C. $n^2 \lg n$ D. n^3

9. Which of the following is not true for the activity scheduling problem?

A. The activities may have various durations.
 B. The greedy solution is optimal.
 C. There may be several optimal solutions.
 D. The goal is to minimize the number of activities chosen.

10. The subset sum problem takes n input values and attempts to find a combination of those values whose sum is m . The worst-case time to extract the solution from the dynamic programming table is:

A. $\Theta(\log m)$ B. $\Theta(n)$ C. $\Theta(m)$ D. $\Theta(mn)$

11. $4^{\lg 7}$ evaluates to which of the following? (Recall that $\lg x = \log_2 x$.)

A. $\sqrt{7}$ B. 7 C. 30 D. 49

12. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\lg n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

- A. $T(n) \in O(\lg n)$ B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

13. The number of calls to `getmin` to build a Huffman code tree for n symbols is:
 A. $\Theta(\log n)$ B. $n - 1$ C. n D. $2n - 2$
14. Which situation for operating on a maxheap uses a `swim`?
 A. `decreaseKey` B. `getmax` C. `insert` D. `heapsort`
15. The time to extract the LCS (for sequences of lengths m and n) after filling in the dynamic programming matrix is in:
 A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

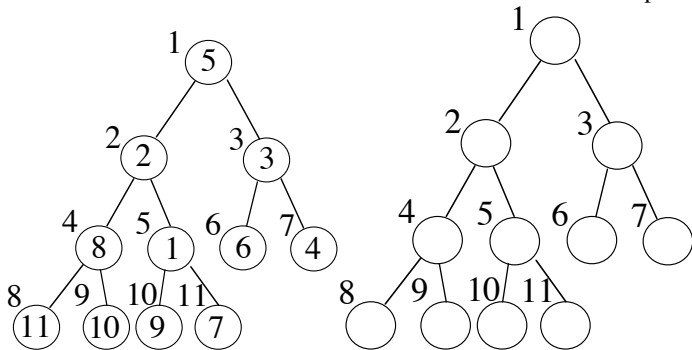
Long Answer

1. Use dynamic programming to find a subset of $\{1, 2, 4, 7, 11\}$ that sums to 13. (No credit for solving by inspection.) 10 points
2. Describe the worst-case input for insertion sort. 5 points
3. Use the recursion-tree method to show that $T(n) = 2T(\frac{n}{4}) + 1$ is in $\Theta(\sqrt{n})$. 10 points
4. Use the substitution method to show that $T(n) = 2T(\frac{n}{4}) + 1$ is in $\Theta(\sqrt{n})$. 10 points
5. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

$p[0]=8$
 $p[1]=6$
 $p[2]=4$
 $p[3]=3$
 $p[4]=5$
 $p[5]=7$

1		2		3		4		5		
1	0	0	192	1	216	1	336	3	???	?
2	-----		0	0	72	2	162	3	303	3
3	-----	-----			0	0	60	3	189	3
4	-----	-----	-----				0	0	105	4
5	-----	-----	-----	-----					0	0

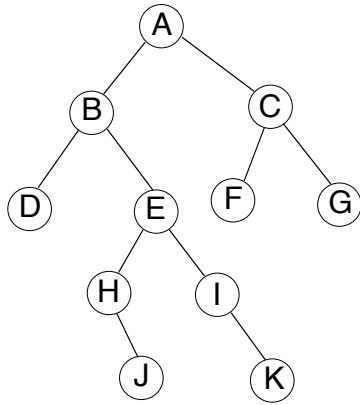
6. Use the efficient construction to convert into a maxHeap. 10 points



CSE 2320
 Test 2
 Summer 2008

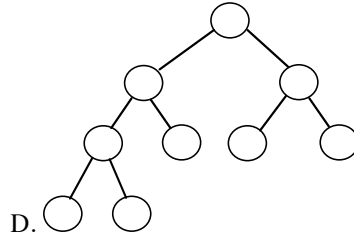
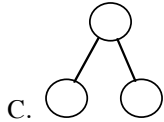
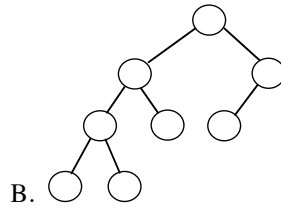
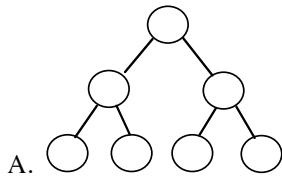
Name _____
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- Multiple Choice. Write your answer to the LEFT of each problem. 3 points each
1. If POP is implemented as `return stack[--SP]`, then PUSH of element X is implemented as:
 A. `return stack[SP++]` B. `stack[SP++] = X` C. `stack[SP--] = X` D. `stack[++SP] = X`
2. Suppose the tree below is a binary search tree whose keys are not shown. Which node will contain the key that is the predecessor of the key stored at H?



- A. A
- B. B
- C. C
- D. G

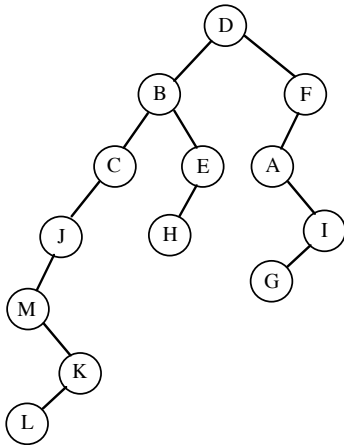
3. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?
 - A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
 - B. The height of the tree is $\Omega(n \log n)$.
 - C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
 - D. There will be $n!$ leaves.
4. What is the worst-case time to perform $\text{MINIMUM}(L)$ for an ordered, doubly-linked list with n nodes?
 - A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
5. Which type of linked list will be the most convenient if $\text{LOGICALPREDECESSOR}$ and LOGICALSUCCESSOR operations will be frequent?
 - A. ordered, doubly-linked
 - B. ordered, singly-linked
 - C. unordered, doubly-linked
 - D. unordered, singly-linked
6. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with n keys and height h is:
 - A. $O(1)$ B. $O(\log n)$ C. $O(h)$ D. $O(n)$
7. The two mandatory pointers in a node for a binary tree are:
 - A. First child and right sibling
 - B. Left child and right child
 - C. Left child and parent
 - D. Left sibling and right sibling
8. In the example of recycling the elements of a list in $O(1)$ time, which situation holds?
 - A. Both lists are circular
 - B. Both lists are not circular
 - C. The garbage list is not circular, the list to be recycled is circular
 - D. The list to be recycled is not circular, the garbage list is circular
9. Which traversal will list the keys in a binary search tree in ascending order?
 - A. Inorder
 - B. Postorder
 - C. Preorder
 - D. All of the above
10. How should the successor of a node without a right child in an unbalanced binary search tree be found?
 - A. Examine the ancestors of the node
 - B. Go left, then proceed to the right
 - C. Go right, then proceed to the left
 - D. Preorder traversal
11. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?



12. In which situation will a sentinel be inappropriate?
- Binary search for a key in an ordered table, to simplify and speed-up code
 - Search for a key in an unordered table, to simplify and speed-up code
 - Search for a key in an unordered linked list, to simplify and speed-up code
 - Red-black tree, to simplify code
13. Which sort treats keys as several digits and uses a counting sort for each position?
- counting
 - insertion
 - merge
 - radix
14. Which binary tree traversal corresponds to the following recursive code?
- ```
void traverse(noderef x)
{
 if (x==null)
 return;
 traverse(x.left);
 traverse(x.right);
 // process x here
}
```
- inorder
  - postorder
  - preorder
  - search for key x
15. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
- 10, 200, 300, 100, 500
  - 100, 1000, 200, 900, 300, 800, 400, 700, 500
  - 200, 300, 400, 700, 600, 500
  - 600, 100, 550, 540, 500

Long Answer

- Give the unbalanced binary search tree that results when the keys 100, 80, 70, 90, 60, 50, 120 are inserted, in the given order, into an initially empty tree. (5 points)
- Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (10 points)

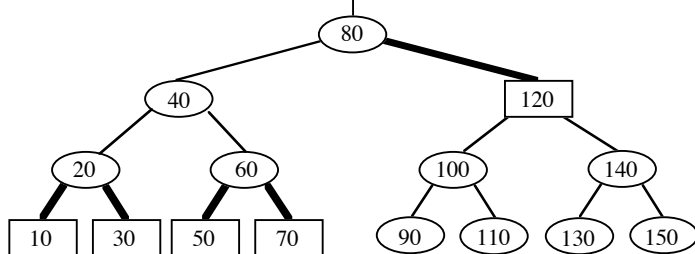


3. Show the result after PARTITION manipulates the following subarray. Be sure to circle which version of PARTITION you applied. (10 points)

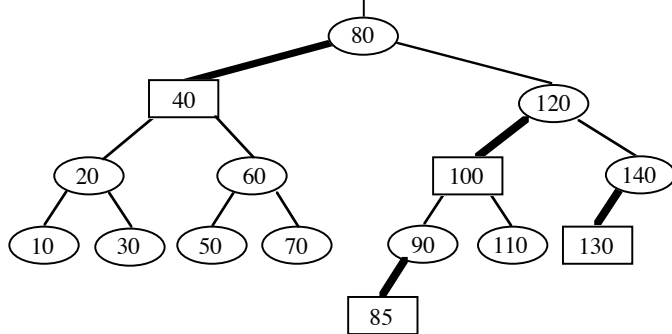
8      2      5      7      4      1      9      0      6      3

Version:                      1                                              2/Sedgwick

4. Twenty million positive integers in the range 0 . . . 99,999,999 are to be sorted by LSD radix sort. Compare the performance for using radix 0 . . . 9999 and radix 0 . . . 9. Show your work. (10 points)
5. Insert 55 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



6. Insert 87 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



CSE 2320

Name \_\_\_\_\_

Test 3

Summer 2008

Last 4 Digits of Student ID # \_\_\_\_\_

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

- The worst-case time for Prim's algorithm implemented with a min-heap is:
 

A.  $\theta(V^2 + E)$       B.  $\theta(E \log V)$       C.  $\theta(V \log V)$       D.  $\theta(V \log E)$
- Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from Y to X, then its type will be:
 

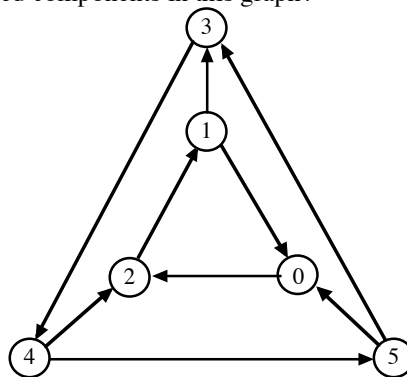
A. Back      B. Cross      C. Forward      D. Tree
- The worst-case time for depth-first search is:
 

A.  $\theta(V \log E)$       B.  $\theta(E \log V)$       C.  $\theta(V \log V)$       D.  $\theta(V + E)$
- Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Y to Z, then its type will be:
 

A. Back      B. Cross      C. Forward      D. Tree
- Which statement is not correct about depth-first search on a directed graph?
 

A. Exploring an edge whose head is colored black will cause the edge to be a back edge.  
 B. Exploring an edge whose head is colored gray will cause the edge to be a back edge.  
 C. Exploring an edge whose head is colored white will cause the edge to be a tree edge.

- D. The run time is  $\Theta(m + n)$ , where  $m$  is the number of edges and  $n$  is the number of vertices.
6. Suppose a directed graph has a path from vertex  $X$  to vertex  $Y$ , but no path from vertex  $Y$  to vertex  $X$ . The relationship between the discovery times is:  
 A.  $\text{discovery}(X) < \text{discovery}(Y)$       B.  $\text{discovery}(X) > \text{discovery}(Y)$   
 C.  $\text{discovery}(X) = \text{discovery}(Y)$       D. could be either A. or B.
7. In Dijkstra's algorithm, the final shortest path distance from the source  $s$  to a vertex  $x$  is known when:  
 A. some vertex  $y$  moves from  $T$  to  $S$  and there is an edge from  $y$  to  $x$ .  
 B.  $x$  has its entry extracted from the heap.  
 C.  $x$  is placed on the heap.  
 D.  $x$  is read from the input file.
8. An adjacency matrix is the most useful representation for which problem?  
 A. Breadth-first search  
 B. Depth-first search  
 C. Finding strongly-connected components  
 D. Warshall's algorithm
9. During a breadth-first search, the status of a white vertex is:  
 A. It has been completely processed.  
 B. It is in the FIFO queue.  
 C. It is in the priority queue.  
 D. It is undiscovered.
10. When a graph is sparse, the best way to find a minimum spanning tree is:  
 A. Floyd-Warshall algorithm  
 B. Prim's algorithm using heap  
 C. Prim's algorithm using T-table  
 D. Warshall's algorithm
11. A topological ordering of a directed graph may be computed by:  
 A. Ordering the vertices by ascending discovery time after DFS  
 B. Ordering the vertices by ascending finish time after DFS  
 C. Ordering the vertices by descending discovery time after DFS  
 D. Ordering the vertices by descending finish time after DFS
12. When finding the strongly connected components, the number of components is indicated by:  
 A. The number of back edges found during the first depth-first search.  
 B. The number of cross edges found during the second depth-first search.  
 C. The number of restarts for the first depth-first search.  
 D. The number of restarts for the second depth-first search.
13. What is the purpose of the first depth-first search when finding strongly connected components?  
 A. To assure that two vertices that are in the same cycle will be output in the same component  
 B. To assure that two vertices with no paths between them are not output in the same component  
 C. To assure that two vertices,  $X$  and  $Y$ , with paths from  $X$  to  $Y$  but not from  $Y$  to  $X$ , are output in different components.  
 D. To make sure that the input graph has no cycles.
14. The number of potential probe sequences when using linear probing with a table with  $m$  entries ( $m$  is prime) is:  
 A.  $O(\log m)$       B.  $m$       C.  $m(m-1)$       D.  $m!$
15. What is the number of strongly connected components in this graph?



A. 1

B. 2

C. 3

D. 4

16. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 0 to 1?

|    |   |   |   |   |
|----|---|---|---|---|
| -1 | 3 | 3 | 3 | 3 |
|----|---|---|---|---|

|    |    |    |    |    |
|----|----|----|----|----|
| -1 | 3  | 3  | 3  | 4  |
| -1 | 1  | 1  | 1  | 4  |
| -1 | 2  | 2  | 2  | 2  |
| -1 | -1 | -1 | -1 | -1 |

A. 0

B. 1

C. 2

D. 3

17. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers  $0, 1, \dots, V-1$ .)
- A. Adjacency lists (ordered):  $\Theta(\log V)$   
 B. Adjacency lists (unordered):  $\Theta(V)$   
 C. Adjacency matrix:  $\Theta(1)$   
 D. Compressed adjacency lists (ordered):  $\Theta(\log V)$
18. The main disadvantage of compressed adjacency lists is:
- A. Directed graphs may not be represented  
 B. It is difficult to change the graph  
 C. They waste space  
 D. Undirected graphs may not be represented
19. Suppose there is exactly one path from vertex 8 to vertex 10 in a directed graph:  
 $8 \rightarrow 7 \rightarrow 3 \rightarrow 5 \rightarrow 10$ . During the scan of which column will Warshall's algorithm record the presence of this path?  
 A. 3                  B. 5                  C. 7                  D. 8
20. Which of the following is not true about probe sequences for an implementation of double hashing?
- A. All slots in the hash table appear in each probe sequence  
 B. Every key has a probe sequence different from the probe sequences for other keys  
 C. The elements of a probe sequence are subscripts in the hash table  
 D. The probe sequence for a key cannot change
21. The expected number of probes for a successful search in hashing by chaining with  $\alpha$  as the load factor is:  
 A.  $\frac{\alpha}{2}$     B.  $\frac{2}{3}\alpha$     C.  $\frac{3}{2}\alpha$     D.  $2\alpha$
22. The time for Warshall's algorithm for a directed graph with  $n$  vertices is in:  
 A.  $\Theta(n)$     B.  $\Theta(n \log n)$     C.  $\Theta(n^2)$     D.  $\Theta(n^3)$

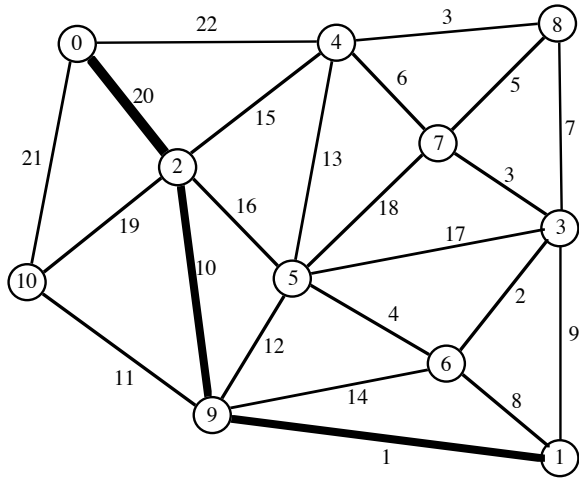
Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using  $h(\text{key}) = \text{key} \% 13$ .

|   |   |   |    |   |     |     |    |    |    |    |    |    |
|---|---|---|----|---|-----|-----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3  | 4 | 5   | 6   | 7  | 8  | 9  | 10 | 11 | 12 |
|   |   |   | 94 |   | 122 | 110 | 20 | 86 | 87 | 62 |    |    |

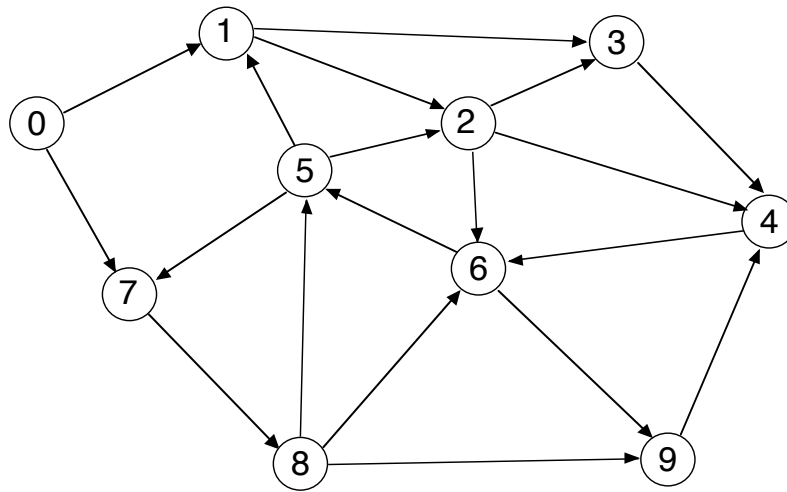
23. 265 would be inserted into which slot of the given table?  
 A. 0                  B. 1                  C. 2                  D. 11
24. 133 would be inserted into which slot of the given table? (without previously inserting 265)  
 A. 0                  B. 4                  C. 6                  D. 11
25. The cycle property for minimum spanning trees may be used to find an MST by:
- A. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.  
 B. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.  
 C. Remove the maximum weight in any cycle until only a tree of edges remains.  
 D. Remove the minimum weight in any cycle until only a tree of edges remains.

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.



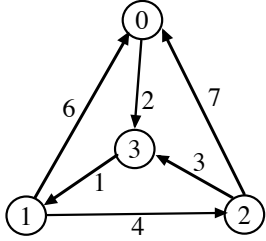
2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points



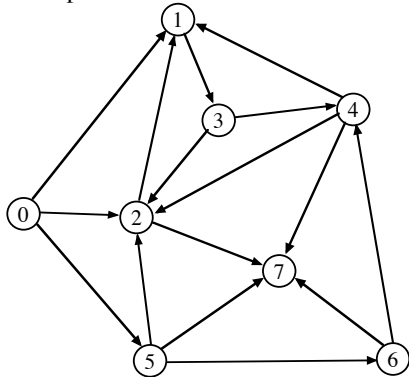
| Vertex | Start    | Finish | Edge | Type | Edge | Type |
|--------|----------|--------|------|------|------|------|
| 0      | <u>1</u> | ___    | 0 1  | ___  | 5 2  | ___  |
| 1      | ___      | ___    | 0 7  | ___  | 5 7  | ___  |
| 2      | ___      | ___    | 1 2  | ___  | 6 5  | ___  |
| 3      | ___      | ___    | 1 3  | ___  | 6 9  | ___  |
| 4      | ___      | ___    | 2 3  | ___  | 7 8  | ___  |
| 5      | ___      | ___    | 2 4  | ___  | 8 5  | ___  |
| 6      | ___      | ___    | 2 6  | ___  | 8 6  | ___  |
| 7      | ___      | ___    | 3 4  | ___  | 8 9  | ___  |
| 8      | ___      | ___    | 4 6  | ___  | 9 4  | ___  |
| 9      | ___      | ___    | 5 1  | ___  |      |      |



3. Demonstrate the Floyd-Warshall algorithm, with successors, for the following graph. The paths indicated in the final matrix must have at least one edge. You are not required to show the intermediate matrices. 10 points.



4. Perform a breadth-first search on the following graph listing the BFS number, shortest path distance (hops) from the source (0), and the predecessor for each vertex. Assume that the adjacency lists are ordered. 10 points



| Vertex | BFS Number | Distance | Predecessor |
|--------|------------|----------|-------------|
| 0      | _____      | _____    | _____       |
| 1      | _____      | _____    | _____       |
| 2      | _____      | _____    | _____       |
| 3      | _____      | _____    | _____       |
| 4      | _____      | _____    | _____       |
| 5      | _____      | _____    | _____       |
| 6      | _____      | _____    | _____       |
| 7      | _____      | _____    | _____       |

5. Consider the following hash table whose keys were stored by double hashing using  $h_1(\text{key}) = \text{key} \% 13$  and  $h_2(\text{key}) = 1 + (\text{key} \% 12)$ .

|   |   |   |     |     |     |     |   |     |     |    |    |     |
|---|---|---|-----|-----|-----|-----|---|-----|-----|----|----|-----|
| 0 | 1 | 2 | 3   | 4   | 5   | 6   | 7 | 8   | 9   | 10 | 11 | 12  |
|   |   |   | 120 | 186 | 187 | 162 |   | 122 | 110 |    |    | 194 |

- a. Give the number of probes needed to find each of the seven stored keys (using double hashing). (7 points)

120 \_\_\_\_\_      186 \_\_\_\_\_      187 \_\_\_\_\_      162 \_\_\_\_\_

122 \_\_\_\_\_      110 \_\_\_\_\_      194 \_\_\_\_\_

- b. Suppose 135 is to be inserted (using double hashing). Which slot will be used? (3 points)