

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to find the maximum of the n elements of an integer array is in:

- A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

2. The goal of the Huffman coding method is:

- A. Minimize the expected bits per symbol.
 B. Find the symbols with high probability of occurring.
 C. Construct a max-heap for the symbols in an alphabet
 D. Maximize the compression for every string.

3. Which of the following is true?

- A. $n^3 \in O(n^2)$ B. $n \log n \in \Omega(n^2)$ C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$ D. $3^n \in O(2^n)$

4. Which of the following facts cannot be proven using one of the limit theorems?

- A. $n^2 \in O(n^3)$ B. $n^2 \in \Omega(n \log n)$ C. $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$ D. $3^n \in \Omega(2^n)$

5. The function $n \log n + \log n$ is in which set?

- A. $\Omega(n^2)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

6. $f(n) = n \lg n$ is in all of the following sets, except

- A. $O(\log n)$ B. $\Theta(\log(n!))$ C. $\Omega(n)$ D. $O(n^2)$

7. The number of calls to `mergeAB` while performing `mergesort` on n items is in:

- A. $\Theta(\log n)$ B. $\Theta(m+n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

8. What is required when calling `union(i, j)` for maintaining disjoint subsets?

- A. i and j are leaders for the same subset B. i and j are in the same subset
 C. i and j are leaders for different subsets D. i is the ancestor of j in one of the trees

9. What is the definition of H_n ?

- A. $\Theta(\sqrt{n})$ B. $\sum_{k=1}^n k$ C. $\ln n$ D. $\sum_{k=1}^n \frac{1}{k}$

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and you already know that $T(n) \in \Omega(1)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

- A. $T(n) \in O(1)$ B. $T(n) \in O(\log n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

11. The time to run the code below is in:

- ```
for (i=n; i>=0; i--)
 for (j=1; j<n; j=j+j)
 sum+=i+j;
```
- A.  $\Theta(n)$     B.  $\Theta(n \log n)$   
 C.  $\Theta(n^2)$     D.  $\Theta(n^3)$

12. Bottom-up heap construction is based on applying `fixDown` in the following fashion:

- A. In ascending slot number order, for each slot that is a parent.  
 B. In descending slot number order, for each slot that is a parent.  
 C.  $\frac{n}{2}$  times, each time from the root of the heap.

- D.  $n - 1$  times, each time from the root of the heap.
13. Which of the following is not true regarding a minheap with 1000 elements?
- Subscript 1 will store the maximum priority.
  - The parent for the node with subscript 500 is stored at subscript 250.
  - The left child for the node with subscript 200 is stored at subscript 400.
  - The right child for the node with subscript 405 is stored at subscript 811.
14. The time to run the code below is in:

```
sum=1;
for (i=1; i<n; i=3+i)
 sum++;
```

A.  $\Theta(\log n)$     B.  $\Theta(\sqrt{n})$   
 C.  $\Theta(n)$         D.  $\Theta(n^2)$

15. Which of the following is solved heuristically by a greedy method?
- Fractional knapsack
  - Huffman code
  - Unweighted interval scheduling
  - 0/1 knapsack

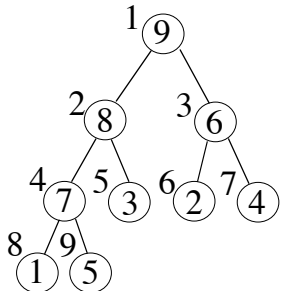
Long Answer

- Prove that if  $f(n) \in \Omega(g(n))$  then  $\frac{1}{f(n)} \in O\left(\frac{1}{g(n)}\right)$ . 5 points
- Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points
 

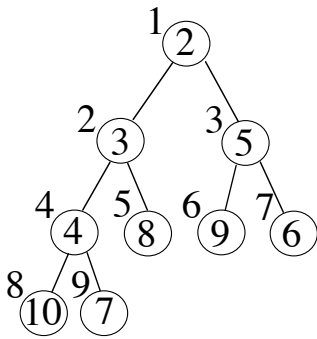
|   |      |
|---|------|
| A | 0.28 |
| B | 0.25 |
| C | 0.15 |
| D | 0.12 |
| E | 0.12 |
| F | 0.07 |
| G | 0.01 |
- Solve the following instance of the activity scheduling problem using the greedy method. 10 points.

| <u>Activity</u> | <u>Start</u> | <u>Finish</u> |
|-----------------|--------------|---------------|
| A               | 1            | 4             |
| B               | 3            | 8             |
| C               | 7            | 12            |
| D               | 11           | 16            |
| E               | 5            | 10            |
| F               | 6            | 9             |
| G               | 2            | 15            |
| H               | 13           | 14            |

4. a. Show the maxheap after performing PQdelmax. 5 points



- b. Show the minheap after changing (PQchange) the priority at subscript 6 to 4. 5 points



5. Use the recursion-tree method to show that  $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$  is in  $\Theta(\sqrt{n} \log n)$ . 10 points

6. Use the substitution method to show that  $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$  is in  $\Theta(\sqrt{n} \log n)$ . 10 points

CSE 2320

Name \_\_\_\_\_

Test 2

Summer 2012

Last 4 Digits of Student ID # \_\_\_\_\_

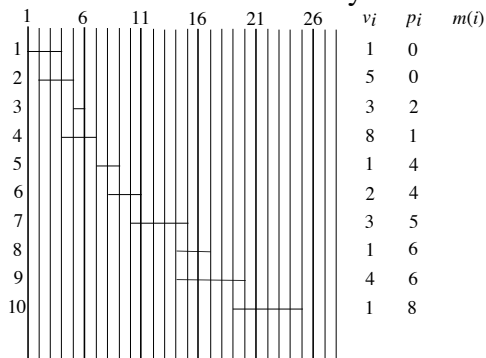
Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Assuming the input has been sorted, Huffman coding may use:
  - A. queues
  - B. dynamic programming
  - C. linked lists
  - D. stacks
2. Which of the following is a longest common subsequence for 1 2 0 1 2 0 and 0 0 1 1 2 2?
  - A. 0 0 1 1
  - B. 1 1 2
  - C. 0 0 1 2
  - D. 0 1 2 0
3. If POP is implemented as  $\text{return stack[--SP]}$ , then PUSH of element X is implemented as:
  - A.  $\text{stack[++SP]} = X$
  - B.  $\text{return stack[SP++]}$
  - C.  $\text{stack[SP++]} = X$
  - D.  $\text{stack[--SP]} = X$
4. What is the worst-case time to perform  $\text{MAXIMUM}(L)$  for a circular, sorted, doubly-linked list with  $n$  nodes?
  - A.  $\Theta(1)$
  - B.  $\Theta(\log n)$
  - C.  $\Theta(n)$
  - D.  $\Theta(n \log n)$
5. For which of the following sorts does the decision tree model not apply?
  - A. QUICKSORT
  - B. Insertion
  - C. LSD Radix Sort
  - D. MERGE-SORT
6. The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:
  - A. to sort the original input
  - B. to search a table that will contain only the LIS elements at termination
  - C. to assure that the final solution is free of duplicate values
  - D. to determine the longest possible increasing subsequence terminated by a particular input value
7. In the example of recycling the elements of a list in  $O(1)$  time, which situation holds?
  - A. Both lists are not circular
  - B. The list to be recycled is circular, the garbage list is not
  - C. Both lists are circular
  - D. The garbage list is circular, the list to be recycled is not
8. What does counting sort count?
  - A. the maximum length among all the strings being sorted
  - B. the number of bytes in the input array
  - C. the number of different input values that have occurred
  - D. the number of occurrences for each possible key value
9. An unsorted integer array with 1500 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?
  - A. If 1481 is returned from PARTITION, we must continue.
  - B. If 1468 is returned from PARTITION, we must continue.

- C. If 1469 is returned from PARTITION, we are done.  
 D. If 1470 is returned from PARTITION, we must continue.
10. Which of the following may be performed in  $\Theta(1)$  worst-case time?  
 A. SEARCH( $L, k$ ) on a sorted, singly linked list  
 B. SEARCH( $L, k$ ) on an unsorted, singly linked list  
 C. LOGICALPREDECESSOR( $L, x$ ) on a sorted, singly linked list  
 D. LOGICALPREDECESSOR( $L, x$ ) on a sorted, doubly linked list
11. Why is it common for a circular queue implementation to waste one table element?  
 A. To perform some loops faster  
 B. To avoid confusing an empty queue with a full queue  
 C. To have a place to store the tail and head values  
 D. To make sure the queue always has at least one element in use
12. Which of the following will not be true regarding the decision tree for MERGESORT for sorting  $n$  input values?  
 A. Every path from the root to a leaf will have  $O(n \log n)$  decisions.  
 B. There will be a path from the root to a leaf with  $\Omega(n^2)$  decisions.  
 C. The height of the tree is  $\Omega(n \log n)$ .  
 D. There will be  $n!$  leaves.
13. The worst-case number of comparisons for finding the  $k$ th largest of  $n$  keys using PARTITION is in which asymptotic set?  
 A.  $\Theta(\log n)$       B.  $\Theta(n)$       C.  $\Theta(n \log n)$       D.  $\Theta(n^2)$
14. Suppose the input to HEAPSORT is always a table of  $n$  ones. The worst-case time will be:  
 A.  $\Theta(\log n)$       B.  $\Theta(n)$       C.  $\Theta(n \log n)$       D.  $\Theta(n^2)$
15. What is minimized in the dynamic programming solution to the subset sum problem?  
 A. The number of input values used to sum to each  $C(i)$       B.  $S_j$   
 C.  $m$       D. The index stored for each  $C(i)$

### Long Answer

1. List the unstable sorts we have studied. (5 points)  
 2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



3. Show the result after PARTITION manipulates the following subarray. You must circle which version of PARTITION you applied. (10 points)

8      2      5      3      6      1      9      0      7      4

Version:                      1                                      2/Sedgewick

4. A billion integers in the range  $0 \dots 999,999$  are to be sorted by LSD radix sort. How much faster will this be done if radix  $0 \dots 999$  is used rather than decimal ( $0 \dots 9$ ) radix? Show your work. (10 points)
5. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

|   |          |          |          |          |          |          |     |     |     |   |
|---|----------|----------|----------|----------|----------|----------|-----|-----|-----|---|
|   | $p[0]=5$ | $p[1]=4$ | $p[2]=6$ | $p[3]=2$ | $p[4]=5$ | $p[5]=6$ |     |     |     |   |
|   | 1        | 2        | 3        | 4        | 5        |          |     |     |     |   |
| 1 | 0        | 0        | 120      | 1        | 88       | 1        | 138 | 3   | ??? | ? |
| 2 | -----    | 0        | 0        | 48       | 2        | 88       | 3   | 156 | 3   |   |
| 3 | -----    | -----    | 0        | 0        | 60       | 3        | 132 | 3   |     |   |
| 4 | -----    | -----    | -----    | 0        | 0        | 60       | 4   |     |     |   |
| 5 | -----    | -----    | -----    | -----    | 0        | 0        |     |     |     |   |

6. Use the dynamic programming solution for subset sums to determine a subset that sums to 15. (10 points)

|       |   |   |   |   |   |    |
|-------|---|---|---|---|---|----|
| $i$   | 0 | 1 | 2 | 3 | 4 | 5  |
| $S_i$ | 0 | 2 | 3 | 5 | 7 | 11 |

CSE 2320

Name \_\_\_\_\_

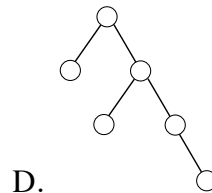
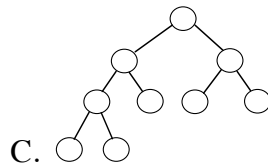
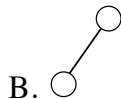
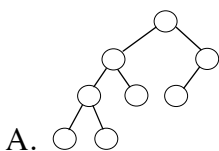
Test 3

Summer 2012

Last 4 Digits of Student ID # \_\_\_\_\_

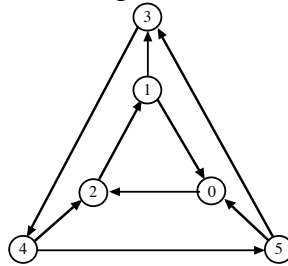
Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. Which edge is chosen in a phase of Kruskal's algorithm?
  - A. An edge that is on a shortest path from the source
  - B. The unprocessed edge  $(x, y)$  of smallest weight such that  $find(x) \neq find(y)$
  - C. The unprocessed edge  $(x, y)$  of smallest weight such that  $find(x) == find(y)$
  - D. An edge of maximum-weight in a cycle (to be excluded)
2. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y. If there is also an edge from X to Y, its type will be:
  - A. Back
  - B. Cross
  - C. Forward
  - D. Tree
3. Which statement is incorrect regarding depth-first search on a directed graph?
  - A. Exploring an edge whose head is colored white will cause the edge to be a tree edge.
  - B. The run time is  $\Theta(m + n)$ , where  $m$  is the number of edges and  $n$  is the number of vertices.
  - C. Exploring an edge whose head is colored black will cause the edge to be a back edge.
  - D. Exploring an edge whose head is colored gray will cause the edge to be a back edge.
4. Suppose a depth-first search is performed on an undirected graph. What is the situation regarding edge types?
  - A. no edge can be a cross edge or a forward edge
  - B. both C and D
  - C. every edge is a tree edge
  - D. there cannot be a back edge
5. Which of the following binary trees can be legally colored as a red-black tree with its root colored red?



6. A topological ordering of a directed graph may be computed by:
- A. Ordering the vertices by descending finish time after DFS

- B. Ordering the vertices by ascending discovery time after DFS  
 C. Ordering the vertices by ascending finish time after DFS  
 D. Ordering the vertices by descending discovery time after DFS
7. Suppose that only numbers in  $1 \dots 1000$  appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
- A. 600, 100, 550, 540, 500  
 B. 10, 200, 300, 100, 500  
 C. 100, 1000, 200, 900, 300, 800, 400, 700, 500  
 D. 200, 300, 400, 700, 600, 500
8. What is the number of strongly connected components in this graph?



- A. 1                      B. 2                      C. 3                      D. 4
9. During a breadth-first search, the status of a gray vertex is:
- A. It has been completely processed.  
 B. It is in the priority queue.  
 C. It is in the FIFO queue.  
 D. It is undiscovered.
10. When a graph is dense, the best way to find a minimum spanning tree is:
- A. Floyd-Warshall algorithm  
 B. Prim's algorithm using heap  
 C. Prim's algorithm using T-table  
 D. Warshall's algorithm
11. The most accurate description of the time to perform a deletion in an unbalanced binary search tree with  $n$  keys and height  $h$  is:
- A.  $O(1)$               B.  $O(\log h)$               C.  $O(h)$               D.  $O(\log n)$
12. Which of the following cannot occur when additional edges are included in a directed graph?
- A. The graph acquires a cycle.  
 B. The number of strong components may remain the same.  
 C. The number of strong components may decrease.  
 D. The number of strong components may increase.
13. What is the purpose of the first depth-first search when finding strongly connected components?
- A. To assure that two vertices,  $X$  and  $Y$ , with paths from  $X$  to  $Y$  but not from  $Y$  to  $X$ , are output in different components.  
 B. To assure that two vertices that are in the same cycle will be output in the same component  
 C. To assure that two vertices with no paths between them are not output in the same component  
 D. To make sure that the input graph has no cycles.
14. The number of potential probe sequences when using double hashing with a table with  $m$  entries ( $m$  is prime) is:
- A.  $m(m-1)$               B.  $m!$               C.  $O(\log m)$               D.  $m$
15. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers  $0, 1, \dots, V-1$ .)
- A. Adjacency lists (ordered):  $\Theta(V)$

- B. Adjacency lists (unordered):  $\Theta(V)$
- C. Adjacency matrix:  $\Theta(1)$
- D. Compressed adjacency lists (ordered):  $\Theta(V)$

16. The following matrix was produced by Warshall's algorithm with successors. How many edges are on the represented path from 3 to 1?

|    |    |    |    |    |
|----|----|----|----|----|
| -1 | 3  | 3  | 3  | 3  |
| -1 | 3  | 3  | 3  | 4  |
| -1 | 1  | 1  | 1  | 4  |
| -1 | 2  | 2  | 2  | 2  |
| -1 | -1 | -1 | -1 | -1 |

- A. 0                      B. 1                      C. 2                      D. 3

17. The fastest method for finding the diameter of a tree (where distance is measured in "hops") is to:

- A. Use breadth-first search.
- B. Use Dijkstra's algorithm.
- C. Use the Floyd-Warshall algorithm.
- D. Use Prim's algorithm.

18. In a binary search tree, which element does not have a successor?

- A. any one of the leaves
- B. the maximum
- C. the minimum
- D. the root

19. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
 if (x==null)
 return;
 traverse(x.left);
 traverse(x.right);
 // process x here
}
```

- A. inorder    B. postorder    C. preorder    D. search for key x

20. Which of the following is not true about probe sequences for an implementation of double hashing?

- A. Two keys could have the same probe sequence
- B. The probe sequence for a key cannot change
- C. All slots in the hash table appear in each probe sequence
- D. The elements of a probe sequence are possible keys for the hash table

21. Suppose a double hash table has  $\alpha = 0.8$  (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

- A. 1.2            B. 2                      C. 5                      D. 10

22. The main disadvantage of compressed adjacency lists is:

- A. Undirected graphs may not be represented
- B. Directed graphs may not be represented
- C. It is difficult to change the graph
- D. They waste space

Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using  $h(\text{key}) = \text{key} \% 13$ .

|   |   |   |    |   |     |     |    |    |    |    |    |    |
|---|---|---|----|---|-----|-----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3  | 4 | 5   | 6   | 7  | 8  | 9  | 10 | 11 | 12 |
|   |   |   | 94 |   | 122 | 110 | 20 | 86 | 87 | 62 |    |    |

23. 143 would be inserted into which slot of the given table?

- A. 0                      B. 1                      C. 2                      D. 11

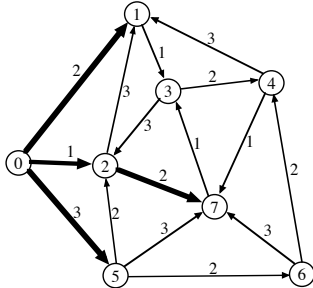
24. 136 would be inserted into which slot of the given table? (without previously inserting 143)

- A. 0                      B. 4                      C. 6                      D. 11

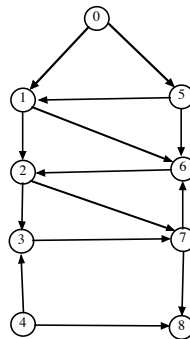
25. Which edge is chosen in a phase of Kruskal's algorithm?
- A. A minimum-weight edge that keeps the result free of cycles
  - B. An edge of maximum-weight in a cycle (to be excluded)
  - C. An edge that is on a shortest path from the source
  - D. A minimum-weight edge connecting T to S.

Long Answer

1. What are the entries in the heap (for *Dijkstra's* algorithm) **before and after** moving the next vertex and edge into the shortest path tree? DO NOT COMPLETE THE ENTIRE TREE!!! Edges already in the shortest path tree are the thick ones. Vertex 0 is the source. 10 points.

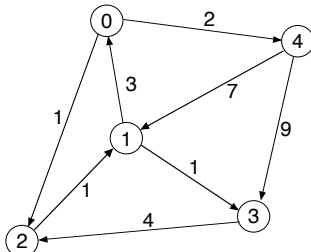


2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are ordered. Write your answer in the tables below. 10 points



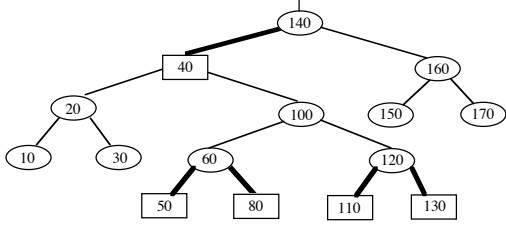
| Vertex | Start    | Finish | Edge | Type | Edge | Type |
|--------|----------|--------|------|------|------|------|
| 0      | <u>1</u> | ___    | 0 1  | ___  | 5 1  | ___  |
| 1      | ___      | ___    | 0 5  | ___  | 5 6  | ___  |
| 2      | ___      | ___    | 1 2  | ___  | 6 2  | ___  |
| 3      | ___      | ___    | 1 6  | ___  | 7 6  | ___  |
| 4      | ___      | ___    | 2 3  | ___  | 7 8  | ___  |
| 5      | ___      | ___    | 2 7  | ___  |      |      |
| 6      | ___      | ___    | 3 7  | ___  |      |      |
| 7      | ___      | ___    | 4 3  | ___  |      |      |
| 8      | ___      | ___    | 4 8  | ___  |      |      |

3. Demonstrate the Floyd-Warshall algorithm, with successors, for the following graph. The paths indicated in the final matrix must have at least one edge. You are not required to show the intermediate matrices. 10 points.





4. Insert 55 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



5. Demonstrate Kruskal's algorithm on this graph. 10 points

