

## Multiple Choice:

1. Write the letter of your answer on the line ( \_\_\_\_\_ ) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. The time to run the code below is in:

```
for (i=n-1; i>=0; i-=2)
  for (j=15; j<100; j+=3)
    sum+=i+j;
```

- \_\_\_\_\_ A.  $\Theta(n \log n)$       B.  $\Theta(n^2)$       C.  $\Theta(n^3)$       D.  $\Theta(n)$

2. A sort is said to be stable when:

- \_\_\_\_\_ A. The expected time and the worst-case time are the same.  
 \_\_\_\_\_ B. Items with the same key will appear in the same order in the output as in the input.  
 \_\_\_\_\_ C. It removes duplicate copies of any key in the final output.  
 \_\_\_\_\_ D. It runs in  $O(n \log n)$  time.

3. Which of the following is false?

- \_\_\_\_\_ A.  $n^3 \in \Omega(n^2)$       B.  $n^2 \in \Omega(n \log n)$   
 \_\_\_\_\_ C.  $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$       D.  $\log \log n \in \Omega(\log n)$

4. Bottom-up maxheap construction is based on applying `maxHeapify` in the following fashion:

- \_\_\_\_\_ A. In ascending slot number order, for each slot that is a parent.  
 \_\_\_\_\_ B. In descending slot number order, for each slot that is a parent.  
 \_\_\_\_\_ C.  $\frac{n}{2}$  times, each time from subscript 1.  
 \_\_\_\_\_ D. In descending slot number order, for each slot that is a leaf.

5. Which of the following functions is not in  $\Omega(n^2)$ ?

- \_\_\_\_\_ A.  $n^2 \lg n$       B.  $n^3$       C.  $n$       D.  $n^2$

6.  $f(n) = n \lg n$  is in all of the following sets, except

- \_\_\_\_\_ A.  $\Omega(\log n)$       B.  $\Theta(\log(n!))$       C.  $O(n)$       D.  $O(n^2)$

7. The number of calls to `merge()` while performing `mergesort` on  $n$  items is in:

- \_\_\_\_\_ A.  $\Theta(\log n)$       B.  $\Theta(1)$       C.  $\Theta(n)$       D.  $\Theta(n \log n)$

8. The time to run the code below is in:

```
for (i=n-5; i>=5; i--)
  for (j=2; j<n; j=2*j+1)
    sum+=i+j;
```

- \_\_\_\_\_ A.  $\Theta(n \log n)$       B.  $\Theta(n^2)$       C.  $\Theta(n^3)$       D.  $\Theta(n)$

9. Which sort takes worst-case  $\Theta(n^2)$  time and is not stable?

- \_\_\_\_\_ A. heap      B. insertion      C. merge      D. selection

10. Suppose you are using the substitution method to establish a  $\Theta$  bound on a recurrence  $T(n)$  and that you already know

that  $T(n) \in \Omega(\log n)$  and  $T(n) \in O(n^2)$ . Which of the following cannot be shown as an improvement?

- \_\_\_\_\_ A.  $T(n) \in \Omega(n^3)$       B.  $T(n) \in O(\log n)$       C.  $T(n) \in O(n)$       D.  $T(n) \in \Omega(n^2)$

11. What is  $n$ , the number of elements, for the largest table that can be processed by binary search using no more than 5 probes?

- \_\_\_\_\_ A. 31      B. 63      C. 64      D. 127

12. Which of the following best approximates  $H_m - H_n$ ? ( $m > n$ )  
 \_\_\_\_\_ A.  $\ln(m/n)$     B.  $\ln(m-n)$     C.  $H_{m-n}$     D.  $1/(m-n)$
13. Which of the following facts can be proven using one of the limit theorems?  
 \_\_\_\_\_ A.  $n^2 \in \Omega(n^3)$     B.  $n^2 \in O(n \log n)$   
 C.  $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$     D.  $3^n \in \Omega(2^n)$
14.  $4^{\lg 7}$  evaluates to which of the following? (Recall that  $\lg x = \log_2 x$ .)  
 \_\_\_\_\_ A.  $\sqrt{7}$     B. 7    C. 25    D. 49
15. When solving the fractional knapsack problem, the items are processed in the following order.  
 \_\_\_\_\_ A. Ascending order of weight    B. Ascending order of \$\$\$/lb  
 C. Descending order of weight    D. Descending order of \$\$\$/lb
16. When did we use  $\sum_{k=0}^t x^k \leq \sum_{k=0}^{\infty} x^k = \lim_{k \rightarrow \infty} \frac{x^k - 1}{x - 1} = \frac{1}{1 - x}$ ?  
 \_\_\_\_\_ A. To define  $H_n$   
 B. For a recursion tree that has the same contribution for each level  
 C. For a recursion tree that has decreasing contributions by each level going away from the root  
 D. For a recursion tree that has increasing contributions by each level going away from the root
17. Which of the following is not true regarding a maxheap with 1000 elements?  
 \_\_\_\_\_ A. Subscript 1 will store the minimum priority.  
 B. The parent for the node with subscript 500 is stored at subscript 250.  
 C. The left child for the node with subscript 200 is stored at subscript 400.  
 D. The right child for the node with subscript 455 is stored at subscript 911.
18. The recursion tree for mergesort has which property?  
 \_\_\_\_\_ A. each level has the same contribution    B. it leads to a definite geometric sum  
 C. it leads to a harmonic sum    D. it leads to an indefinite geometric sum
19. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.  
 \_\_\_\_\_ A. Ascending order of finish time    B. Descending order of interval length  
 C. Ascending order of start time    D. Descending order of finish time
20. The time for the following code is in which set?  

```

for (i=0; i<n; i++)
  for (j=2; j<n; j++)
  {
    c[i][j] = 0;
    for (k=0; k<n; k++)
      c[i][j] += a[i][k]*b[k][j];
  }

```

 \_\_\_\_\_ A.  $\Theta(n)$     B.  $\Theta(n \log n)$     C.  $\Theta(n^2)$     D.  $\Theta(n^3)$
21. The goal of the Huffman coding method is:  
 \_\_\_\_\_ A. Construct a max-heap for the symbols in an alphabet  
 B. Minimize the expected bits per symbol.  
 C. Find the symbols with high probability of occurring.  
 D. Maximize the compression for every string.
22. Suppose you are given a large table with  $n$  integers in descending order, possibly with repeated values. How much time is needed to determine the minimum value?  
 \_\_\_\_\_ A.  $\Theta(1)$     B.  $\Theta(\log n)$     C.  $\Theta(n)$     D.  $\Theta(n \log n)$
23. The number of calls to `heapExtractMin` to build a Huffman code tree for  $n$  symbols is:  
 \_\_\_\_\_ A.  $\Theta(\log n)$     B.  $n - 1$     C.  $n$     D.  $2n - 2$
24. Which technique allows interfacing a priority queue with a dictionary?

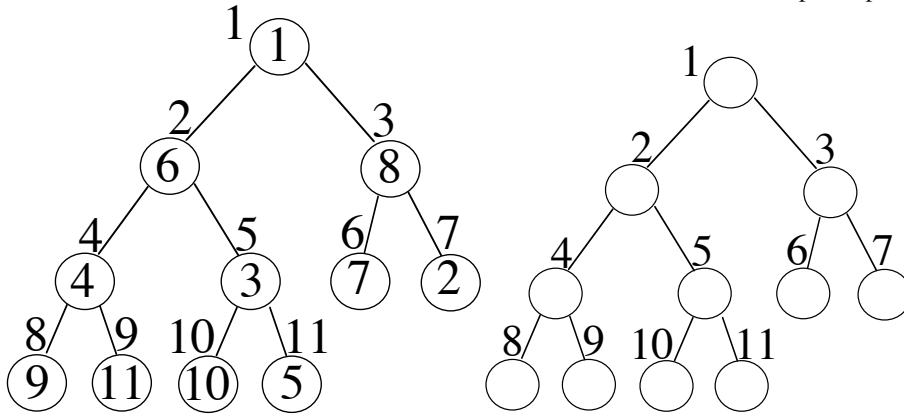
- \_\_\_\_\_ A. Array of handles                      B. Binary search tree  
 C. maxHeapChange                      D. Binary search

25. What is the value of  $\sum_{k=0}^{t-1} 2^k$  ?

- \_\_\_\_\_ A.  $2^k$                       B.  $2^t - 1$                       C.  $2^{t+1} - 1$                       D.  $2^{t+1} + 1$

Long Answer

1. Use the efficient construction from Notes 05 to convert into a maxheap. 10 points



2. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

- A 0.06
- B 0.24
- C 0.17
- D 0.34
- E 0.09
- F 0.05
- G 0.05

3. Suppose an `int` array `a` contains `m` zeroes followed by `n` ones, where `m` and `n` are unknown non-negative values. The size of the array is given to you as a non-negative value `p`, i.e. `p==m+n`. Give C code to determine `m` in  $O(\log p)$  time using *binary search*. (Only the code for this task, setting the value of `m`, is needed. I/O, declarations, a `return`, etc. are unnecessary. Your code must stay within the legal subscripts for array `a`.) 10 points

4. Use the recursion-tree method to show that  $T(n) = 8T\left(\frac{n}{2}\right) + n^3$  is in  $\Theta(n^3 \log n)$ . 10 points

5. Use the substitution method to show that  $T(n) = 8T\left(\frac{n}{2}\right) + n^3$  is in  $O(n^3 \log n)$ . (You do not need to show that

$T(n)$  is in  $\Omega(n^3 \log n)$ .) 10 points

CSE 2320

Name \_\_\_\_\_

Test 2

Summer 2018

Multiple Choice:

1. Write the letter of your answer on the line ( \_\_\_\_\_ ) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. In the example of recycling the elements of a list in  $O(1)$  time, which situation holds?

- \_\_\_\_\_ A. Both lists are circular  
 B. Both lists are not circular  
 C. The list to be recycled is circular, the garbage list is not  
 D. The garbage list is circular, the list to be recycled is not

2. What is the worst-case time to perform `MINIMUM(L)` for a sorted, doubly-linked list with `n` nodes?

- \_\_\_\_\_ A.  $\Theta(1)$  B.  $\Theta(\log n)$  C.  $\Theta(n)$  D.  $\Theta(n \log n)$

3. Which binary tree traversal corresponds to the following recursive code?

```

void traverse(noderef x)
{
  if (x==null)
    return;
  // process x here
  traverse(x.left);
  traverse(x.right);
}

```

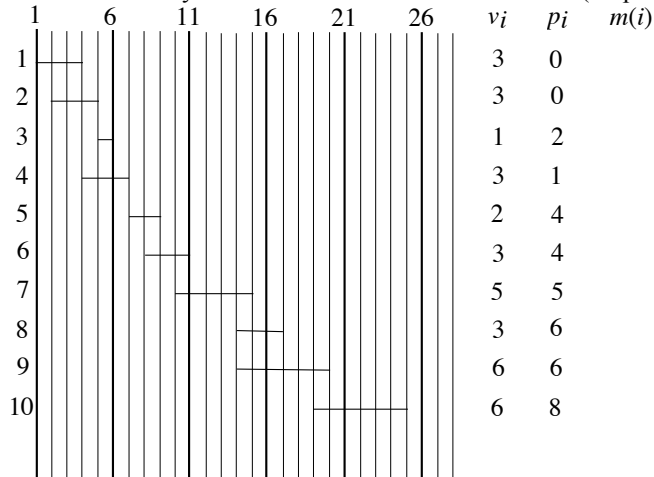
- \_\_\_\_\_ A. inorder B. postorder C. preorder D. search for key x
4. Suppose that only numbers in 1 . . . 100 appear as keys in a binary search tree. While searching for 50, which of the following sequences of keys could not be examined?  
 \_\_\_\_\_ A. 10, 30, 70, 60, 50 B. 100, 20, 80, 30, 50  
 C. 1, 100, 20, 70, 50 D. 10, 40, 70, 30, 50
5. What does counting sort count?  
 \_\_\_\_\_ A. the number of bytes in the input array  
 B. the number of occurrences for each possible key value  
 C. the number of different input values that have occurred  
 D. the maximum length among all the strings being sorted
6. Which of the following is not true regarding dynamic programming?  
 \_\_\_\_\_ A. It is a form of divide-and-conquer B. It is a form of exhaustive search  
 C. A cost function must be defined D. The backtrace may be based on recomputing the cost function
7. The time to extract the LCS (for sequences of lengths  $m$  and  $n$ ) after filling in the dynamic programming matrix is in:  
 \_\_\_\_\_ A.  $\Theta(n)$  B.  $\Theta(m+n)$  C.  $\Theta(n \log n)$  D.  $\Theta(mn)$
8. The queue for breadth-first rat-in-a-maze stores  
 \_\_\_\_\_ A. all maze positions that have walls B. maze positions that must be in the final path  
 C. maze positions that have been reached D. the current path being explored
9. For which of the following sorts does the decision tree model not apply?  
 \_\_\_\_\_ A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT
10. Given a pointer to a node, the worst-case time to delete the node from a singly-linked list with  $n$  nodes in ascending order is:  
 \_\_\_\_\_ A.  $\Theta(1)$  B.  $\Theta(\log n)$  C.  $\Theta(n \log n)$  D.  $\Theta(n)$
11. Memoization is associated with which technique?  
 \_\_\_\_\_ A. top-down dynamic programming B. circular lists  
 C. greedy methods D. bottom-up dynamic programming
12. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as:  
 \_\_\_\_\_ A. return stack[SP++] B. stack[SP++] = X C. stack[--SP] = X D. stack[++SP] = X
13. The cost function for the optimal matrix multiplication problem is:  
 \_\_\_\_\_ A.  $C(i,j) = \min_{i \leq k < j} \{C(i,k) + C(k,j) + P_{i-1}P_kP_j\}$  B.  $C(i,j) = \min_{i \leq k < j} \{C(i,k) + C(k+1,j) + P_iP_kP_j\}$   
 C.  $C(i,j) = \min_{i \leq k < j} \{C(i,k) + C(k+1,j) + P_{i-1}P_kP_j\}$  D.  $C(i,j) = \max\{C(i,j-1), C(i-1,j)\}$  if  $x_i \neq y_j$
14. The worst-case number of comparisons for finding the  $k$ th largest of  $n$  keys using PARTITION is in which asymptotic set?  
 \_\_\_\_\_ A.  $\Theta(\log n)$  B.  $\Theta(n)$  C.  $\Theta(n \log n)$  D.  $\Theta(n^2)$
15. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?  
 \_\_\_\_\_ A. The head points to the first element and the tail points to the last element.  
 B. The tail points to the first element and the head points to the last element.  
 C. Like a circular queue, the maximum number of items is determined at initialization.  
 D. One node is always wasted.

#### Long Answer

- Give the unbalanced binary search tree that results when the keys 50, 30, 40, 60, 70, 90, 20, 80 are inserted, ***in the given order***, into an initially empty tree. (5 points)
- A billion integers in the range  $0 \dots 2^{32} - 1$  will be sorted by LSD radix sort. How much faster is this done using radix  $0 \dots 2^8 - 1$  rather than  $0 \dots 2^4 - 1$ ? Show your work. (10 points)
- Show the result after PARTITION (Version 1) manipulates the following subarray. Recall that both pointers start at the left end of the subarray. (10 points)

8      2      5      3      4      1      9      0      7      6

4. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. (10 points)



5. Use the dynamic programming solution for subset sums to determine a subset that sums to 18. Be sure to give the complete table that would be produced. (10 points)

$i$	0	1	2	3	4	5
$S_i$	0	2	3	5	7	11

6. Complete the following example of the efficient dynamic programming technique for finding a longest common subsequence. Be sure to provide the backtrace for your LCS using arrows in the matrix. (10 points)

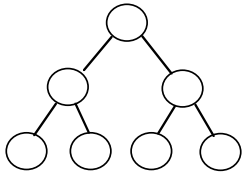
		0	1	1	0	1	0
	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1
0	0	1	1	1	2	2	2
0	0						
1	0						
0	0						
1	0						

CSE 2320  
 Test 3  
 Summer 2018

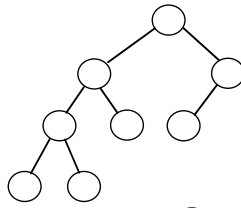
Name \_\_\_\_\_  
 Your name as it appears on your UTA ID Card

Multiple Choice:

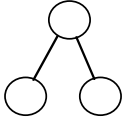
- Write the letter or value for your answer on the line ( \_\_\_\_\_ ) to the LEFT of each problem.
  - CIRCLED ANSWERS DO NOT COUNT.
  - 2 points each
- Suppose the compressed adjacency list representation is used for a directed graph with  $n$  vertices and  $m$  edges. The last subscript for the tailTab is:  
 \_\_\_\_\_ A.  $n$       B.  $n + 1$  C.  $m$                       D.  $m + 1$
  - The expected number of probes for an unsuccessful search in hashing by chaining with  $\alpha$  as the load factor is:  
 \_\_\_\_\_ A.  $\alpha$     B.  $2\alpha$                       C.  $\frac{\alpha}{2}$                       D.  $\frac{2}{3}\alpha$
  - Which of the following binary trees has *exactly* one legal coloring as a red-black tree?



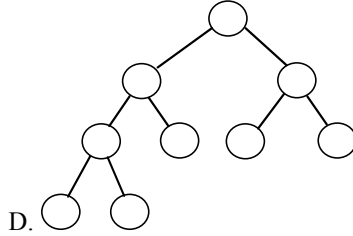
A.



B.



C.



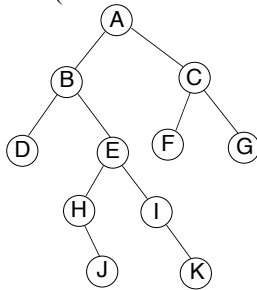
D.

4. Which of the following cannot occur when additional edges are included in a directed graph?

- \_\_\_\_\_ A. The number of strong components may remain the same.
- \_\_\_\_\_ B. The number of strong components may decrease.
- \_\_\_\_\_ C. The number of strong components may increase.
- \_\_\_\_\_ D. The graph acquires a cycle.

5. For a double hash table with  $\alpha = 0.8$  (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

6. Suppose the tree below is a binary search tree whose keys and subtree sizes are not shown. Which node will contain the key with rank 8? (Write the node's letter on the line.)



7. What is required when calling `union(i, j)` for maintaining disjoint subsets?

- \_\_\_\_\_ A.  $i$  and  $j$  are leaders for the same subset
- \_\_\_\_\_ B.  $i$  and  $j$  are in the same subset
- \_\_\_\_\_ C.  $i$  and  $j$  are leaders for different subsets
- \_\_\_\_\_ D.  $i$  is the ancestor of  $j$  in one of the trees

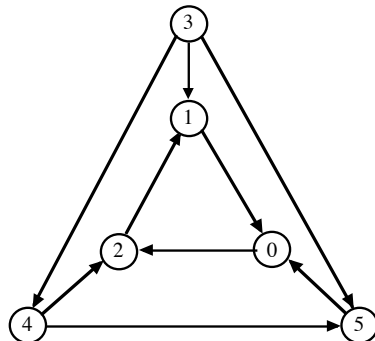
8. During a breadth-first search, the status of a gray vertex is:

- \_\_\_\_\_ A. It has been completely processed.
- \_\_\_\_\_ B. It is in the FIFO queue.
- \_\_\_\_\_ C. It is in the priority queue.
- \_\_\_\_\_ D. It is undiscovered.

9. The cycle property for minimum spanning trees may be used to find an MST by:

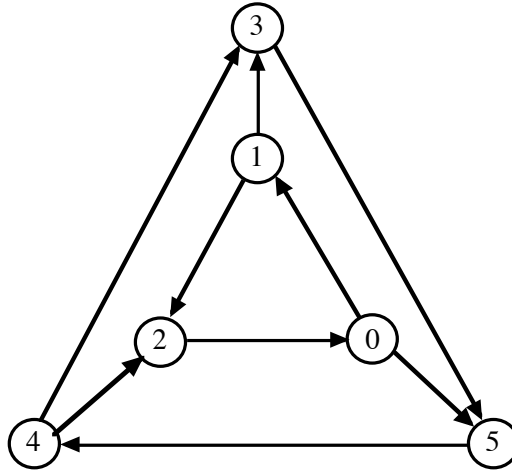
- \_\_\_\_\_ A. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
- \_\_\_\_\_ B. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
- \_\_\_\_\_ C. Remove the maximum weight edge in any cycle until only a tree of edges remains.
- \_\_\_\_\_ D. Remove the minimum weight edge in any cycle until only a tree of edges remains.

10. What is the number of strongly connected components in this graph?



\_\_\_\_\_

11. Which algorithm maintains multiple subtrees?  
 \_\_\_\_\_ A. Dijkstra's B. Kruskal's C. Prim's D. Warshall's
12. A topological ordering of a directed graph may be computed by:  
 \_\_\_\_\_ A. Ordering the vertices by descending finish time after DFS  
 B. Ordering the vertices by ascending discovery time after DFS  
 C. Ordering the vertices by ascending finish time after DFS  
 D. Ordering the vertices by descending discovery time after DFS
13. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?  
 \_\_\_\_\_ A. hashing with chaining B. open addressing  
 C. ordered linked list D. red-black tree
14. What is the number of strongly connected components in this graph?



15. The worst-case time for Prim's algorithm implemented with a minheap is:  
 \_\_\_\_\_ A.  $\theta(V + E)$  B.  $\theta(E \log V)$  C.  $\theta(V \log V)$  D.  $\theta(V \log E)$
16. Suppose the compressed adjacency list representation is used for a directed graph with  $n$  vertices and  $m$  edges. The value stored at the last entry of the tailTab is:  
 \_\_\_\_\_ A.  $n$  B.  $n + 1$  C.  $m$  D.  $m + 1$
17. The number of potential probe sequences when using double hashing with a table with  $m$  entries ( $m$  is prime) is:  
 \_\_\_\_\_ A.  $O(\log m)$  B.  $m$  C.  $m(m - 1)$  D.  $m!$
18. For which graph representation is querying for the presence of an edge supported by binary search?  
 \_\_\_\_\_ A. Adjacency lists (ordered) B. Adjacency lists (unordered)  
 C. Adjacency matrix D. Compressed adjacency lists (ordered)
19. The maximum number of rotations while inserting a key into a red-black tree is:  
 \_\_\_\_\_ A. 1 B. 2 C. 3 D. the black-height
20. Suppose a node  $x$  in an unbalanced binary search tree has two children, each storing one key. What is the first step to delete  $x$ ?  
 \_\_\_\_\_ A. Find the successor of  $x$  B. Inorder traversal  
 C. Rotate  $x$  so it becomes a leaf D. Splice the parent of  $x$  to either child of  $x$
21. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex  $X$  to vertex  $Y$  and a path of tree edges from vertex  $X$  to  $Z$ . If there is also an edge from  $Y$  to  $Z$ , then its type will be:  
 \_\_\_\_\_ A. Tree B. Back C. Cross D. Forward
22. During depth-first search on an undirected graph, a cycle is indicated by which edge type?  
 \_\_\_\_\_ A. Back B. Cross C. Forward D. Tree
23. Which edge is chosen in a phase of Kruskal's algorithm?  
 \_\_\_\_\_ A. The unprocessed edge  $(x, y)$  of smallest weight such that  $\text{find}(x) \neq \text{find}(y)$   
 B. An edge of maximum-weight in a cycle (to be excluded)  
 C. An edge that is on a shortest path from the source  
 D. The unprocessed edge  $(x, y)$  of smallest weight such that  $\text{find}(x) = \text{find}(y)$

Problems 24 and 25 refer to the following hash table whose keys are stored by linear probing using  $h(\text{key}) = \text{key} \% 13$ .

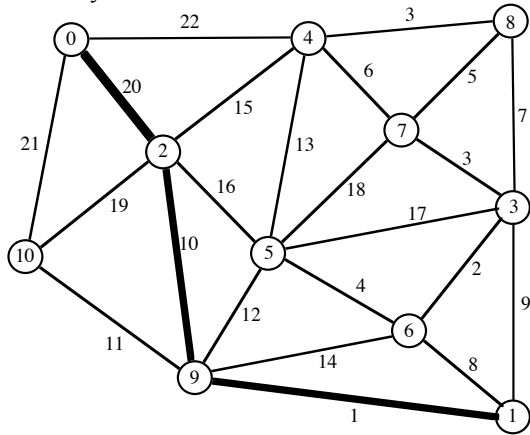
0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

24. 148 would be inserted into which slot of the given table?

25. 133 would be inserted into which slot of the given table? (Assume 148 has already been inserted.)

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? **DO NOT COMPLETE THE ENTIRE MST!!!** Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.



2. Consider the following hash table whose keys were stored by double hashing using  $h_1(\text{key}) = \text{key} \% 17$  and  $h_2(\text{key}) = 1 + (\text{key} \% 16)$ .

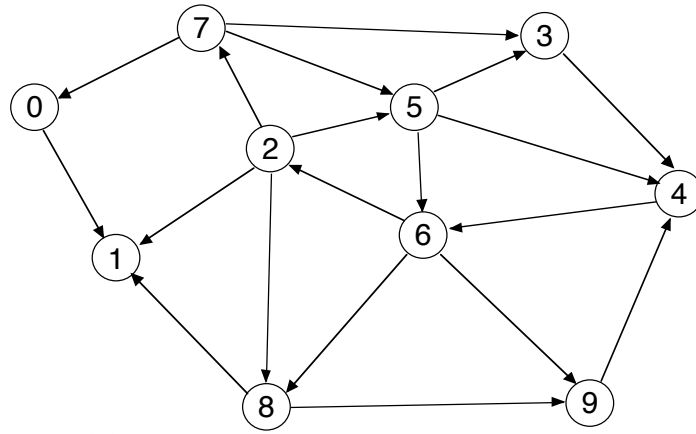
0	-1
1	800
2	-1
3	-1
4	701
5	-1
6	601
7	-1
8	501
9	-1
10	401
11	-1
12	301
13	-1
14	201
15	-1
16	101

- a. Suppose 2001 is to be inserted (using double hashing). Which slot will be used? (5 points)  
 b. Suppose 2002 is to be inserted (using double hashing) *after* 2001 has been stored. Which slot will be used? (5 points)
3. Demonstrate the Floyd-Warshall algorithm, **with successors**, for the following input adjacency matrix. (999 represents infinity) The paths indicated in the final matrix must have **at least one** edge. You **are not** required to show the intermediate matrices. 10 points.

	0	1	2	3	4
0	999	999	11	12	4
1	999	8	6	999	5
2	20	999	15	999	999
3	999	999	5	999	999
4	999	999	3	999	4

4. Perform depth-first search on the following graph, including discovery/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume the adjacency lists are **ordered**. Write your answer in the tables below. 10 points





Vertex	Start	Finish	Edge	Type	Edge	Type
0	<u>1</u>	---	0 1	---	6 2	---
1	---	---	2 1	---	6 8	---
2	---	---	2 5	---	6 9	---
3	---	---	2 7	---	7 0	---
4	---	---	2 8	---	7 3	---
5	---	---	3 4	---	7 5	---
6	---	---	4 6	---	8 1	---
7	---	---	5 3	---	8 9	---
8	---	---	5 4	---	9 4	---
9	---	---	5 6	---		

5. Insert 9 into the given red-black tree. Be sure to indicate the cases that you used (10 points)

