Short Answer. 5 points each
1. Give the recurrence that describes the time needed for mergesort.
2. Under what condition does counting sort run in linear time?
3. Give a function that is in $O(n)$, but not in $\Theta(n)$.
4. Give the definition of $\Omega$-notation.
5. If $f(n) \in O(2^n)$, then is $f(n) \in O(3^n)$?
6. Evaluate the sum:
   $$\sum_{k=1}^{20} \left( \frac{1}{k} - \frac{1}{k+1} \right)$$

7. How may an unstable sort be forced to behave in a stable fashion?
8. Use integrals to give lower and upper bounds on
   $$\sum_{k=5}^{20} k$$

9. Suppose that BUILD-HEAP is applied to a heap $A$ with 21 nodes. What will be the first call to HEAPIFY?
10. What value is returned by PARTITION?

Long Answer. 15 points each
1. Use iteration to show that $T(n) = 3T(n/3) + n^2$ is in $\Theta(n^2)$
2. Use substitution to show that $T(n) = 3T(n/3) + n^2$ is in $\Theta(n^2)$
3. Explain the decision tree lower bound for key comparison sorts by giving a. the number of leaves in a decision tree and b. the height of the decision tree.
4. List the sorts that were discussed in class. For each sort, indicate its worst-case time and whether it is stable or unstable.

Short Answer. 5 points each
1. Give the tree that results from performing a left rotation at node X.

2. Use linear probing with the hash function $h'(k) = k \mod 6$ to store the keys 0, 6, 12, 5, 11, 9. The insertions are to be performed in the given order.
3. Explain how to determine the successor of a node in a binary search tree when the node does not have a right child.
4. Give an upper bound on the expected number of probes for unsuccessful search with uniform hashing when the hash table has 10,000 slots and presently holds 9,000 keys.
5. What is a sentinel?

Long Answer. 15 points each
1. If possible, give a legal red-black coloring for the following tree by listing the keys of the black nodes.

2. Explain the additional details that are needed to support deletions in open-addressing techniques.
3. Suppose that you have been asked to design code for supporting a dynamic set. What questions might you ask about the application in determining what data structure to use?

4. Insert 70 into the following red-black tree. Be sure to indicate the cases that are used.

```
20
   /
  10  40
   /    |
30  60  50  80
```

5. Insert 40 into the following red-black tree. Be sure to indicate the cases that are used.

```
20
   /
  10  70
   /    |
50  30  80
```

6. Delete 70 from the following red-black tree. Be sure to indicate the cases that are used.

```
60
   /
20  70
   /    |
10  40  30  50
```

7. Delete 40 from the following red-black tree. Be sure to indicate the cases that are used.

```
40
   /
20  60
   /    |
10  30  50  80
```

Multiple Choice. 5 points each. Write the letter of your answer to the left of each problem.

1. Which expression best describes the run time for the KMP algorithm?
   A. O(n)
   B. O(lg n)
   C. O(m + n)
   D. O(mn)
2. Which expression best describes the run time for the Longest Common Subsequence algorithm?
   A. $O(n)$
   B. $O(lg n)$
   C. $O(m + n)$
   D. $O(mn)$

3. The key difference between the two fail link constructions for KMP is:
   A. The second construction runs in time faster than linear.
   B. The second construction avoids fail links that match a text character against the same pattern character.
   C. The second construction allows the matcher to run in time faster than linear.
   D. The first construction always gives a fail link table with fewer -1 entries than the second construction.

4. Which problem is not solved by a greedy algorithm?
   A. Depth-first search
   B. Minimum spanning tree
   C. Huffman coding
   D. Single-source shortest path

5. Which problem is optimally solved by a greedy algorithm?
   A. Fractional knapsack
   B. Longest common subsequence
   C. Optimal matrix multiplication order
   D. 0/1 knapsack

6. Which statement is not correct about depth-first search on a directed graph?
   A. Exploring an edge whose head is colored black will cause the edge to be a back edge.
   B. Exploring an edge whose head is colored gray will cause the edge to be a back edge.
   C. Exploring an edge whose head is colored white will cause the edge to be a tree edge.
   D. The run time is $\theta(m + n)$, where $m$ is the number of edges and $n$ is the number of vertices.

7. What is the size of the maximum bipartite matching for the following graph?
   A. 2
   B. 3
   C. 4
   D. 5

Long Answer. Points for each problem are given in parenthesis.
1. Give both KMP fail link tables for the pattern `babacabaa`. (15)
2. Determine a maximum flow in the following network and the minimum cut. Be sure to give each augmenting path, the amount of additional flow that it provides, and the residual graph after each augmenting path is recorded. You may choose each augmenting path in any matter that you choose. (15)
3. Use dynamic programming to determine the longest common subsequence of \textit{abcdabcd} and \textit{dcbadcba}. (10)

4. Demonstrate Warshall's algorithm on the successor/predecessor matrix (your choice) for the following graph. Do NOT use the version that only gives the binary reachability matrix. (15)

5. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. (10)