CSE 5311 Name _____
Summer 2003
Test 1

Closed Book Questions

Multiple Choice - WRITE your answer to the LEFT of each problem. 4 points each.

1. Which of the follow is not true regarding CASCADING-CUT?

   A. It calls CONSOLIDATE.
   B. It may set or clear the marks on some nodes.
   C. It is used by DECREASE-KEY.
   D. It is used by DELETE-KEY.

2. Which of the following statements is true?

   A. A particular AVL tree may have balance factors assigned in only one way.
   B. A particular red-black tree may only be colored in one way.
   C. A rotation takes $\Omega(\lg n)$ time.
   D. A treap can be colored as a red-black tree.

3. Which property holds for binomial heaps?

   A. DECREASE-KEY takes O(1) time.
   B. MINIMUM takes O(1) time.
   C. Performing n INSERT operations into an empty heap will take O(lg n) time.
   D. The number of trees is based on the binary representation of the number of stored items.

4. What is the main contribution of leftist heaps?

   A. The minimum is found in O(1) time.
   B. The amortized complexity of DECREASE-KEY is O(1)
   C. The height of the tree is O(lg n).
   D. They were the first heap to support UNION in O(lg n) time.

5. Assuming that $|E| > |V|$, the time for Dijkstra's algorithm using a Fibonacci heap is:

   A. $O(|E|)$
   B. $O(|E| \log |E|)$
   C. $O(|E| \log |V|)$
   D. $O(|V| \log |V| + |E|)$

6. An optimal binary search tree may be found in time:

   A. O(lg n)
   B. O(n)
   C. $O(n^2)$
   D. $O(n^3)$

7. The perfect hashing method discussed in class depends on which fact?

   A. The sum $1 + 1/x + 1/x^2 + 1/x^3 + 1/x^4 + \ldots < 1/(1-x)$ when $x < 1$
   B. $\ln n < H_n < \ln n + 1$
   C. The expected number of probes for a successful search in Brent's method is less than 2.5
   D. The probability of collisions among n keys stored in a hash table of size n^2 is less than 1/2

8. What is the basis for the potential function for comparing the move-to-front strategy to the optimal off-line strategy?

   A. Dynamic programming
   B. Inversions
   C. Ranks
   D. The expected fixed optimal cost

9. Performing selection in worst-case linear time is based on separating the n input values into how many groups?

   A. 5
   B. lg n
   C. n/lg n
   D. n/5

10. The expected number of random coupons needed to obtain at least one each of n coupon types is.

    A. n
    B. H_n
    C. n ln n
    D. n^2

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11. Suppose a gambling game involves a sequence of rolls from a standard six-sided die. A player wins $1 when the value rolled is different from the previous roll. If a sequence has 601 rolls, what is the expected amount paid out? (5 points)

12. Suppose a gambling game involves a sequence of rolls from a standard six-sided die. A player wins k dollars when the value k rolled is smaller than the previous roll. If a sequence has 601 rolls, what is the expected amount paid out? (5 points)

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1. Use the master method to derive an asymptotic bound on $L(n) = 2L(n/2) + \log n$. (10 points)

2. a. Suppose that the root of a tree in a binomial heap has 5 children. What is the minimum number of nodes that may appear in the tree? (3 points)

   b. What is the minimum number of nodes that may appear in a leftist heap whose rightmost path has 5 nodes? (5 points)

   c. Suppose that the root of a tree in a Fibonacci heap has 5 children. What is the minimum number of nodes that may appear in the tree? (7 points)

3. Give the treap (min-heap ordering) for the following keys and priorities (15 points)

key	priority
10	33
20	12
30	1
40	31
50	17
60	100
70	5
80	22

4. Consider the following information for constructing an optimal binary search tree.

```
n=6;
q[0]=0.01;
key[1]=10;
p[1]=0.29;
q[1]=0.02;
key[2]=20;
p[2]=0.1;
q[2]=0.03;
key[3]=30;
p[3]=0.2;
q[3]=0.04;
key[4]=40;
p[4]=0.2;
q[4]=0.0;
key[5]=50;
p[5]=0.02;
q[5]=0.04;
key[6]=60;
p[6]=0.02;
q[6]=0.03;
w[0][0]=0.010000
w[0][1]=0.320000
w[0][2]=0.450000
w[0][3]=0.690000
w[0][4]=0.890000
w[0][5]=0.950000
w[0][6]=1.000000
w[1][1]=0.020000
w[1][2]=0.150000
w[1][3]=0.390000
w[1][4]=0.590000
w[1][5]=0.650000
w[1][6]=0.700000
w[2][2]=0.030000
w[2][3]=0.270000
w[2][4]=0.470000
w[2][5]=0.530000
w[2][6]=0.580000
w[3][3]=0.040000
w[3][4]=0.240000
w[3][5]=0.300000
w[3][6]=0.350000
w[4][4]=0.000000
w[4][5]=0.060000
w[4][6]=0.110000
w[5][5]=0.040000
w[5][6]=0.090000
w[6][6]=0.030000
```

```
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 1 thru 3
Building c(1,4) using roots 3 thru 3
Building c(2,5) using roots 3 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 1 thru 3
Building c(1,5) using roots 3 thru 4
Building c(2,6) using roots 4 thru 4
Building c(0,5) using roots 3 thru 3
Building c(1,6) using roots 3 thru 4
Building c(0,6) using roots ? thru ?
Counts - root trick 29 without root trick 50
Average probe length is ????????
trees in parenthesized prefix
c(0,0) cost 0.000000
c(1,1) cost 0.000000
c(2,2) cost 0.000000
c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.320000 10
c(1,2) cost 0.150000 20
c(2,3) cost 0.270000 30
c(3,4) cost 0.240000 40
c(4,5) cost 0.060000 50
c(5,6) cost 0.090000 60
c(0,2) cost 0.600000 10(,20)
c(1,3) cost 0.540000 30(20,)
c(2,4) cost 0.710000 30(,40)
c(3,5) cost 0.360000 40(,50)
c(4,6) cost 0.170000 60(50,)
c(0,3) cost 1.230000 10(,30(20,))
c(1,4) cost 0.980000 30(20,40)
c(2,5) cost 0.860000 40(30,50)
c(3,6) cost 0.520000 40(,60(50,))
c(0,4) cost 1.730000 30(10(,20),40)
c(1,5) cost 1.160000 30(20,40(,50))
c(2,6) cost 1.020000 40(30,60(50,))
c(0,5) cost 1.910000 30(10(,20),40(,50))
c(1,6) cost 1.370000 30(20,40(,60(50,)))
c(0,6) cost ???????? ????????????????????
```

Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

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Test 2

Closed Book Questions

Multiple Choice - WRITE your answer to the LEFT of each problem. 4 points each.

1. When performing bin packing using the ''next fit'' technique, the total number of items placed in the bins past the optimal bins (1 .. OPT) is bounded by:

   A. $1 + \in$
   B. 2
   C. OPT - 1
   D. OPT

2. Each successive time an edge (x, y) becomes critical in the Edmonds-Karp method, the distances for x and y along the augmenting path increase by at least this much:

   A. 1

B. 2
C. 3
D. capacity - flow on the edge where this value is minimized.

3. The length of the TSP tour found by the triangle inequality technique achieves what minimization ratio?

A. 0.5
B. $1 + \in$
C. 1.5
D. 2

4. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A.

A. The reduction takes polynomial time.
B. The reduction has an inverse that takes each instance of problem B to an instance of problem A.
C. Problem A is NP-complete
D. The reduction must be consistent for the decision results for each instance of problem A and the corresponding instance of problem B.

5. Which of the following uses the most angular comparisons in the worst case to find a 2-d convex hull?

A. Ford-Fulkerson
B. Graham scan
C. Jarvis march
D. Karp-Rabin

6. An entry in the Z array contains what value?

A. A pointer or subscript for some suffix of a string.
B. The length of the longest prefix of a string that matches the string starting at this position.
C. The link for continued attempts at matching within this pattern.
D. The number of occurences of the first suffix symbol within the corresponding suffix.

7. The method for constructing a suffix array using radix sort takes worst-case time:

A. $\theta(\lg n)$
B. $\theta(n)$
C. $\theta(n \lg n)$
D. $\theta(n^2)$

8. Among the problems listed, this problem achieves the best approximation ratio:

A. Edge coloring
B. Set cover
C. TSP with the triangle inequality
D. Vertex cover

9. Articulation points are found by:

A. 2-d closest pairs
B. Convex hull
C. Depth-first search
D. Strongly connected components

10. Edge connectivity can be computed using this many instances of network flows:

A. V - 1
B. V
C. E - 1
D. E

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11. Explain how the relabel (‘‘lift’’) operation is applied. 10 points

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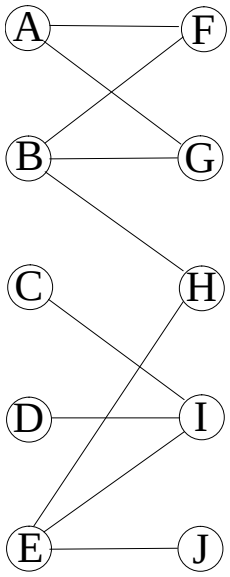
1. Fill in the Z table. 10 points

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	b	a	b	a	c	a	b	a	b	a	c	a	b	a	b

2. List the vertices in a minimum vertex cover for this graph. 10 points.



3. In the k-subset-sum problem, it must be decided whether there is a subset with *exactly* k values chosen from a given set (that may include repeated values and zeroes) that sum to a given value. Show that this problem is NP-complete. 15 points.

4. Solve the following instance of Longest Common Subsequences using the method based on the Longest Strictly Increasing Subsequence problem. 15 points.

0	1	2	3	4	5	6	7	8	9	10	11
a	b	c	d	a	b	c	d	a	b	c	d
d	c	b	a	d	c	b	a	d	c	b	a