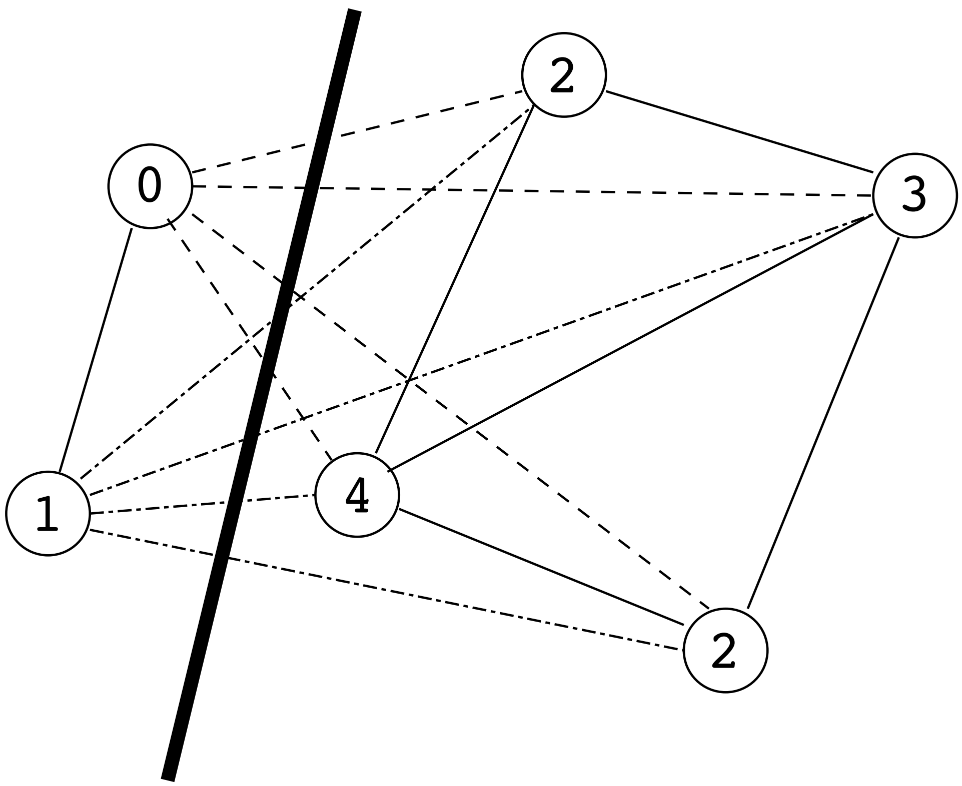
1. Show that deciding whether an undirected graph is 5-colorable is NP-complete by a simple reduction from the 3-colorability problem. In addition to your proof, give an example of your reduction on a 3-colorable graph.

This is related to the proof of vertex coloring not having an efficient heuristic approximation.

Membership in NP is trivial - after guessing a partition of the set of vertices into color classes, it is easy to verify that no color class includes a pair of adjacent vertices.

Reduction: The complete graph on two vertices (K_2) requires two colors. For each vertex in the instance of 3-coloring, connect an edge to both vertices in a K_2 instance. The original instance is 3-colorable iff the result of the reduction is 5-colorable.



2. Prove that the *set packing* problem is NP-complete.

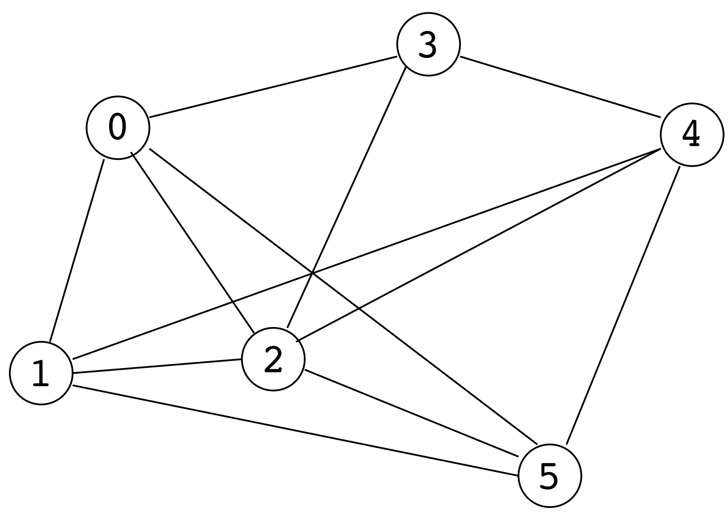
Hint 1: There is a straightforward reduction from k-clique.
Hint 2: It is often helpful to give an example of the reduction used.

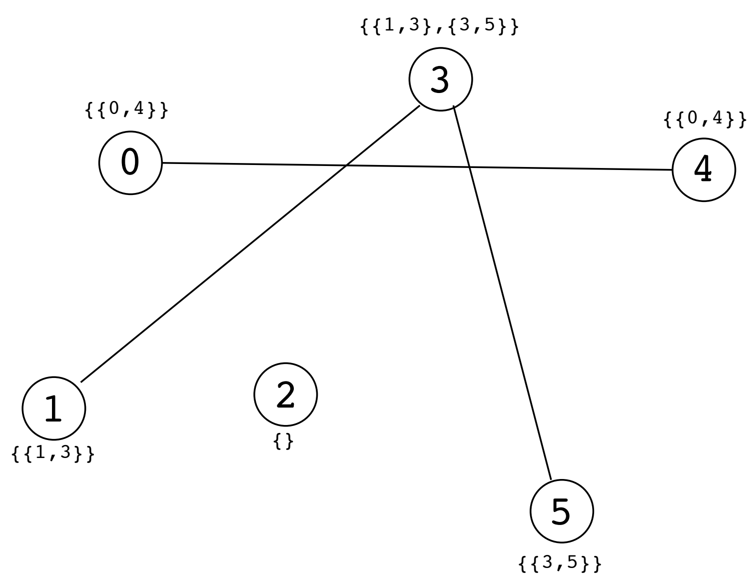
INSTANCE: Collection C of finite sets, positive integer $K \leq |C|$.
QUESTION: Does C contain at least K mutually (i.e. pairwise) disjoint sets?

Membership in NP is trivial - after guessing a set packing solution of size K, it is easy to verify that no pair of chosen sets have a common element.

Reduction: Suppose each vertex in an instance of k-clique has a label. Let $K = k$ and there is a set in C for each vertex. The elements of a vertex's set will be unordered pairs corresponding to the two vertices defining the edges incident to the vertex in the complement graph (as defined for vertex cover in Notes 11).

Example: Is there a 4-clique?





3. The *hitting set* problem gives a collection C of subsets of a set S and a positive integer k. We would like to know if there is a subset S' of S with $|S'| \leq k$ such that S' contains at least one element from each subset in C. Give a proof that *hitting set* is NP-complete by using the fact that vertex cover is NP-complete.

Membership in NP is trivial - after guessing a hitting set of size k, it is easy to verify coverage of each element of C.

Suppose the instance of vertex cover (V, E) requires no more than k vertices in the cover. For the instance of hitting set, let $S = V$, each element of C will be the unordered pair of vertices for an edge in E, and k is the same as the vertex cover instance.