CSE 5311
Test 1 - Closed Book
Spring 2017

Name Key
Last name, then first name as on your UTA ID card

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

25x2=50

1. To support computing the number of keys that are smaller than a query key, an augmented binary search tree stores the following at every node:

C

A. the count of the number of keys in the entire tree
B. the sum of all keys in the left subtree
C. the count of the number of keys stored in the subtree rooted by this node
D. the sum of all keys stored in the subtree rooted by this node

2. Which of the following is not true regarding the amortized analysis of binary tree traversals?

C

A. INIT had an amortized cost of 0.
B. SUCC had an actual cost determined by the number of edges followed.
C. SUCC had an amortized cost of 1.
D. The potential was defined with regard to the type of traversal being performed.

3. Suppose you already have 15 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 15 you already have?

C

A. 2 B. 3 C. 4 D. 5

4. When is path compression used?

B

A. After an insertion into a splay tree.
B. With a FIND operation.
C. After an insertion into any type of balanced binary search tree.
D. With a UNION operation.

5. When performing selection in worst-cast linear time for n numbers, roughly how many column medians are computed in the first round?

A

A. $\frac{n}{5}$ B. m, the median-of-medians C. $0.7n$ D. $W\left(\frac{n}{5}\right)$

6. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of +2?

A

A. none of them
B. only the leaves
C. only the root
D. the leaves and the root

7. How many inversions are there for the lists 1, 2, 5, 4, 3 and 2, 5, 4, 3, 1?

C ____ A. 2 B. 3 C. 4 D. 5

8. Which priority queue is defined using the notion of null path length?

A ____ A. Leftist heap B. Binomial heap C. Pairing heap D. Binary heap

9. Pairing heaps are a practical alternative to:

C ____ A. binary heaps B. binomial heaps C. Fibonacci heaps D. leftist heaps

10. Which property does not hold for binomial heaps?

C ____
A. The number of trees is based on the binary representation of the number of stored items.
B. DECREASE-KEY takes $O(\log n)$ time.
C. MINIMUM takes $O(1)$ time.
D. Performing n INSERT operations into an empty heap will take $O(n)$ time.

11. In the worst case, the number of rotations for inserting a key in a treap with n keys is:

D ____ A. $\theta(n \log n)$ B. $\theta(1)$ C. $\theta(\log n)$ D. $\theta(n)$

12. Assuming a random n-permutation is provided, the expected number of hires for the hiring problem is:

A ____ A. H_n B. 2 C. $\sqrt{n}$ D. $\ln \ln n$

13. What is minimized in the dynamic programming solution to the subset sum problem?

C ____
A. The number of input values used to sum to each $C(i)$
B. S_j
C. The index stored for each $C(i)$
D. m

14. What is the worst-case number of rotations when performing deletion on an AVL tree?

C ____ A. $\Theta(n)$ B. $\Theta(1)$ C. $\Theta(\log n)$ D. No rotations are ever used

15. The master method summarizes the main characteristics of what other technique.

C ____ A. asymptotic analysis B. potential functions C. recursion trees D. substitution method

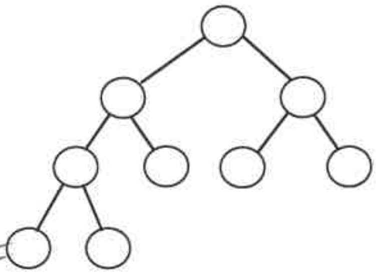
16. Which of the following is not true regarding Bloom filters?

D ____
A. Several hash functions are used.
B. The optimal bit array size depends on the number of items and the false positive probability.
C. An indication that a candidate element is not in the set is always correct.
D. They are an especially compact dictionary representation.

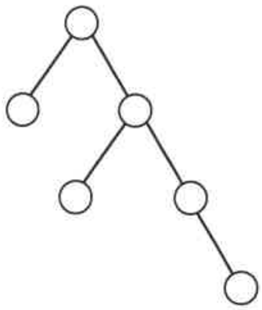
17. Which of the following binary trees has *multiple* legal colorings as a red-black tree?

B____ A.

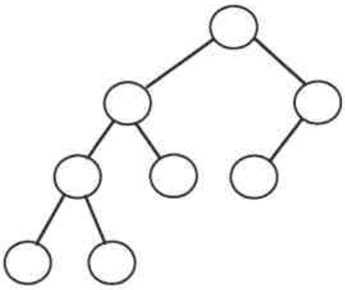
B.



C.



D.



18. Sorting the set of input edges is a property of which minimum spanning tree technique?

B____ A. Boruvka B. Kruskal C. Path-based (Warshall) D. Prim

19. When using Brent's rehash during insertion, the number of previously inserted keys that may move is:

A____ A. 1 B. 2 C. $\frac{1}{\alpha}$ D. H_m, where m is the number of stored keys

20. The goal of the optimal matrix multiplication problem is to:

C____ A. Minimize the number of C(i,j) instances evaluated.
B. Minimize the number of matrix multiplications.
C. Minimize the number of scalar multiplications.
D. Minimize the number of scalar additions.

21. The main difference between MTF and OPT for self-organizing linear lists is:

C____ A. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
B. OPT counts inversions
C. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time
D. MTF can do transpositions

22. Which minimum spanning tree algorithm may use a priority queue?

C____ A. Boruvka B. Kruskal C. Prim D. Warshall

23. Warshall's algorithm on a directed graph with n vertices uses this much time:

A A. $\Theta(n^3)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

24. The perfect hashing method discussed in class depends on which fact?

D A. $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad 0 < x < 1$

B. $\ln n < H_n < \ln n + 1$

C. The expected number of probes for a successful search in Brent's method is less than 2.5.

D. The probability of collisions among n keys stored in a hash table of size n^2 is less than 0.5.

25. When performing selection in worst-cast linear time, roughly what fraction of the set of n keys is kept (in the worst case) for the next round?

D A. 10% B. 20% C. 30% D. 70%

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Name Key
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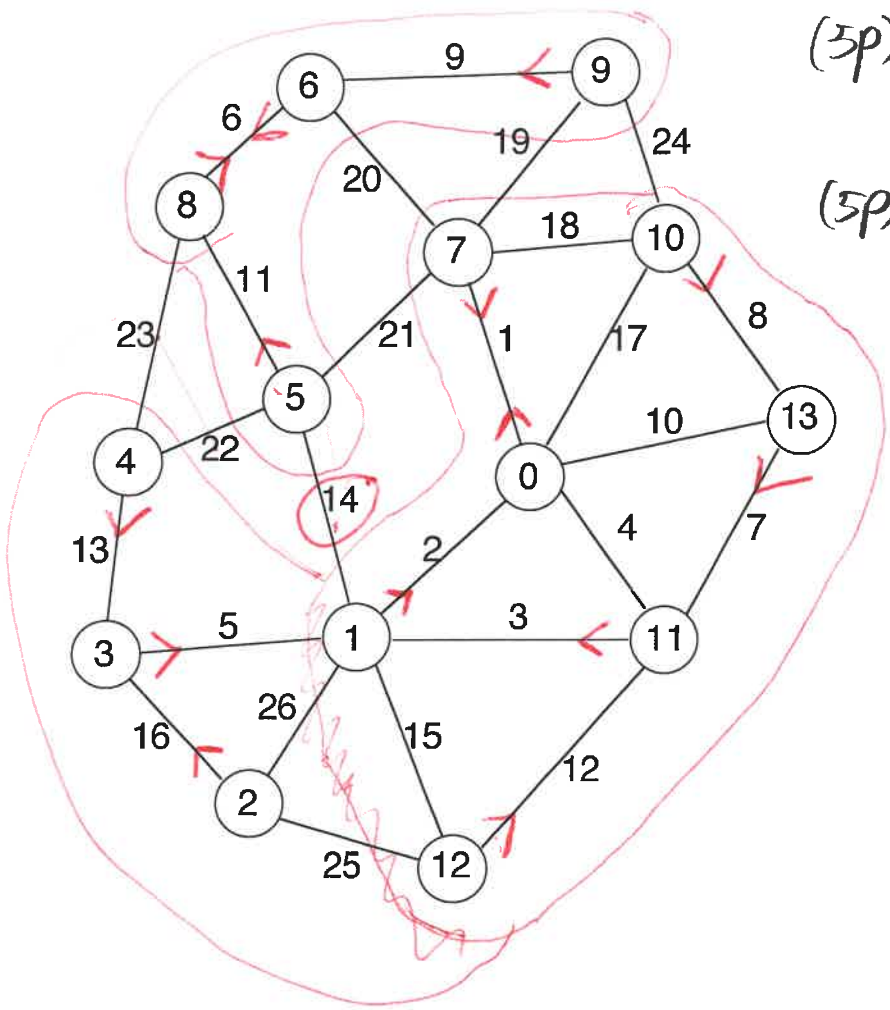
1. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (10 points)



(5p) > = first phase

(5p) 14 is the only edge in the second phase

2. The hash table below was created using double hashing with Brent's rehash. The initial slot ($h_1(key)$) and rehashing increment ($h_2(key)$) are given for each *key*. Show the result from inserting 7097 using Brent's rehash when $h_1(7097) = 6$ and $h_2(7097) = 6$. (10 points)

Partial Credit

	key	$h_1(key)$	$h_2(key)$
0	7000	0	5
1			
2	7002	2	1
3			
4	7081	4	2
5	7005	5	4
6	7006	6	5

0 5 3

2 3

4 6 1

5 2 6 3

6 4 2 0 5 3

4p

7097 6 5 4 3

new 1 2 3 4

old 5/6 3/5 2/5 0/4 – total additional probes

3p

	key
0	7000
1	
2	7002
3	7097
4	7081
5	7005
6	7006

Answer (3) (3p)

Indicate the additional number of probes for searching for each key once after 7097 is inserted:

Answer 4 (3p)

4x1

cost formula — pass [illegible]
Knuth trick —
Tree — 3

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```
n=6;
q[0]=0.0;
key[1]=10;
p[1]=0.1;
q[1]=0.04;
key[2]=20;
p[2]=0.2;
q[2]=0.02;
key[3]=30;
p[3]=0.01;
q[3]=0.12;
key[4]=40;
p[4]=0.03;
q[4]=0.09;
key[5]=50;
p[5]=0.12;
q[5]=0.03;
key[6]=60;
p[6]=0.2;
q[6]=0.04;
w[0][0]=0.000000
w[0][1]=0.140000
w[0][2]=0.360000
w[0][3]=0.490000
w[0][4]=0.610000
w[0][5]=0.760000
w[0][6]=1.000000
w[1][1]=0.040000
w[1][2]=0.260000
w[1][3]=0.390000
w[1][4]=0.510000
w[1][5]=0.660000
w[1][6]=0.900000
w[2][2]=0.020000
w[2][3]=0.150000
w[2][4]=0.270000
w[2][5]=0.420000
w[2][6]=0.660000
w[3][3]=0.120000
w[3][4]=0.240000
w[3][5]=0.390000
w[3][6]=0.630000
w[4][4]=0.090000
w[4][5]=0.240000
w[4][6]=0.480000
w[5][5]=0.030000
w[5][6]=0.270000
w[6][6]=0.040000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 2 thru 2
Building c(1,4) using roots 2 thru 4
Building c(2,5) using roots 4 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 2 thru 2
Building c(1,5) using roots 2 thru 4
Building c(2,6) using roots 4 thru 5
Building c(0,5) using roots 2 thru 4
Building c(1,6) using roots 4 thru 5
Building c(0,6) using roots ? thru ?
Counts - root trick 33 without root
    trick 50
Average probe length is ????
trees in parenthesized prefix
c(0,0) cost 0.000000
c(1,1) cost 0.000000
c(2,2) cost 0.000000
c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.140000 1
c(1,2) cost 0.260000 2
c(2,3) cost 0.150000 3
c(3,4) cost 0.240000 4
c(4,5) cost 0.240000 5
c(5,6) cost 0.270000 6
c(0,2) cost 0.500000 2(1,)
c(1,3) cost 0.540000 2(,3)
c(2,4) cost 0.420000 4(3,)
c(3,5) cost 0.630000 4(,5)
c(4,6) cost 0.720000 6(5,)
c(0,3) cost 0.780000 2(1,3)
c(1,4) cost 0.930000 2(,4(3,))
c(2,5) cost 0.810000 4(3,5)
c(3,6) cost 1.140000 5(4,6)
c(0,4) cost 1.170000 2(1,4(3,))
c(1,5) cost 1.440000 4(2(,3),5)
c(2,6) cost 1.350000 5(4(3,),6)
c(0,5) cost 1.710000 2(1,4(3,5))
c(1,6) cost 2.100000 5(2(,4(3,)),6)
c(0,6) cost ???????? ????????
```

Trick
k=1 c[0,0] + c[1,6] X (no key)
k=6 X

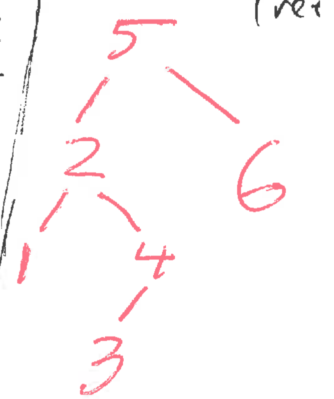
1 ≤ 2 ≤ 5 ✓
2: c(0,1) + c(2,6) + w[0][6]
.14 1.35 + 1.0 = 2.49

2 ≤ 3 ≤ 5 ✓
3: c(0,2) + c(3,6)
.5 1.14 + 1.0 = 2.64

2 ≤ 4 ≤ 6 ✓
4: c(0,3) + c(4,6)
.78 .72 + 1.0 = 2.5

2 ≤ 5 ≤ 6 ✓
5: c(0,4) + c(5,6)
1.17 .27 + 1.0 = 2.44 ✓

Tree



Proof Wrong ~ 1-2p (partial)

4. Use the substitution method to show that $T(n) = T\left(\frac{n}{2}\right) + n^2$ is in $O\left(n^2\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(n^2\right)$.) (10 points)

Suppose $T(k) \leq ck^2$ for $k < n$ −(3p)

So, $T\left(\frac{n}{2}\right) \leq c\frac{n^2}{4}$ − (4p)

$T(n) = T\left(\frac{n}{2}\right) + n^2$

$\leq c\frac{n^2}{4} + n^2$

$= cn^2 - \frac{3}{4}cn^2 + n^2$

$\leq cn^2$ for $c \geq \frac{4}{3}$ −(3p)

"−2p"

5. Show the result of deleting 30 from this red-black tree (preserving red-black properties). Be sure to indicate the cases that were used. (10 points)