

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

 $15 \times 3p = 45p$

1. What is the nature of the linear-space method for the longest common subsequence problem?
B A. Build a suffix array and lcp array for the concatenated input sequences
 B. Recursive divide-and-conquer
 C. Radix sort
 D. Use a polynomial for the signature function
2. The length of the TSP tour found by the triangle inequality technique achieves what minimization ratio?
D A. 0.5 B. $1 + \epsilon$ (e.g. a PTAS) C. 1.5 D. 2
3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?
D A. Delaunay triangulation B. Euclidean minimum spanning tree
 C. Voronoi diagram D. Convex hull
4. While constructing a suffix array by using the original Manber-Myers radix sort construction, sequence symbols are used in which of the radix sorts?
D A. All of them B. None of them C. The last one D. The first one
5. What is the minimum increase in the tail's distance from the source between the first and second times that an edge becomes critical in the Edmonds-Karp method?
B A. 1 B. 2 C. $(V-2)/2$ D. VE
6. Under what condition does an instance of stable marriages have only one solution?
C A. Every male preference list is identical to some female preference list
 B. No female appears at the beginning of multiple male preference lists
 C. The male-optimal solution and female-optimal solution are the same
 D. There is only one rotation
7. When performing bin packing using the first-fit decreasing technique, the total number of items placed in the bins past the optimal bins ($1 \dots OPT$) is bounded by:
C A. $1 + \epsilon$ B. 2 C. $OPT - 1$ D. OPT

8. Which of the following is a deficiency of the maximum capacity path technique?

- D
- A. Augmenting paths will be discovered in descending incremental flow increase order.
 - B. Flow decomposition must be applied.
 - C. An augmenting path is blocked if it introduces a cycle of flow.
 - D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.

9. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A?

- A
- A. The reduction has an inverse that takes each instance of problem B to an instance of problem A.
 - B. The reduction takes polynomial time.
 - C. The reduction must be consistent for the decision results for each instance of problem A and the corresponding instance of problem B.
 - D. Problem A is NP-complete.

10. The four russians' concept is to:

- C
- A. Implement longest common subsequences using linear space
 - B. Pack bits into an efficient storage unit
 - C. Trade-off between enumerating situations and referencing these situations
 - D. Trade-off between scalar additions and multiplications

11. Pareto optimality was the solution criteria for which problem?

- A
- A. House allocation
 - B. Stable marriages
 - C. Stable roommates
 - D. Stable marriages with incomplete preference lists

12. The technique for approximating a subset cover proceeds by:

- B
- A. Choosing the subset with the largest fraction of its elements uncovered
 - B. Choosing the subset with the largest number of uncovered elements
 - C. Choosing the subset with the smallest fraction of its elements uncovered
 - D. Choosing the subset with the smallest number of uncovered elements

13. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:

- D
- A. V
 - B. $O(VE)$
 - C. f
 - D. E

14. The incircle test is useful for finding:

- C
- A. Closest pair of points
 - B. Convex hull
 - C. Delaunay triangulation
 - D. Doubly-connected edge list

15. Kasai's linear-time LCP construction is based on which fact?

- C
- A. $\text{lcp}[\text{rank}[i]] = \text{lcp}[\text{rank}[i-1]]$
 - B. $\text{lcp}[\text{rank}[i-1]] \geq \text{lcp}[\text{rank}[i]] - 1$
 - C. $\text{lcp}[\text{rank}[i]] \geq \text{lcp}[\text{rank}[i-1]] - 1$
 - D. $\text{lcp}[\text{rank}[i]] = \text{lcp}[\text{rank}[i-1]] - 1$

16. What is the input and the output for the smallest enclosing disk problem? Be precise! (5 points)

Input: 2P

A ^{1P} set of ^{1P} 2-d points.

(A set of points in the plane.)

Output: 3P

A smallest enclosing disk ^{1P} specified by two or three ^{2P} points on its boundary.

101101011011010110101

n is 22

i sa suffix

lcp[rank]

lcp s rank

| | | | | | | | |
|----|----|-----------------------|--|----|---|----|----|
| 0 | 21 | | | -1 | 1 | 16 | 11 |
| 1 | 19 | 01 | | 0 | 0 | 7 | 10 |
| 2 | 17 | 0101 | | 2 | 1 | 20 | 9 |
| 3 | 12 | 010110101 | | 4 | 1 | 13 | 8 |
| 4 | 4 | 01011011010110101 | | 7 | 0 | 4 | 7 |
| 5 | 14 | 0110101 | | 2 | 1 | 17 | 6 |
| 6 | 9 | 011010110101 | | 7 | 0 | 8 | 5 |
| 7 | 1 | 01101011011010110101 | | 10 | 1 | 21 | 4 |
| 8 | 6 | 011011010110101 | | 5 | 1 | 15 | 8 |
| 9 | 20 | 1 | | 0 | 0 | 6 | 7 |
| 10 | 18 | 101 | | 1 | 1 | 19 | 6 |
| 11 | 16 | 10101 | | 3 | 1 | 12 | 5 |
| 12 | 11 | 1010110101 | | 5 | 0 | 3 | 4 |
| 13 | 3 | 101011011010110101 | | 8 | 1 | 14 | 3 |
| 14 | 13 | 10110101 | | 3 | 0 | 5 | 2 |
| 15 | 8 | 1011010110101 | | 8 | 1 | 18 | 1 |
| 16 | 0 | 101101011011010110101 | | 11 | 1 | 11 | 3 |
| 17 | 5 | 1011011010110101 | | 6 | 0 | 2 | 2 |
| 18 | 15 | 110101 | | 1 | 1 | 10 | 1 |
| 19 | 10 | 11010110101 | | 6 | 0 | 1 | 0 |
| 20 | 2 | 1101011011010110101 | | 9 | 1 | 9 | 0 |
| 21 | 7 | 11011010110101 | | 4 | 0 | | -1 |

1. Fill in the blanks in the following instance of a suffix array with lcp values and ranks. As usual, s[21] is NULL ('\0'). (15 points)

| i | sa | suffix | lcp | s | rank | lcp[rank] |
|----|----------|------------------------------|-----|---|-----------|-----------|
| 0 | 21 | | -1 | 1 | 16 | <u>11</u> |
| 1 | 19 | 01 | 0 | 0 | 7 | <u>10</u> |
| 2 | 17 | 0101 | 2 | 1 | <u>20</u> | <u>9</u> |
| 3 | 12 | <u>010</u> 110101 | 4 | 1 | 13 | <u>8</u> |
| 4 | 4 | 01011011010110101 | 7 | 0 | 4 | 7 |
| 5 | 14 | 0110101 | 2 | 1 | 17 | 6 |
| 6 | 9 | <u>0110</u> 10110101 | 7 | 0 | 8 | 5 |
| 7 | 1 | <u>0110</u> 1011011010110101 | 10 | 1 | <u>21</u> | 4 |
| 8 | 6 | 011011010110101 | 5 | 1 | 15 | 8 |
| 9 | 20 | 1 | 0 | 0 | 6 | 7 |
| 10 | 18 | 101 | 1 | 1 | 19 | 6 |
| 11 | 16 | 10101 | 3 | 1 | 12 | 5 |
| 12 | 11 | <u>1010</u> 110101 | 5 | 0 | 3 | 4 |
| 13 | <u>3</u> | 101011011010110101 | 8 | 1 | 14 | 3 |
| 14 | 13 | 10110101 | 3 | 0 | 5 | 2 |
| 15 | <u>8</u> | 1011010110101 | 8 | 1 | 18 | 1 |
| 16 | 0 | 101101011011010110101 | 11 | 1 | 11 | <u>3</u> |
| 17 | 5 | <u>10110</u> 11010110101 | 6 | 0 | 2 | <u>2</u> |
| 18 | 15 | 110101 | 1 | 1 | 10 | 1 |
| 19 | 10 | 11010110101 | 6 | 0 | 1 | 0 |
| 20 | 2 | 1101011011010110101 | 9 | 1 | 9 | 0 |
| 21 | 7 | 11011010110101 | 4 | 0 | -1 | |

15 x 14

2. Use the Gale-Shapley algorithm to determine the male-optimal solution for the following instance of the stable marriages problem. In addition, show the preference lists at termination, i.e. you are to use the MEGS technique. Note that the preference lists are given left-to-right. (10 points)

male preference lists are:

1: ~~1~~ ~~2~~ 3 ~~4~~ 5

2: 2 ~~3~~ ~~4~~ 5 ~~1~~

3: ~~3~~ 4 ~~2~~ 5 ~~1~~

4: 1 ~~3~~ ~~2~~ 5 ~~4~~

5: ~~3~~ 5 ~~4~~ ~~2~~ ~~1~~

female preference lists are:

1: 4 ~~3~~ ~~5~~ ~~2~~ ~~1~~

2: 2 ~~3~~ ~~5~~ ~~4~~ ~~1~~

3: 1 ~~2~~ ~~5~~ ~~4~~ ~~3~~

4: 3 ~~2~~ ~~4~~ ~~5~~ ~~1~~

5: 2 4 3 1 5

(spare, same as above) male preference lists are:

1: 1 2 3 4 5

2: 2 3 4 5 1

3: 3 4 2 5 1

4: 1 3 2 5 4

5: 3 5 4 2 1

(spare, same as above) female preference lists are:

1: 4 3 5 2 1

2: 2 3 5 4 1

3: 1 2 5 4 3

4: 3 2 4 5 1

5: 2 4 3 1 5

* 7p - stable matching
* 3p - list deletions

Fail link table 1 5p

0 c -1

1 a 0

2 b 0

3 a 0

4 c 0

5 a 1

6 c 2

7 a 1

8 b 2

9 a 3

10 c 4

11 a 5

12 b 6

13 a 3

14 c 4

15 a 5

16 c 6

17 a 7

18 b 8

19 a 9

20 c 10

21 a 11

22 c 12

23 a 7

24 b 8

Fail link table 2 5p

0 c -1

1 a 0

2 b 0

3 a 0

4 c -1

5 a 0

6 c 2

7 a 0

8 b 0

9 a 0

10 c -1

11 a 0

12 b 6

13 a 0

14 c -1

15 a 0

16 c 2

17 a 0

18 b 0

19 a 0

20 c -1

21 a 0

22 c 12

23 a 0

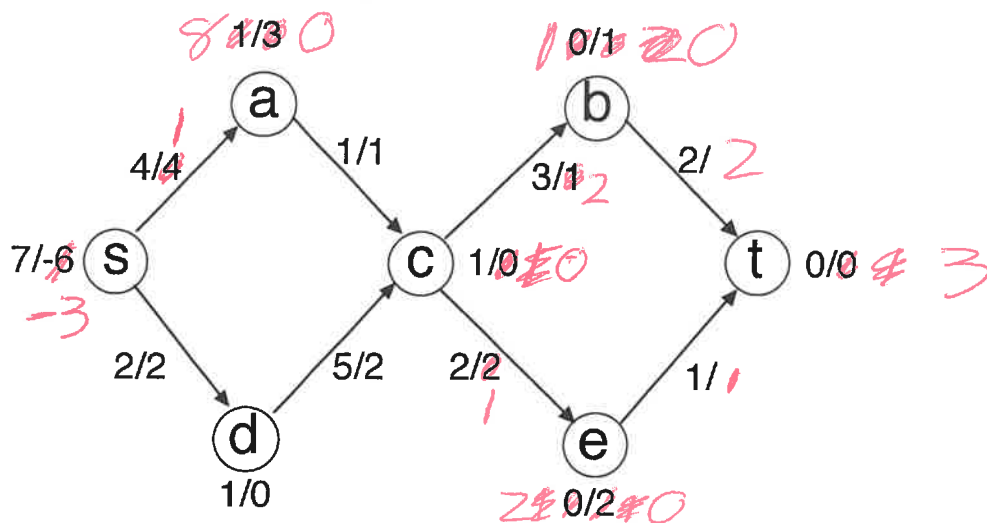
24 b 0

-1p per error

3. Give the result of both KMP methods for the following pattern. (10 points)

0 c
1 a
2 b
3 a
4 c
5 a
6 c
7 a
8 b
9 a
10 c
11 a
12 b
13 a
14 c
15 a
16 c
17 a
18 b
19 a
20 c
21 a
22 c
23 a
24 b

4. List the lift and push operations to complete the maximum flow. In addition, give a minimum cut. Edges are labeled with capacity/flow, while vertices are labeled with height/excess. (15 points)



- Lift e from 0 to 1 1
- Push 1 from e to t 0.5
- Lift e from 1 to 2 1.5
- Push 1 from e to c 0.5
- ~~Lift c fro~~
- Push 1 from c to b 0.5
- Lift b from 0 to 1 1
- Push 2 from b to t 0.5
- Lift a from 1 to 8 1
- Push 3 from a to s 0.5

$$1.5 + 2.5 + 1.5 + 1.5 = 7$$

TP

(different ordering) *

Minimum cut

S : s, a

3p

T : t, b, c, d, e