CSE 5311
Test 1 - Closed Book
Spring 2018

Name Key 1
Last name, then first name as on your UTA ID card

Multiple Choice/Short Answer:

1. Write your answer on the line (______) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. To support computing the number of keys that are smaller than a query key, an augmented binary search tree stores the following at every node:

C A. the count of the number of keys in the entire tree
B. the sum of all keys in the left subtree
C. the count of the number of keys stored in the subtree rooted by this node
D. the sum of all keys stored in the subtree rooted by this node

2. Which of the following is not true regarding the amortized analysis of binary tree traversals?

A A. INIT had an amortized cost of 1.
B. SUCC had an actual cost determined by the number of edges followed.
C. SUCC had an amortized cost of 2.
D. The potential was defined with regard to the type of traversal being performed.

3. Suppose you already have 16 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the ~~15~~ 16 you already have?

5 ← Write your answer on the line

4. Mapping a multi-way tree to a binary tree (left child, right sibling representation) is part of which priority queue approach?

D A. binary heaps B. binomial heaps C. leftist heaps D. pairing heaps

5. The balance factors in an AVL tree are computed as:

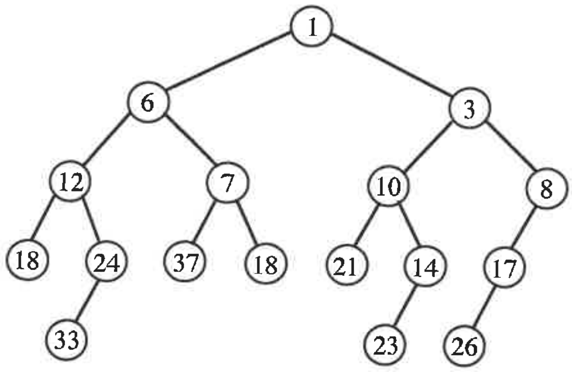
A A. $height_{right} - height_{left}$
B. same as the null path length in a leftist heap
C. the difference of the number of nodes in the left and right subtrees
D. the distance from a node to the root

6. How many inversions are there for the lists 1, 2, 4, 5, 3 and 2, 5, 4, 3, 1?

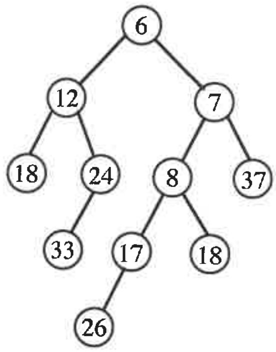
5 ← Write your answer on the line

7. Which of the following is not a legal leftist heap?

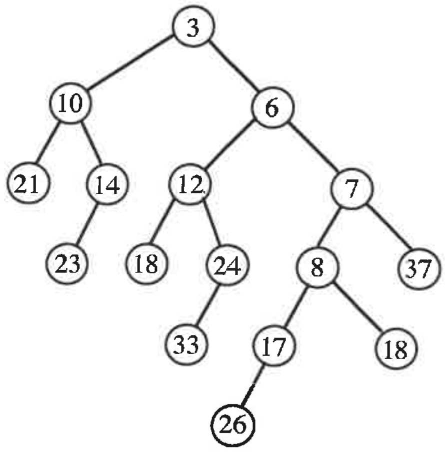
C ____ A.



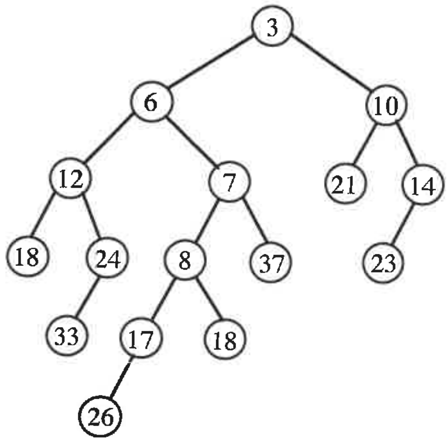
B.



C.



D.



8. Lower cost for DECREASE-KEY operations is associated with:

D ____ A. binary heaps   B. binomial heaps   C. leftist heaps   D. pairing heaps

9. Which property does not hold for binomial heaps?

C ____ A. Performing n INSERT operations into an empty heap will take $O(n)$ time.
B. The number of trees is based on the binary representation of the number of stored items.
C. DECREASE-KEY takes $O(1)$ time.
D. MINIMUM takes $O(\log n)$ time.

10. In the worst case, the number of rotations for inserting a key in a treap with n keys is:

D ____ A. $\theta(n \log n)$ B. $\theta(1)$ C. $\theta(\log n)$ D. $\theta(n)$

11. Which minimum spanning tree algorithm was modified to test for multiple MSTs?

B ____ A. Boruvka   B. Kruskal   C. Prim   D. Warshall

12. What is the worst-case number of rotations when performing insertion on an AVL tree?

B ____ A. $\Theta(n)$ B. $\Theta(1)$ C. $\Theta(\log n)$ D. No rotations are ever used

13. The master method summarizes the main characteristics of what other technique.

C A. asymptotic analysis B. potential functions C. recursion trees D. substitution method

14. Which of the following is not true regarding Bloom filters?

D
A. Several hash functions are used.
B. The optimal bit array size depends on the number of items and the false positive probability.
C. An indication that a candidate element is not in the set is always correct.
D. They are an especially compact dictionary representation.

15. When are Fibonacci trees used?

C
A. Constructing a priority queue with excellent amortized complexity for DECREASE-KEY.
B. Defining the potential function for splay trees.
C. Demonstrating worst-case behaviors for AVL trees.
D. Demonstrating worst-case behaviors for red-black trees.

16. Sorting the set of input edges is a property of which minimum spanning tree technique?

B A. Boruvka B. Kruskal C. Path-based (Warshall) D. Prim

17. Dynamic optimality is a concept involving the comparison of

C
A. a key-comparison based data structure to hashing.
B. amortized complexity to actual complexity.
C. an online data structure to an offline data structure.
D. an online data structure to a fixed, unchanging data structure.

18. The goal of the optimal matrix multiplication problem is to:

C
A. Minimize the number of C(i,j) instances evaluated.
B. Minimize the number of matrix multiplications.
C. Minimize the number of scalar multiplications.
D. Minimize the number of scalar additions.

19. Warshall's algorithm on a directed graph with n vertices uses this much time:

A A. $\Theta(n^3)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

20. When performing selection in worst-cast linear time, roughly what fraction of the set of n keys is kept (in the worst case) for the next round?

D A. 10% B. 20% C. 30% D. 70%

21. Suppose you roll three standard six-sided dice. What is the probability that the sum of the three values rolled does not exceed 10? Show your work. (10 points)

Sum of first two dice	# of possibilities	Ways for 3rd dice to not exceed 10	
2	1	6	6
3	2	6	12
4	3	6	18
5	4	5	20
6	5	4	20
7	6	3	18
8	5	2	10
9	4	1	4

Σ 108

$$\frac{\text{Ways to not exceed 10}}{\text{Ways to roll 3 dice } (216)}$$

.5

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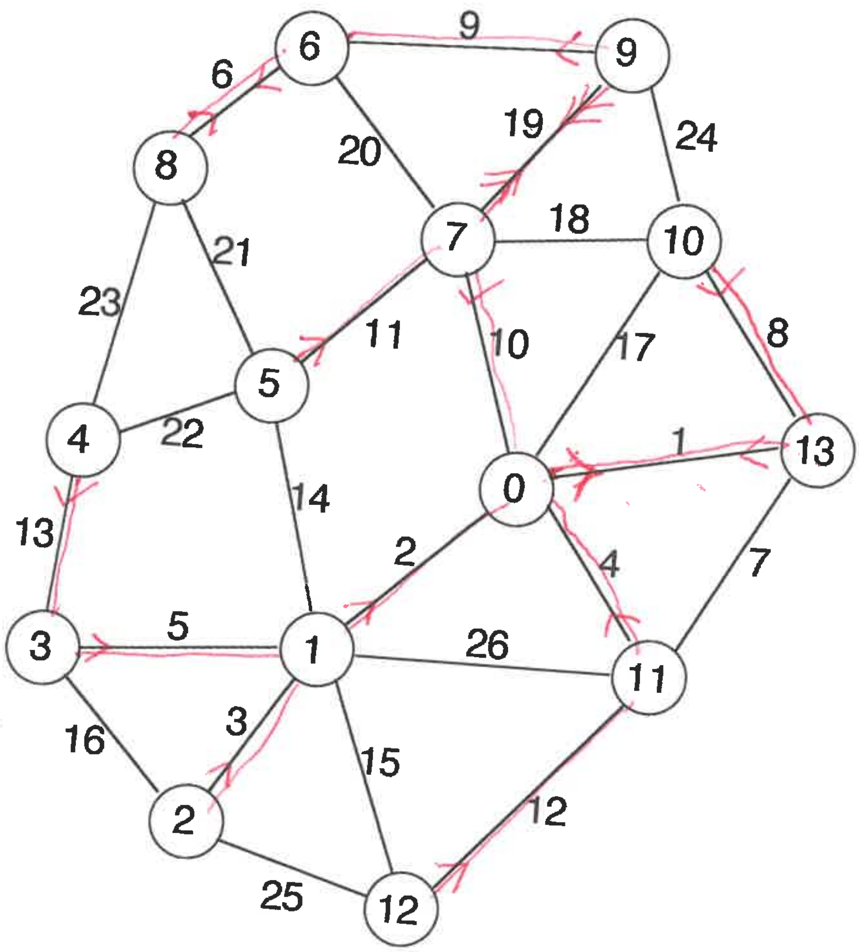
1. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (10 points)



2. The hash table below was created using double hashing with Brent's rehash. The initial slot ($h_1(key)$) and rehashing increment ($h_2(key)$) are given for each *key*. Show the result from inserting 7097 using Brent's rehash when $h_1(7077) = 0$ and $h_2(7077) = 4$. (10 points)

	key	$h_1(key)$	$h_2(key)$
0	9999	3	4
1	5561	3	6
2	6666	2	1
3	7780	3	5
4	8886	3	1
5			
6			

	key
0	9999
1	5561
2	6666
3	7780
4	7077
5	8886
6	

Indicate the additional number of probes for searching for all keys once after ~~7097~~ 7077 is inserted:

3

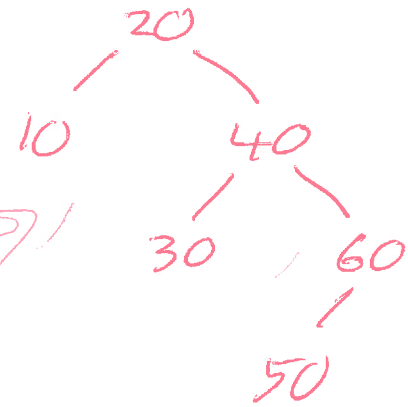
____

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```
n=6;
q[0]=0.12;
key[1]=10;
p[1]=0.12;
q[1]=0.09;
key[2]=20;
p[2]=0.2;
q[2]=0.03;
key[3]=30;
p[3]=0.1;
q[3]=0.04;
key[4]=40;
p[4]=0.2;
q[4]=0.0;
key[5]=50;
p[5]=0.01;
q[5]=0.04;
key[6]=60;
p[6]=0.03;
q[6]=0.02;
w[0][0]=0.120000
w[0][1]=0.330000
w[0][2]=0.560000
w[0][3]=0.700000
w[0][4]=0.900000
w[0][5]=0.950000
w[0][6]=1.000000
w[1][1]=0.090000
w[1][2]=0.320000
w[1][3]=0.460000
w[1][4]=0.660000
w[1][5]=0.710000
w[1][6]=0.760000
```

```
w[2][2]=0.030000
w[2][3]=0.170000
w[2][4]=0.370000
w[2][5]=0.420000
w[2][6]=0.470000
w[3][3]=0.040000
w[3][4]=0.240000
w[3][5]=0.290000
w[3][6]=0.340000
w[4][4]=0.000000
w[4][5]=0.050000
w[4][6]=0.100000
w[5][5]=0.040000
w[5][6]=0.090000
w[6][6]=0.020000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 1 thru 2
Building c(1,4) using roots 2 thru 4
Building c(2,5) using roots 4 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 2 thru 2
Building c(1,5) using roots 2 thru 4
Building c(2,6) using roots 4 thru 4
Building c(0,5) using roots 2 thru 2
Building c(1,6) using roots 2 thru 4
Building c(0,6) using roots ? thru ?
Counts - root trick 31 without root
    trick 50
Average probe length is ???
```

```
trees in parenthesized prefix
c(0,0) cost 0.000000
c(1,1) cost 0.000000
c(2,2) cost 0.000000
c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.330000 10
c(1,2) cost 0.320000 20
c(2,3) cost 0.170000 30
c(3,4) cost 0.240000 40
c(4,5) cost 0.050000 50
c(5,6) cost 0.090000 60
c(0,2) cost 0.880000 10(,20)
c(1,3) cost 0.630000 20(,30)
c(2,4) cost 0.540000 40(30,)
c(3,5) cost 0.340000 40(,50)
c(4,6) cost 0.150000 60(50,)
c(0,3) cost 1.200000 20(10,30)
c(1,4) cost 1.200000 20(,40(30,))
c(2,5) cost 0.640000 40(30,50)
c(3,6) cost 0.490000 40(,60(50,))
c(0,4) cost 1.770000 20(10,40(30,))
c(1,5) cost 1.350000 20(,40(30,50))
c(2,6) cost 0.790000 40(30,60(50,))
c(0,5) cost 1.920000 20(10,40(30,50))
c(1,6) cost 1.540000 40(20(,30),60(50,))
c(0,6) cost ??? ?????????????????
```



2: c(0,1) + c(2,6) + w[0][7]
.33 .79 1.0 — 2.12

3: c(0,2) + c(3,6) + w
.88 .49 1.0 — 2.37

4: c(0,3) + c(4,6) + w
1.2 .15 1.0 — 2.35

#4

$T(n) \Rightarrow \sqrt{n}$

$T\left(\frac{n}{4}\right) \Rightarrow \frac{\sqrt{n}}{2} \cdot 2 \Rightarrow \sqrt{n}$

$T\left(\frac{n}{16}\right) \Rightarrow \frac{\sqrt{n}}{4} \cdot 4 \Rightarrow \sqrt{n}$

⋮

$T(1)$

$1 + \log_4 n$

$\#\text{leaves} = 2^{\log_4 n}$

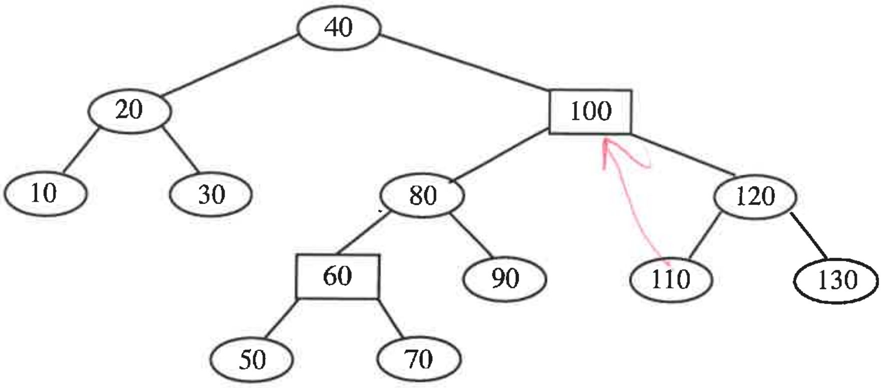
$= n^{\log_4 2}$

$= \sqrt{n}$

$\sqrt{n}\log_4 n + \sqrt{n} = \Theta(\sqrt{n}\log n)$

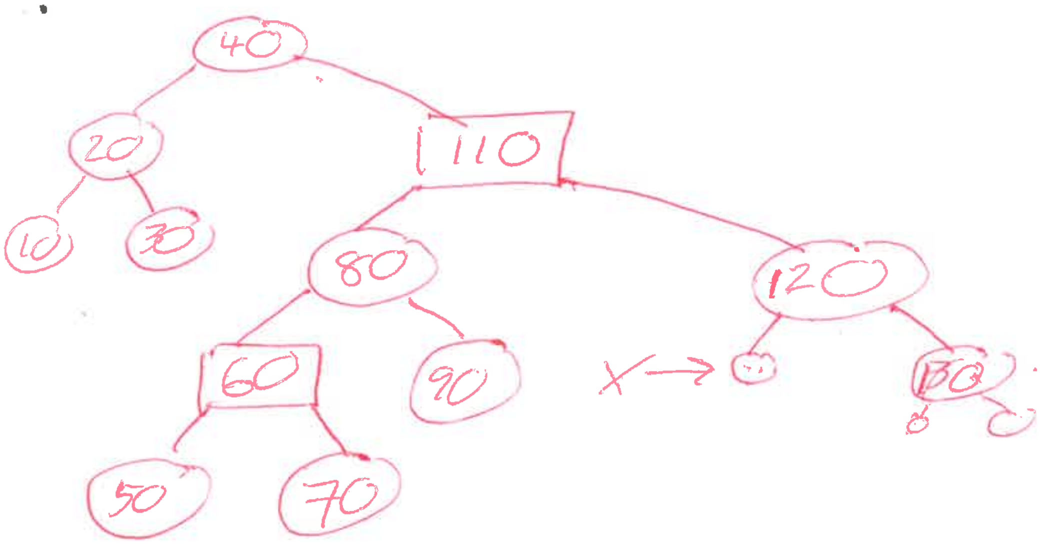
4. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ is in $\Theta\left(\sqrt{n}\log n\right)$. (10 points)

5. Show the result of deleting 100 from this red-black tree (preserving red-black properties). Be sure to indicate the cases that were used. (10 points)

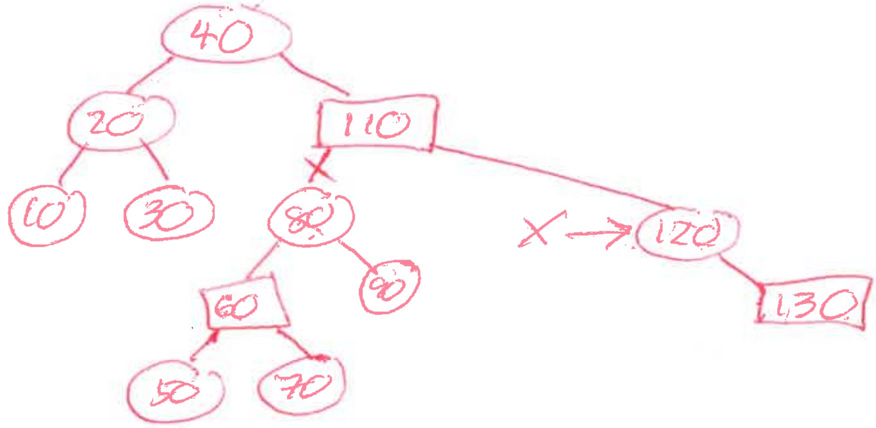


Delete 100

#5



2 ⇒



4 ⇒