CSE 5311
Test 1 - Closed Book
Spring 2019

Name Key 1
Last name, then first name as on your UTA ID card

Multiple Choice/Short Answer:

1. Write your answer, letter or value, on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:

C A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

2. Which of the following is not true regarding the amortized analysis of binary tree traversals?

A A. INIT had an amortized cost of 1.
B. SUCC had an actual cost determined by the number of edges followed.
C. SUCC had an amortized cost of 2.
D. The potential was defined with regard to the type of traversal being performed.

3. Suppose you already have 10 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 10 you already have?

2 ← Write your answer on the line $\frac{20}{20-10}$

4. Mapping a multi-way tree to a binary tree (left child, right sibling representation) is part of which priority queue approach?

D A. binary heaps B. binomial heaps C. leftist heaps D. pairing heaps

5. The balance factors in an AVL tree are computed as:

A A. $\text{height}_{\text{right}}$ - $\text{height}_{\text{left}}$
B. same as the null path length in a leftist heap
C. the difference of the number of nodes in the left and right subtrees
D. the distance from a node to the root

6. How many inversions are there for the lists 3, 1, 2, 4, 5 and 2, 5, 4, 3, 1?

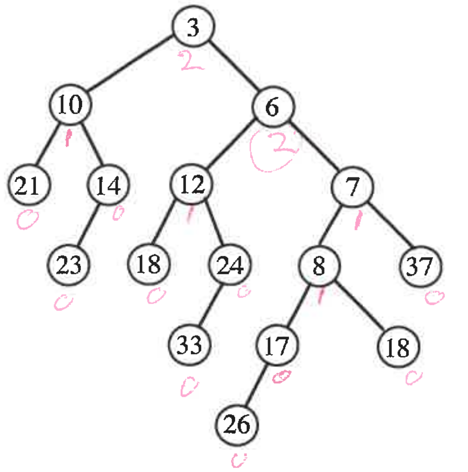
7 ← Write your answer on the line

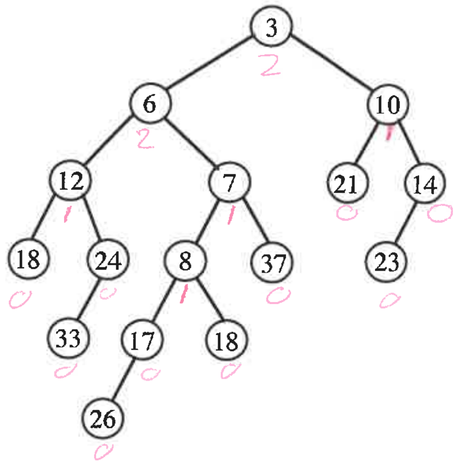
3 ⇒ 3
1 ⇒ 3
2 ⇒ 0
4 ⇒ 1

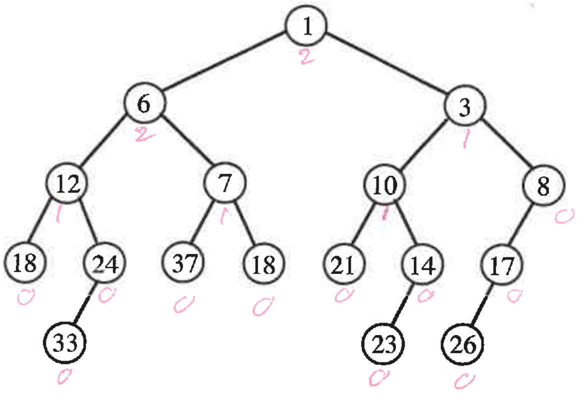
7. Assuming a random n-permutation is provided, the expected number of hires for the hiring problem is:

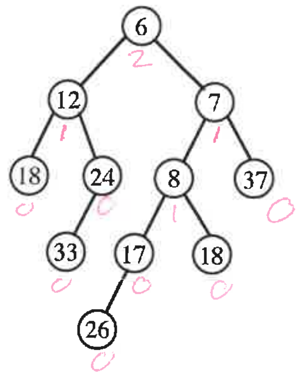
A A. H_n B. 2 C. $\sqrt{n}$ D. $\ln \ln n$

8. Which of the following is not a legal leftist heap?

A A. 

B. 

C. 

D. 

9. Which property does not hold for binomial heaps?

C
A. Performing n INSERT operations into an empty heap will take $O(n)$ time.
B. The number of trees is based on the binary representation of the number of stored items.
C. DECREASE-KEY takes $O(1)$ time.
D. MINIMUM takes $O(\log n)$ time.

10. The perfect hashing method discussed in class depends on which fact?

D
A. $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad 0 < x < 1$
B. $\ln n < H_n < \ln n + 1$
C. The expected number of probes for a successful search in Brent's method is less than 2.5.
D. The probability of collisions among n keys stored in a hash table of size n^2 is less than 0.5.

11. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of 0?

B
A. none of them
B. only the leaves
C. only the root
D. the leaves and the root

12. What is the worst-case number of rotations when performing deletion on an AVL tree?

C A. $\Theta(n)$ B. $\Theta(1)$ C. $\Theta(\log n)$ D. No rotations are ever used

13. The master method summarizes the main characteristics of what other technique.

C A. asymptotic analysis B. potential functions C. recursion trees D. substitution method

14. Which of the following is not true regarding Bloom filters?

A
A. They are an especially compact dictionary representation.
B. Several hash functions are used.
C. The optimal bit array size depends on the number of items and the false positive probability.
D. An indication that a candidate element is not in the set is always correct.

15. When are Fibonacci trees used?

D
A. Constructing a priority queue with excellent amortized complexity for DECREASE-KEY.
B. Demonstrating worst-case behaviors for red-black trees.
C. Defining the potential function for splay trees.
D. Demonstrating worst-case behaviors for AVL trees.

16. When performing selection in worst-cast linear time for n numbers, roughly how many column medians are computed in the first round?

A A. $\frac{n}{5}$ B. m, the median-of-medians C. $0.7n$ D. $W\left(\frac{n}{5}\right)$

17. The main difference between MTF and OPT for self-organizing linear lists is:

B
A. OPT can do transpositions
B. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time
C. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
D. MTF counts inversions

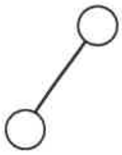
18. When of the following is not an application of union-find trees?

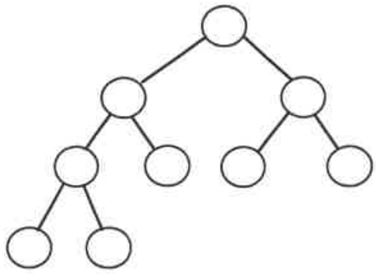
A
A. Finding the median of a set of numbers
B. Kruskal's algorithm
C. Off-line least common ancestors
D. Maximum cardinality k-coloring of a set of intervals

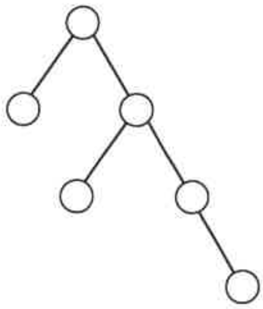
19. What is required when calling `union(i,j)` for maintaining disjoint subsets?

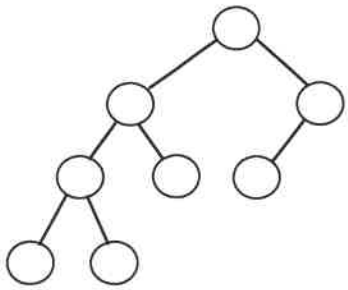
A
A. `i` and `j` are roots for different subsets
B. `i` is the ancestor of `j` in one of the trees
C. `i` and `j` are roots for the same subset
D. `i` and `j` are in the same subset

20. Which of the following binary trees has *multiple* legal colorings as a red-black tree?

B ____ A. 

B. 

C. 

D. 

21. When performing selection in worst-cast linear time, roughly what fraction of the set of n keys is kept (in the worst case) for the next round?

C ____ A. 20% B. 30% C. 70% D. 80%

22. When using Brent's rehash, the number of previously inserted keys that may move is:

A ____ A. 1 B. 2 C. $\frac{1}{\alpha}$ D. H_m, where m is the number of stored keys

23. Which priority queue implementation generalizes binary heaps by increasing the branching?

B ____ A. Binomial heaps B. d-heaps C. Pairing heaps D. Leftist heaps

24. What is minimized in the dynamic programming solution to the subset sum problem?

C ____ A. The number of input values used to sum to each $C(i)$ B. S_j
C. The index stored for each $C(i)$ D. m

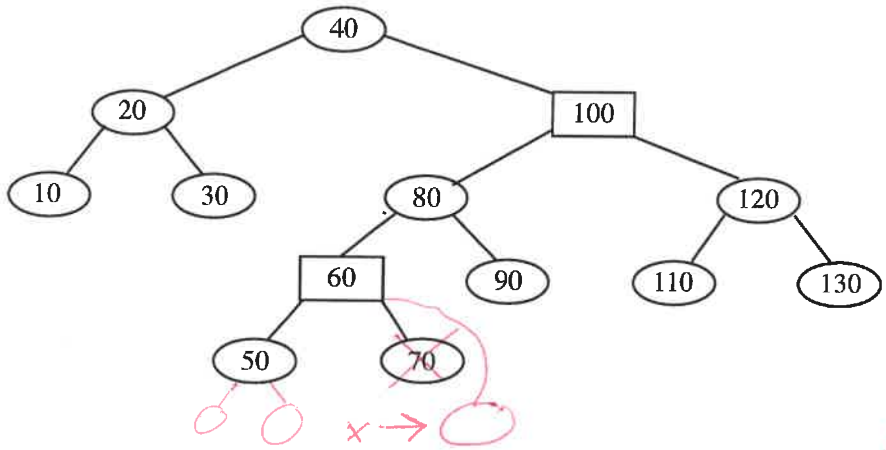
25. The recursion tree for mergesort has which property?

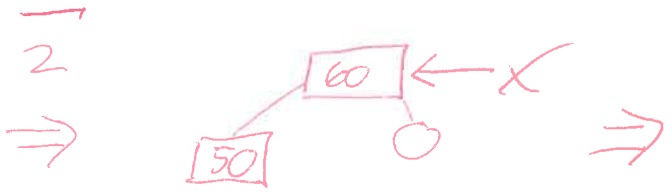
C ____ A. it leads to a definite geometric sum B. it leads to a harmonic sum
C. each level has the same contribution D. it leads to an indefinite geometric sum

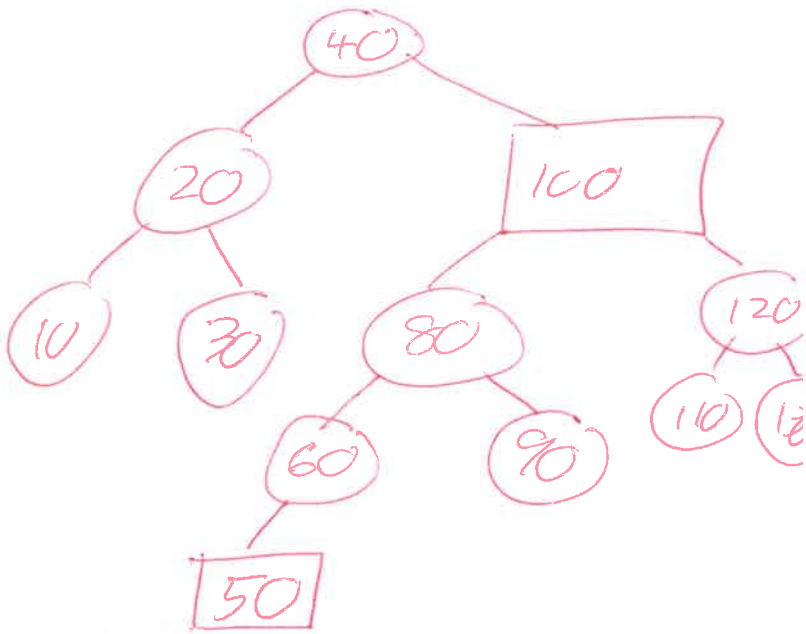
CSE 5311
Test 1 - Open Book
Spring 2019

Name ______________________________
Last name, then first name as on your UTA ID card

1. Show the result of deleting 70 from this red-black tree (preserving red-black properties). Be sure to indicate the cases that were used. (10 points)







2. The hash table below was created using double hashing with Brent's rehash. The initial slot ($h_1(key)$) and rehashing increment ($h_2(key)$) are given for each *key*. Show the result from inserting 284 using Brent's rehash when $h_1(284) = 4$ and $h_2(284) = 3$. (10 points)

	key	$h_1(key)$	$h_2(key)$	
0	171	3	4	0 4 1
1				
2	912	2	1	2 3 4 5 6
3	801	3	4	3 0 4 1
4	102	4	1	4 5 6
5	212	2	3	5 1
6				

284	4	0	3	6
New probes	1	2	3	4
Old probes	2	2	3	0
Σ	3	4	6	4

	key
0	171
1	
2	912
3	801
4	284
5	212
6	102

Indicate the additional number of probes for searching for all keys once after 284 is inserted:

3

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

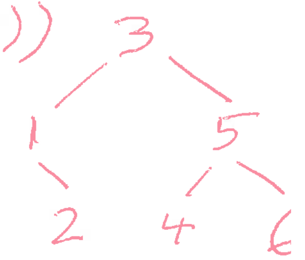
```
n=6
q[0]=0.03;
key[1]=1;
p[1]=0.20;
q[1]=0.01;
key[2]=2;
p[2]=0.15;
q[2]=0.03;
key[3]=3;
p[3]=0.10;
q[3]=0.02;
key[4]=4;
p[4]=0.10;
q[4]=0.03;
key[5]=5;
p[5]=0.15;
q[5]=0.05;
key[6]=6;
p[6]=0.10;
q[6]=0.03;
w[0][0]=0.030000
w[0][1]=0.240000
w[0][2]=0.420000
w[0][3]=0.540000
w[0][4]=0.670000
w[0][5]=0.870000
w[0][6]=1.000000
w[1][1]=0.010000
w[1][2]=0.190000
w[1][3]=0.310000
w[1][4]=0.440000
w[1][5]=0.640000
w[1][6]=0.770000
w[2][2]=0.030000
```

```
w[2][3]=0.150000
w[2][4]=0.280000
w[2][5]=0.480000
w[2][6]=0.610000
w[3][3]=0.020000
w[3][4]=0.150000
w[3][5]=0.350000
w[3][6]=0.480000
w[4][4]=0.030000
w[4][5]=0.230000
w[4][6]=0.360000
w[5][5]=0.050000
w[5][6]=0.180000
w[6][6]=0.030000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 1 thru 2
Building c(1,4) using roots 2 thru 4
Building c(2,5) using roots 4 thru 5
Building c(3,6) using roots 5 thru 5
Building c(0,4) using roots 2 thru 3
Building c(1,5) using roots 3 thru 4
Building c(2,6) using roots 4 thru 5
Building c(0,5) using roots 2 thru 3
Building c(1,6) using roots 3 thru 5
Building c(0,6) using roots ? thru ?
Counts - root trick 33 without root
    trick 50
Average probe length is ???
trees in parenthesized prefix
c(0,0) cost 0.000000
```

```
c(1,1) cost 0.000000
c(2,2) cost 0.000000
c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.240000 1
c(1,2) cost 0.190000 2
c(2,3) cost 0.150000 3
c(3,4) cost 0.150000 4
c(4,5) cost 0.230000 5
c(5,6) cost 0.180000 6
c(0,2) cost 0.610000 1(,2)
c(1,3) cost 0.460000 2(,3)
c(2,4) cost 0.430000 4(3,)
c(3,5) cost 0.500000 5(4,)
c(4,6) cost 0.540000 5(,6)
c(0,3) cost 0.930000 2(1,3)
c(1,4) cost 0.780000 3(2,4)
c(2,5) cost 0.860000 4(3,5)
c(3,6) cost 0.810000 5(4,6)
c(0,4) cost 1.340000 2(1,4(3,))
c(1,5) cost 1.330000 3(2,5(4,))
c(2,6) cost 1.220000 5(4(3,),6)
c(0,5) cost 1.970000 2(1,4(3,5))
c(1,6) cost 1.730000 5(3(2,4),6)
c(0,6) cost ??? ?????????????????
```

2 5

3(1(,2),5(4,6))



2: 1.0 + c(0,1) + c(2,6) = 2.46
(.24, 1.22)

3: 1.0 + c(0,2) + c(3,6) = **2.42**
(.61, .81)

4: 1.0 + c(0,3) + c(4,6) = 2.47
(.93, .54)

5: 1.0 + c(0,4) + c(5,6) = 2.52
(1.34, .18)

4. Use the substitution method to show that $T(n) = 3T\left(\frac{n}{3}\right) + n$ is in $O(n \log n)$. (You do not need to show that $T(n)$ is in $\Omega(n \log n)$.) (10 points)

Suppose $T(k) \le c k \log_3 k$ for $k < n$

$T\left(\frac{n}{3}\right) \le c \frac{n}{3} \log_3 \frac{n}{3}$

$= c \frac{n}{3} \log_3 n - c \frac{n}{3}$

$T(n) \le 3\left[c \frac{n}{3} \log_3 n - c \frac{n}{3}\right] + n$

$= cn \log_3 n - cn + n$

$\le cn \log_3 n$ for $c \ge 1$

5. Give the range of possible heights for an AVL tree with 100 keys. Your answer should be two natural numbers giving the minimum and maximum heights. (A tree with one node has height 0.) Show your work! (10 points)

Complete tree of height h has $2^{h+1}-1$ nodes

minimum

6

$h=5 \Rightarrow 2^6-1=63$

$h=6 \Rightarrow 2^7-1=127$

maximum

8

Fib Trees

height	nodes
0	1
1	2
2	4
3	7
4	12
5	20
6	33
7	54
→ 8	88
9	(143)

since 100 is smaller, must be → 8