

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. Capacity scaling is a reasonable substitute for:

- C A. Edmonds-Karp B. Ford-Fulkerson
C. Maximum capacity paths D. Push-relabel

2. Which of the following is not used when efficiently counting the number of occurrences of a pattern in a text represented with a suffix array?

- B A. lcp array B. rank array C. binary search D. pattern symbols

3. Which of the following is not true regarding Strassen's algorithm?

- A A. It is not possible to have an asymptotically faster algorithm.
B. It requires more space than the everyday method.
C. It uses $\Theta(n^{\lg 7})$ scalar additions when multiplying two $n \times n$ matrices.
D. It uses $\Theta(n^{\lg 7})$ scalar multiplications when multiplying two $n \times n$ matrices.

4. What is the nature of the linear-space method for the longest common subsequence problem?

- B A. Build a suffix array and lcp array for the concatenated input sequences
B. Recursive divide-and-conquer
C. Radix sort
D. Use a polynomial for the signature function

5. The technique for approximating a subset cover proceeds by:

- D A. Choosing the subset with the smallest fraction of its elements uncovered
B. Choosing the subset with the smallest number of uncovered elements
C. Choosing the subset with the largest fraction of its elements uncovered
D. Choosing the subset with the largest number of uncovered elements

6. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?

- D A. Delaunay triangulation B. Euclidean minimum spanning tree
C. Voronoi diagram D. Convex hull

7. Which of the following is not represented in a DCEL?

- C A. Edges B. Faces C. Sweep-line D. Vertices

8. The four russians' concept is to:

- B A. Pack bits into an efficient storage unit
 B. Trade-off between enumerating situations and referencing these situations
 C. Implement longest common subsequences using linear space
 D. Trade-off between scalar additions and multiplications

9. When coloring the edges of a graph, a dc-path gets inverted because:

- C A. All edges in the fan will be colored with d or c.
 B. d is a free color for all fan vertices.
 C. d is the free color for two fan vertices.
 D. We are trying to minimize the number of colors used by the path.

10. The length of the TSP tour found by the triangle inequality technique achieves what minimization ratio?

- D A. 0.5 B. $1 + \epsilon$ (e.g. a PTAS) C. 1.5 D. 2

11. Top-trading cycles is the solution approach for which problem?

- B A. Stable roommates B. House allocation
 C. Stable marriages D. Stable marriages with incomplete preference lists

12. The reduction from 3-sat to 3-colorability will give a graph requiring four colors when:

- B A. Removing one clause from the 3-sat instance will leave a satisfiable set of clauses.
 B. The 3-sat instance is unsatisfiable.
 C. The 3-sat instance is satisfiable.
 D. The 3-sat instance is a tautology.

13. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:

- D A. V B. $O(VE)$ C. f D. E

14. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A?

- A A. The reduction has an inverse that takes each instance of problem B to an instance of problem A.
 B. The reduction takes polynomial time.
 C. The reduction must be consistent for the decision results for each instance of problem A and the corresponding instance of problem B.
 D. Problem A is NP-complete.

15. The incircle test is useful for finding:

- C A. Closest pair of points B. Convex hull
 C. Delaunay triangulation D. Doubly-connected edge list

16. Describe the initialization for push/relabel techniques. You may give an example. (5 points)

Heights -

Source - # of vertices

All others - zero

Saturate edges leaving source /

Head vertex excess is capacity of edge

Excess at source is the sum of the
 exiting flows, but negated

All other flows/excesses are zero

1. Fill in the blanks in the following instance of a suffix array with lcp values and ranks. As usual, s[21] is NULL ('\0'). (15 points)

i	sa	suffix	lcp	s	rank	lcp[rank]
0	21		-1	1	16	<u>11</u>
1	19	01	0	0	7	<u>10</u>
2	17	0101	2	1	<u>20</u>	<u>9</u>
3	12	<u>010</u> 110101	4	1	13	<u>8</u>
4	4	01011011010110101	7	0	4	7
5	14	0110101	2	1	17	6
6	9	<u>0110</u> 10110101	7	0	8	5
7	1	<u>0110</u> 1011011010110101	10	1	<u>21</u>	4
8	6	011011010110101	5	1	15	8
9	20	1	0	0	6	7
10	18	101	1	1	19	6
11	16	10101	3	1	12	5
12	11	<u>1010</u> 110101	5	0	3	4
13	<u>3</u>	101011011010110101	8	1	14	3
14	13	10110101	3	0	5	2
15	<u>8</u>	1011010110101	8	1	18	1
16	0	101101011011010110101	11	1	11	<u>3</u>
17	5	<u>10110</u> 11010110101	6	0	2	<u>2</u>
18	15	110101	1	1	10	1
19	10	11010110101	6	0	1	0
20	2	1101011011010110101	9	1	9	0
21	7	11011010110101	4	0	-1	

```

debug: lifting 1 from 0 to 1
debug: pushing 10 units from 1 to 2
debug: pushing 6 units from 1 to 3
debug: pushing 3 units from 1 to 4
debug: lifting 1 from 1 to 6
debug: pushing 1 units from 1 to 0
debug: lifting 2 from 0 to 1
debug: pushing 4 units from 2 to 4
debug: lifting 2 from 1 to 7
debug: pushing 6 units from 2 to 1
debug: lifting 3 from 0 to 1
debug: pushing 6 units from 3 to 4
debug: pushing 6 units from 1 to 0
total flow is 13

```

Preflow-push time 0.000090

flows along edges:

```

0->1 has 13
1->2 has 4
1->3 has 6
1->4 has 3
2->4 has 4
3->4 has 6

```

vertex height excess

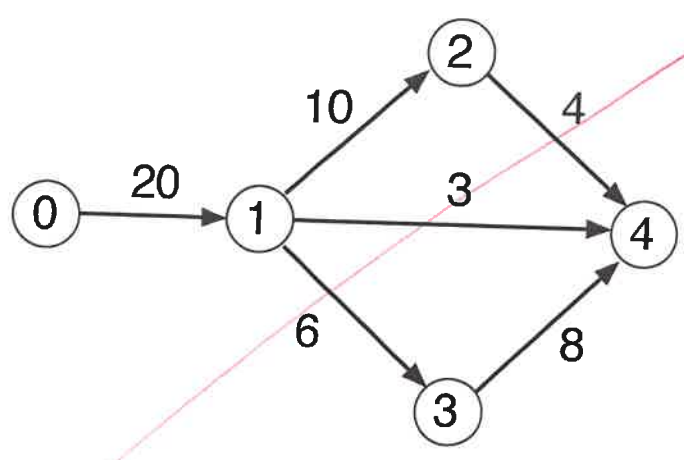
```

  0      5     -13
  1      6      0
  2      7      0
  3      1      0
  4      0     13

```

tail	head	capacity	flow
0	1	20	13
1	2	10	4
1	3	6	6
1	4	3	3
2	4	4	4
3	4	8	6

2. **List** the lift and push operations to solve for the maximum flow. Vertex 0 is the source and vertex 4 is the sink. In addition, give a minimum cut. (15 points)



Cut

Fail link table 1

0 a -1
1 c 0
2 a 0
3 b 1
4 a 0
5 c 1
6 a 2
7 c 3
8 a 2
9 b 3
10 a 4
11 c 5
12 a 6
13 b 7
14 a 4
15 c 5
16 a 6
17 c 7
18 a 8
19 b 9
20 a 10
21 c 11
22 a 12
23 c 13
24 a 8
25 b 9

Fail link table 2

0 a -1
1 c 0
2 a -1
3 b 1
4 a -1
5 c 0
6 a -1
7 c 3
8 a -1
9 b 1
10 a -1
11 c 0
12 a -1
13 b 7
14 a -1
15 c 0
16 a -1
17 c 3
18 a -1
19 b 1
20 a -1
21 c 0
22 a -1
23 c 13
24 a -1
25 b 1

1 point off
per error
limited to 5
pts per each
table

3. Give the result of both KMP failure link construction methods for the following sequence. (10 points)

0 a

1 c

2 a

3 b

4 a

5 c

6 a

7 c

8 a

9 b

10 a

11 c

12 a

13 b

14 a

15 c

16 a

17 c

18 a

19 b

20 a

21 c

22 a

23 c

24 a

25 b

A B 7 included in MST
B C 6 included in MST
C D 3 included in MST
E I 2 included in MST
E F 9 included in MST
G H 1 included in MST
J K 4 included in MST
L M 5 included in MST
numTrees is 5

B L 8 included in MST
F H 10 included in MST
K L 12 included in MST
numTrees is 2

D E 13 included in MST
numTrees is 1

Sum of weights of spanning edges 80

4. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (10 points)

