AVL Trees
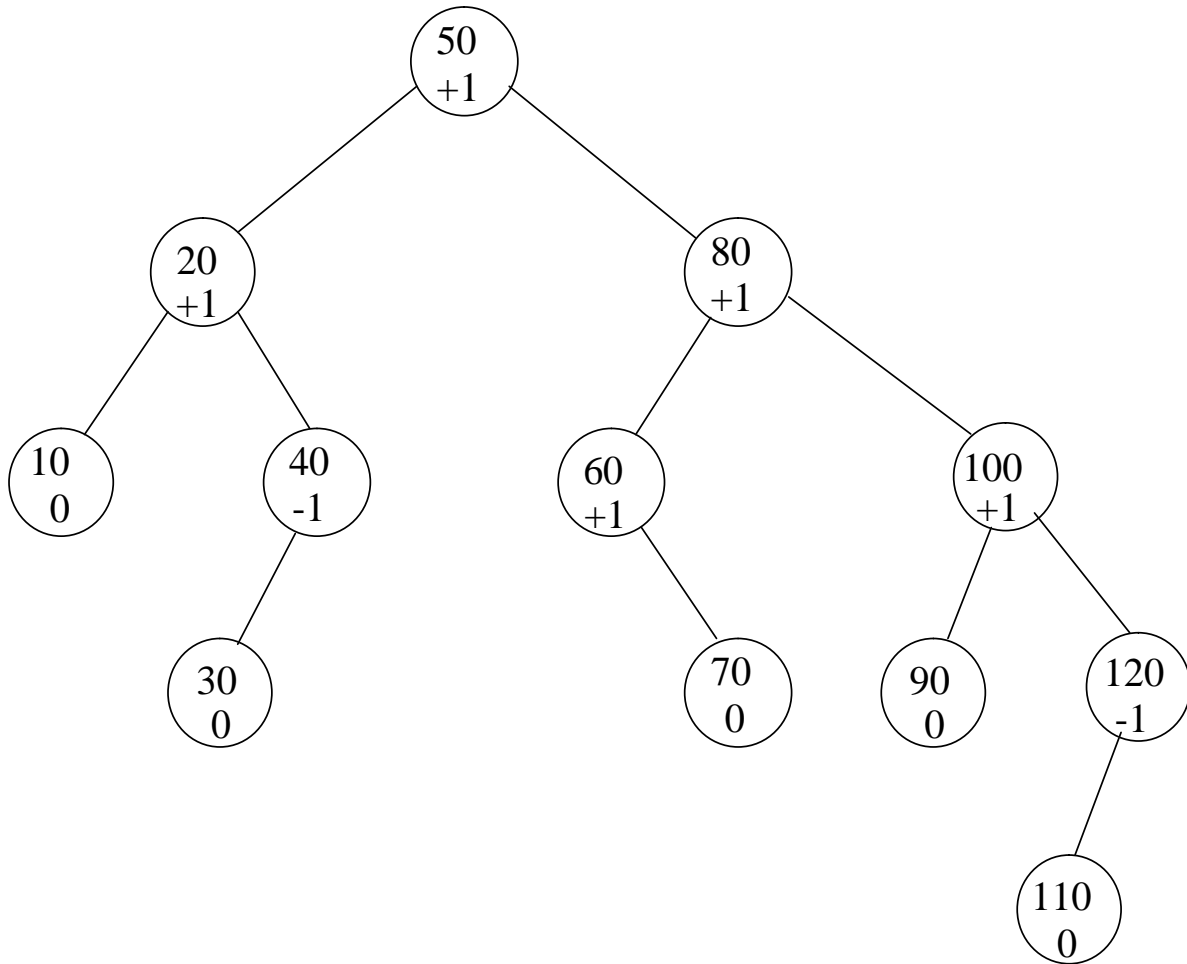
1. Each node stores the difference of the heights (known as the <u>balance factor</u>) of the right and left subtrees rooted by the children:

$$height_{right} - height_{left}$$

```
                          50
                          +1

          20                          80
          +1                          +1

    10          40            60              100
    0           -1            +1              +1

              30                  70      90        120
              0                   0       0         -1

                                                 110
                                                  0
```

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2. A balance factor must be +1, 0, -1 (leans right, "balanced", leans left).

3. An insert is implemented by:

   a. Attaching a leaf

   b. Rippling changes to balance factor:

      1. Right child ripple

         Parent.Bal = 0 ⟹ +1 and ripple to parent

         Parent.Bal = -1 ⟹ 0 to complete insertion

         Parent.Bal = +1 ⟹ +2 and ROTATION to complete insertion

      2. Left child ripple

         Parent.Bal = 0 ⟹ -1 and ripple to parent
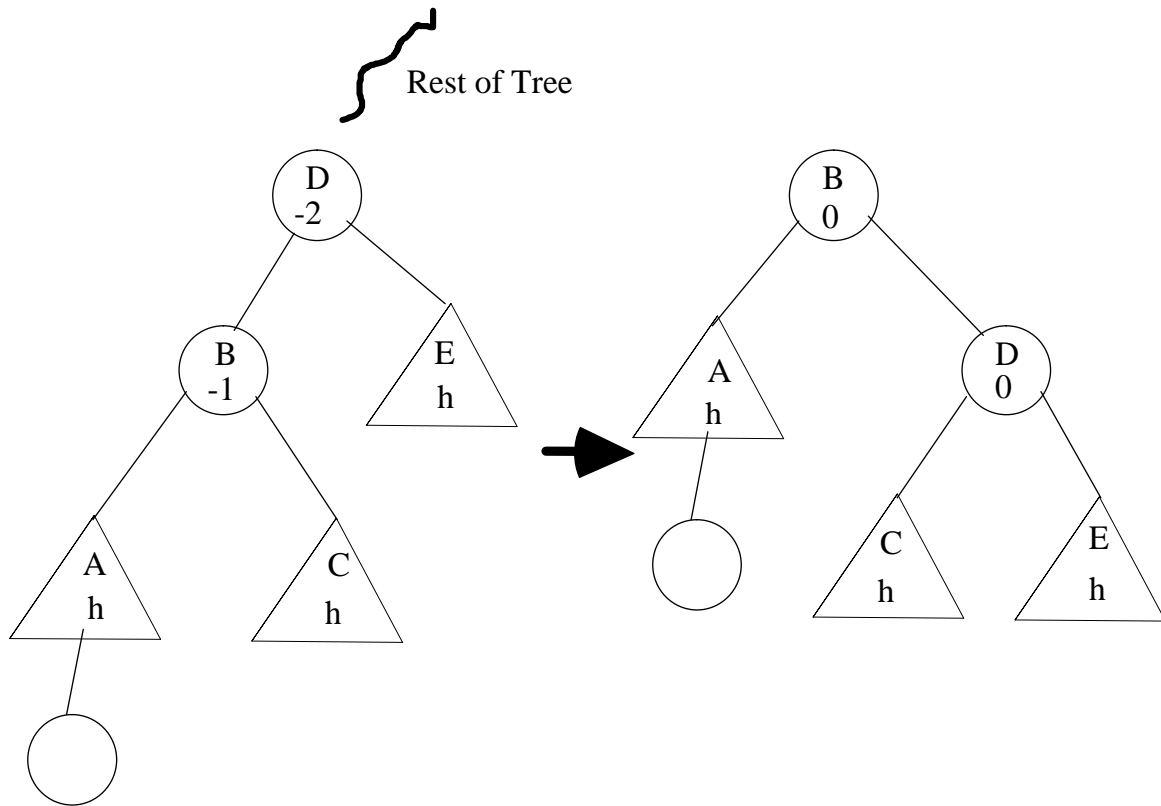
         Parent.Bal = +1 ⟹ 0 to complete insertion

         Parent.Bal = -1 ⟹ -2 and ROTATION to complete insertion

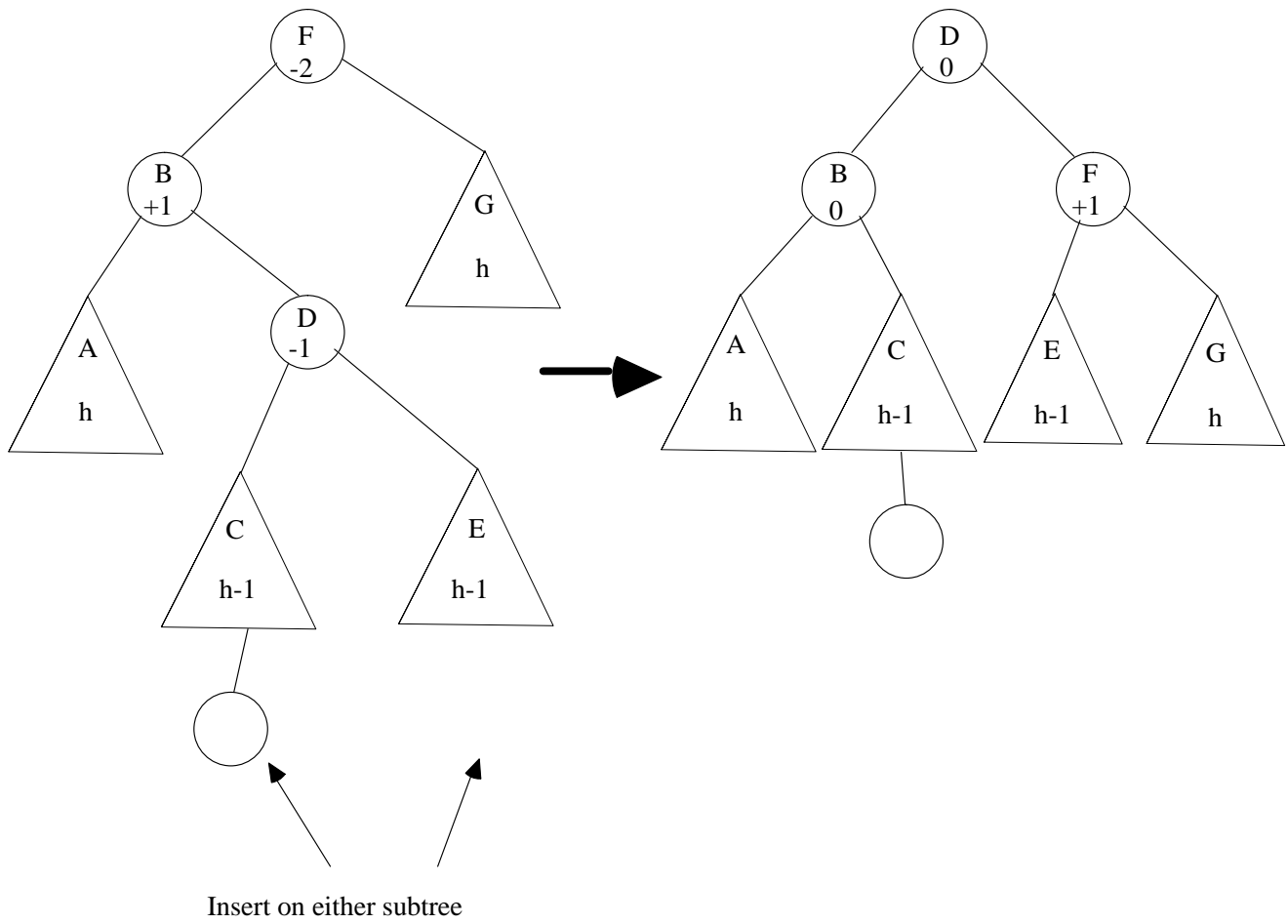===

4. Rotations

   a. Single (LL) - right rotation at D

   Rest of Tree

   Restores height of subtree to pre-insertion number of levels

   RR case is symmetric

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b. Double (LR)

Insert on either subtree

Restores height of subtree to pre-insertion number of levels

RL case is symmetric

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