CSE 5311: Homework 1

1. Find the optimal binary search tree for the following keys and probabilities. Draw the optimal subtree for all members of each family.

key	q_i	p_i
	.01	
5	.02	.03
10	.08	.16
15	.20	.20
20	.2	.1

2. Prove that the optimal static list for n elements with Zipf's distribution has an average probe length of n/H_n.

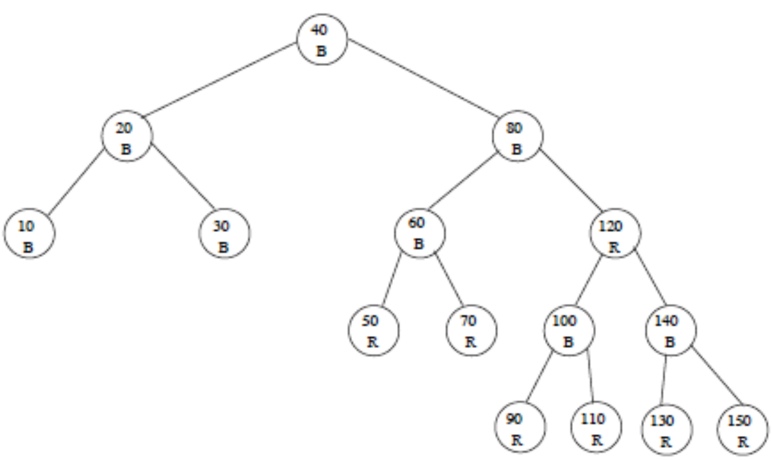
3. Find an asymptotic upper bound on the recurrence $T(n) = T(n/5) + n$ by using substitution.

4. Find an asymptotic upper bound on the recurrence $T(n) = 3T(n/4) + n/2$ by iteration.

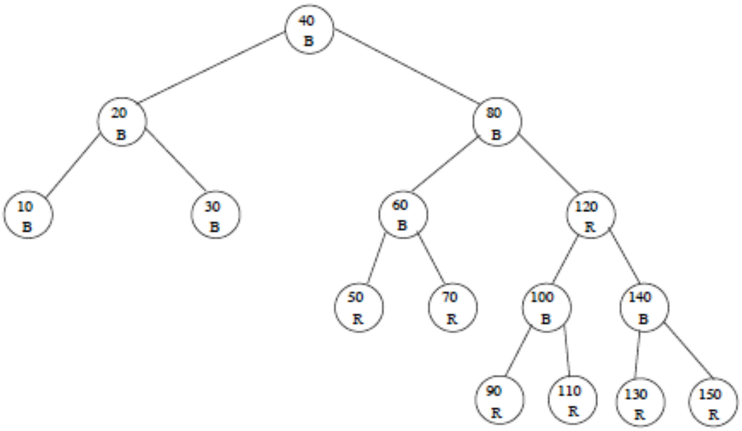
5. Use the substitution method to show that $f(n) = n$ is the asymptotic upper and lower bound on the recurrence $T(n) = 3T(n/3) + 1$.

6. Use the iteration method to show that $f(n)=n$ is an asymptotic upper bound on $T(n) = 3T(n/3) + 1$.

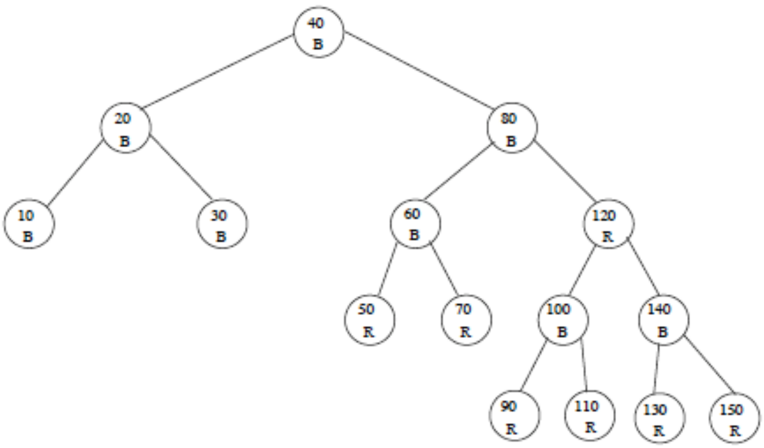
7. Insert 160 into the following red-black tree. Indicate the steps needed to restore the red-black tree properties.



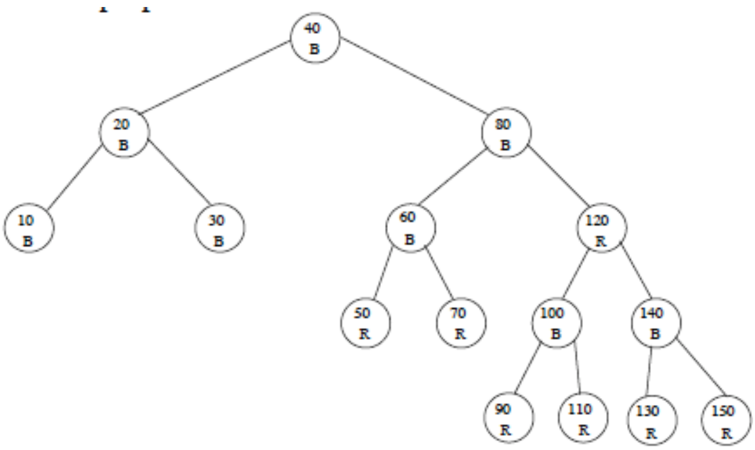
8. Insert 95 into the following red-black tree. Indicate the steps needed to restore the red-black tree properties.



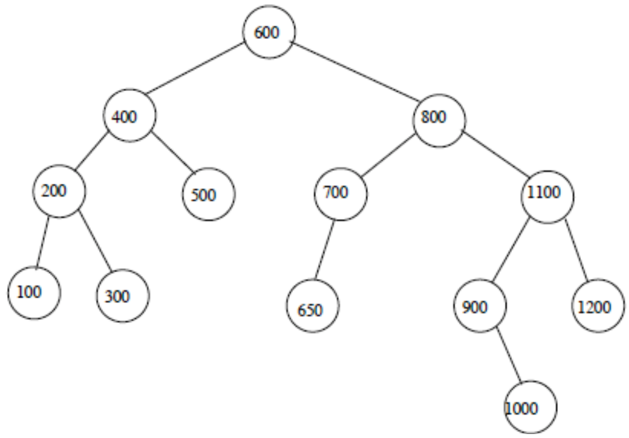
9. Delete 80 from the following red-black tree. Indicate the steps needed to restore the red-black tree properties.



10. Delete 30 from the following red-black tree. Indicate the steps needed to restore the red-black tree properties.



11. Suppose you are given the following AVL tree:



First, insert: 850, 875, 750, 725, 950, 450, 550, 575, 425, 475, 412

Then, delete: 412, 1100, 550, 800

12. Use the master method to give tight asymptotic bounds for:

    a. $T(n)=4T(n/2) + n$
    b. $T(n)=4T(n/2) + n^2$
    c. $T(n)=4T(n/2) + n^3$

13. The recurrence $T(n)=7T(n/2) + n^2$ describes the running time of an algorithm A. A competing algorithm A' has a running time of $T'(n)=aT'(n/4)+n^2$. What is the largest integer value for *a* such that A' is asymptotically faster than A?

14. C-1

15. 9.3-1

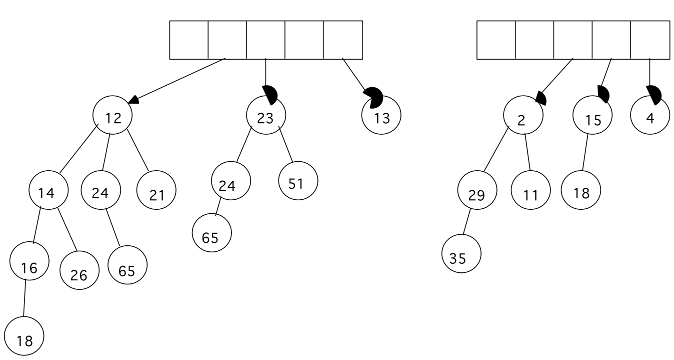
16. 11.4-1.

17. 14.2-2 (black-heights)

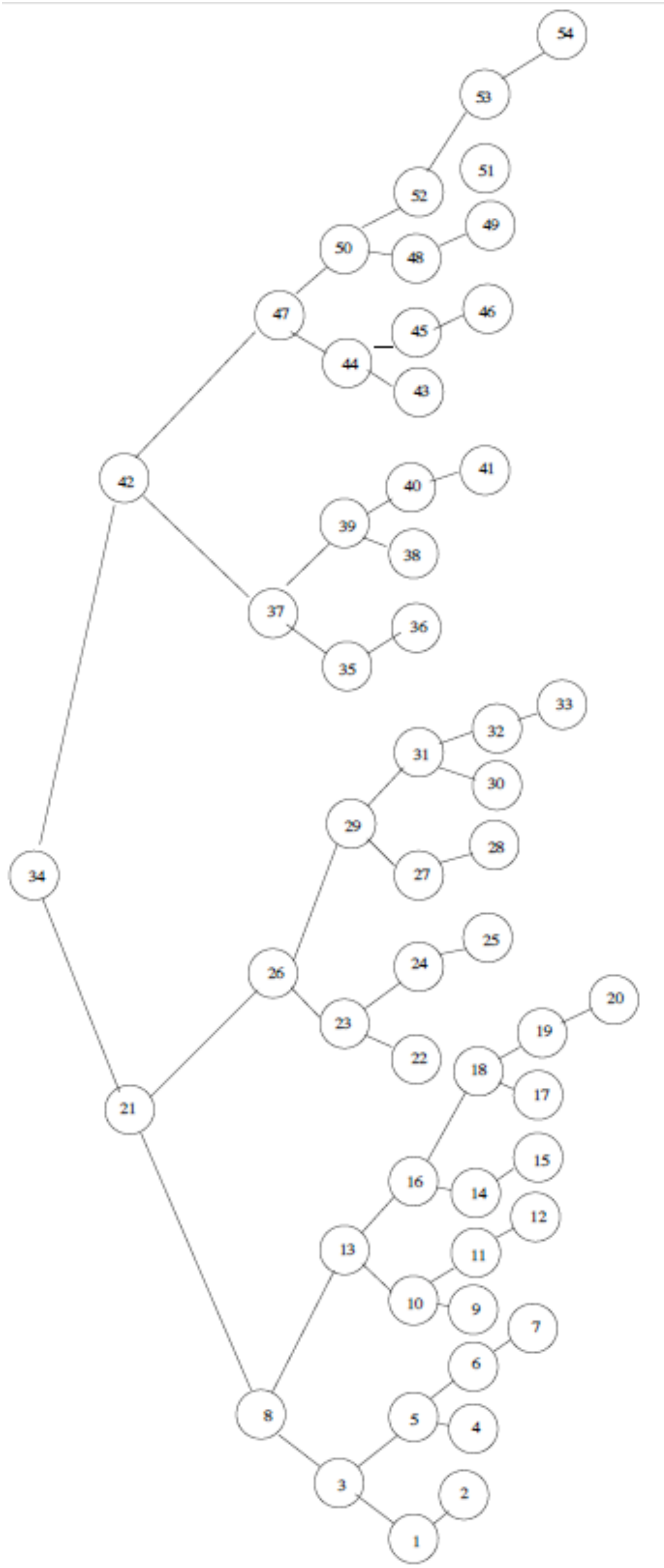
18. 14.2-2 (depths)

19. Why does insertion into a binomial queue take O(1) amortized time?

20. Merge the following binomial queues:



21. Delete 36 from the following AVL tree. Show all rotations you performed.



22. Consider the following information for constructing an optimal binary search tree.

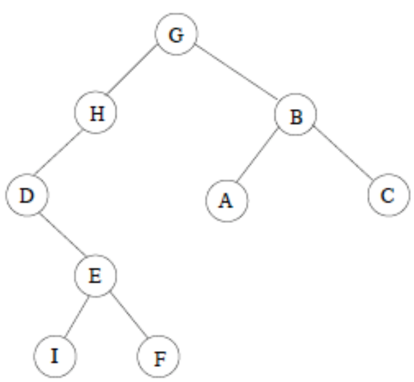
n = 5
q[0] = 0.2
key[1] = 10

p[1] = 0.09
q[1] = 0.2
key[2] = 20
p[2] = 0.1
q[2] = 0.03
key[3] = 30
p[3] = 0.2
q[3] = 0.04
key[4] = 40
p[4] = 0.02
q[4] = 0.01
key[5] = 50
p[5] = 0.05
q[5] = 0.06
w[0,0]= 0.2000
w[0,1]= 0.4900
w[0,2]= 0.6200
w[0,3]= 0.8600
w[0,4]= 0.8900
w[0,5]= 1.0000
w[1,1]= 0.2000
w[1,2]= 0.3300
w[1,3]= 0.5700
w[1,4]= 0.6000
w[1,5]= 0.7100
w[2,2]= 0.0300
w[2,3]= 0.2700
w[2,4]= 0.3000
w[2,5]= 0.4100
w[3,3]= 0.0400
w[3,4]= 0.0700
w[3,5]= 0.1800
w[4,4]= 0.0100
w[4,5]= 0.1200
w[5,5]= 0.0600
building c(0,2) using roots 1 thru 2
building c(1,3) using roots 2 thru 3
building c(2,4) using roots 3 thru 4
building c(3,5) using roots 4 thru 5
building c(0,3) using roots 1 thru 2
building c(1,4) using roots 2 thru 3
building c(2,5) using roots 3 thru 5
building c(0,4) using roots 2 thru 2
building c(1,5) using roots 2 thru 3
building c(0,5) using roots ???? thru ????
Average probe length is ????
trees in parenthesized prefix
C(0 0) cost 0.0000
C(1 1) cost 0.0000
C(2 2) cost 0.0000
C(3 3) cost 0.0000

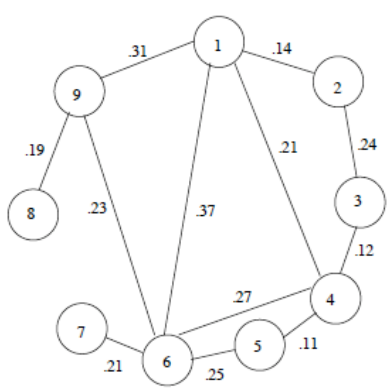
C(4 4) cost	0.0000
C(5 5) cost	0.0000
C(0 1) cost	0.4900 10
C(1 2) cost	0.3300 20
C(2 3) cost	0.2700 30
C(3 4) cost	0.0700 40
C(4 5) cost	0.1200 50
C(0 2) cost	0.9500 10(,20)
C(1 3) cost	0.8400 20(,30)
C(2 4) cost	0.3700 30(,40)
C(3 5) cost	0.2500 50(40,)
C(0 3) cost	1.6200 20(10,30)
C(1 4) cost	0.9700 20(,30(,40))
C(2 5) cost	0.6600 30(,50(40,))
C(0 4) cost	1.7500 20(10,30(,40))
C(1 5) cost	1.2900 30(20,50(40,))
C(0 5) cost	????? ??????

Construct the final optimal binary search tree and give its cost.

23. Show that postorder traversal of a binary tree takes θ(1) amortized time per node processed ("initialize" and "successor" operations) by giving a simple potential function. Demonstrate that your function works on the following tree. Now do a similar derivation for preorder traversal.



24. Find the MST of the following graph using the method based on Warshall's algorithm:



25. 23.2-4

26. 23.2-5