CSE 5311
Test 1 - Closed Book
Spring 2004

Name ______________________________

Student ID # _________________________

Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. To reduce the probability of having any collisions to < 0.5 when hashing n keys, the table should have at least this number of elements.

   A. n
   B. n ln n
   C. $n^2$
   D. $n^3$

2. Path compression is used in which algorithm?

   A. DECREASE-KEY (e.g. CASCADING-CUT) for Fibonacci heaps
   B. FIND for disjoint sets
   C. UNION for disjoint sets
   D. UNION for Fibonacci heaps

3. When performing selection in worst-cast linear time, roughly what fraction of the set of n keys is kept (in the worst case) for the next round?

   A. 10%
   B. 20%
   C. 30%
   D. 70%

4. Which Fibonacci heap operation has O(log n) actual cost?

   A. FIB-HEAP-DECREASE-KEY
   B. FIB-HEAP-DELETE
   C. FIB-HEAP-EXTRACT-MIN
   D. FIB-HEAP-UNION

5. The main difference between MTF and OPT for self-organizing linear lists is:

   A. MTF counts inversions
   B. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time
   C. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time
   D. OPT can do transpositions

6. The minimum number of nodes in a tree in a Fibonacci heap where the root has 7 children is:

   A. 34
   B. 61
   C. 121
   D. 128

7. The maximum value of the potential function when comparing MTF and OPT on a list with $n$ elements is:

   A. $n$
   B. $\frac{n^2-n}{2}$
   C. $\frac{n^2+n}{2}$
   D. $n^2-n$

8. In the worst case, the number of rotations for inserting a key in a treap with n keys is:

   A. $\theta(1)$
   B. $\theta(\log n)$
   C. $\theta(n)$
   D. $\theta(n \log n)$

9. Slow convergence toward the optimal fixed ordering is a property of which technique?

   A. Count
   B. Move-ahead-k
   C. Move-to-front
   D. Transpose

10. Sorting the edges is a property of which minimum spanning tree technique?

    A. Boruvka
    B. Kruskal
    C. Prim
    D. Path-based (Warshall)

Long Answer

1. Suppose there are 20 coupon types for the coupon collecting problem and you have already obtained 18 of the coupon types. How many boxes of cereal do you expect (mathematically) to open to get the remaining two coupon types? (5 points)

2. Suppose there are 20 coupon types for the coupon collecting problem. Each of the coupon types is identified by an integer in the range 1 to 20. If you are given two random cereal boxes, what is the probability (ignoring the order in which the coupons were obtained) that the two coupons have consecutive numbers, i.e. some i and i + 1? (5 points)

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1. Suppose you wish to implement a verifier to test if a given subset (with V - 1 edges) of the set of edges for a graph is a minimum spanning tree. Assuming you already have the code below for Kruskal's algorithm, explain how it should be modified to perform this verification. You may assume that each `struct` in `edgeTab` has a field `subset` that flags whether this edge is in the alleged MST. (15 points)

```
. . .

main()
{
. . .

qsort(edgeTab,numEdges,sizeof(edgeType),weightAscending);
for (i=0;i<numEdges;i++)
{
  root1=find(edgeTab[i].tail);
  root2=find(edgeTab[i].head);
  if (root1==root2)
    printf("%d %d %d discarded\n",edgeTab[i].tail,edgeTab[i].head,
      edgeTab[i].weight);
  else
  {
    printf("%d %d %d included\n",edgeTab[i].tail,edgeTab[i].head,
      edgeTab[i].weight);
    makeEquivalent(root1,root2);
  }
}
if (numTrees!=1)
  printf("MST does not exist\n");
}
```

2. Evaluate the following recurrences using the master method. (15 points)

a. $T(n) = T(0.7n) + n$

b. $T(n) = T(0.7n) + 1$

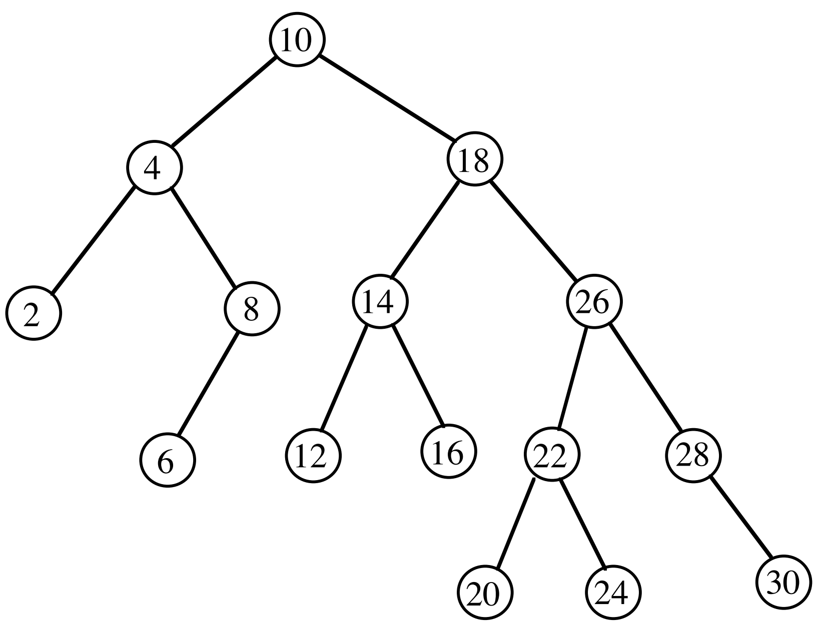
c. $T(n) = 16T\left(\frac{n}{2}\right) + n^3$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```
n=6;            q[2]=0.03;      key[5]=50;
q[0]=0.01;      key[3]=30;      p[5]=0.02;
key[1]=10;      p[3]=0.2;       q[5]=0.04;
p[1]=0.19;      q[3]=0.04;      key[6]=60;
q[1]=0.02;      key[4]=40;      p[6]=0.12;
key[2]=20;      p[4]=0.2;       q[6]=0.03;
p[2]=0.1;       q[4]=0.0;       w[0][0]=0.010000
```

```
w[0][1]=0.220000
w[0][2]=0.350000
w[0][3]=0.590000
w[0][4]=0.790000
w[0][5]=0.850000
w[0][6]=1.000000
w[1][1]=0.020000
w[1][2]=0.150000
w[1][3]=0.390000
w[1][4]=0.590000
w[1][5]=0.650000
w[1][6]=0.800000
w[2][2]=0.030000
w[2][3]=0.270000
w[2][4]=0.470000
w[2][5]=0.530000
w[2][6]=0.680000
w[3][3]=0.040000
w[3][4]=0.240000
w[3][5]=0.300000
w[3][6]=0.450000
w[4][4]=0.000000
w[4][5]=0.060000
w[4][6]=0.210000
w[5][5]=0.040000
w[5][6]=0.190000
w[6][6]=0.030000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 1 thru 3
Building c(1,4) using roots 3 thru 3
Building c(2,5) using roots 3 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 2 thru 3
Building c(1,5) using roots 3 thru 4
Building c(2,6) using roots 4 thru 4
Building c(0,5) using roots 3 thru 3
Building c(1,6) using roots 3 thru 4
Building c(0,6) using roots ? thru ?
Counts - root trick 29 without root
    trick 50
Average probe length is ????
trees in parenthesized prefix
c(0,0) cost 0.000000
c(1,1) cost 0.000000
c(2,2) cost 0.000000
c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.220000 10
c(1,2) cost 0.150000 20
c(2,3) cost 0.270000 30
c(3,4) cost 0.240000 40
c(4,5) cost 0.060000 50
c(5,6) cost 0.190000 60
c(0,2) cost 0.500000 10(,20)
c(1,3) cost 0.540000 30(20,)
c(2,4) cost 0.710000 30(,40)
c(3,5) cost 0.360000 40(,50)
c(4,6) cost 0.270000 60(50,)
c(0,3) cost 1.080000 20(10,30)
c(1,4) cost 0.980000 30(20,40)
c(2,5) cost 0.860000 40(30,50)
c(3,6) cost 0.720000 40(,60(50,))
c(0,4) cost 1.530000 30(10(,20),40)
c(1,5) cost 1.160000 30(20,40(,50))
c(2,6) cost 1.220000 40(30,60(50,))
c(0,5) cost 1.710000 30(10(,20),40(,50))
c(1,6) cost 1.610000 40(30(20,),60(50,))
c(0,6) cost ???????? ??????????????
```

4. Insert 23 into the following AVL tree to preserve the AVL properties. (10 points)

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Test 2 - Closed Book
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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following problems is not NP-complete? (Assume $P \neq NP$)

   A. 3-satisfiability
   B. Testing if a graph is 3-colorable
   C. Testing if a table is in sorted order
   D. Testing if the number of colors needed to edge color a graph is the degree of the graph

2. How many times will -1 occur in the style 1 fail link table for the pattern `abcabd`?

   A. 1
   B. 2
   C. 3
   D. 4

3. How many times will -1 occur in the style 2 fail link table for the pattern `abcabd`?

   A. 1
   B. 2
   C. 3
   D. 4

4. Suppose that you have devised a reduction from a known NP-complete problem A to a problem B in NP. Later you discover that the reduction takes exponential time. What conclusion may be drawn?

   A. $P \neq NP$
   B. Problem B is not NP-complete.
   C. You need a reduction that takes linear time to prove that B is NP-complete.
   D. You need a reduction that takes polynomial time to prove that B is NP-complete.

5. Which of the following is not part of the Gale-Shapley algorithm?

   A. Men issue proposals
   B. Finding rotations
   C. Women break engagements to accept proposals from their more-preferred men
   D. Women accept proposals if unengaged

6. Which of the following is not true regarding Strassen's algorithm?

   A. It is not possible to have an asymptotically faster algorithm.
   B. It requires more space than the everyday method.
   C. It uses $\Theta\left(n^{\lg 7}\right)$ scalar additions when multiplying two n x n matrices.
   D. It uses $\Theta\left(n^{\lg 7}\right)$ scalar multiplications when multiplying two n x n matrices.

7. Which algorithm is defined using the notions of left-turn and right-turn?

   A. Closest points in 2-d space

B. Graham scan
C. Jarvis march
D. Suffix array construction

8. Which of the following is not a condition for performing a push operation from u to v?

A. the height at u is one more than the height at v
B. (u, v) has capacity > flow
C. u has excess > 0
D. v has a path to the sink in the residual graph

9. Which of the following does not have an approximation algorithm?

A. Bin packing
B. Edge coloring
C. Traveling salesperson
D. Vertex covering

10. The length of a longest strictly increasing subsequence for `1 4 3 2 3 4 3 6 3 4` is:

A. 4
B. 5
C. 6
D. 7

11. Which longest common subsequence method is potentially the most time-consuming?

A. Compact version of dynamic programming
B. Method based on subsequence indices and longest strictly increasing subsequence
C. Ordinary dynamic programming

12. Consider the technique for determining articulation points using depth-first search. If a vertex has no predecessors (first vertex discovered for a "restart"), it can be an articulation point only if

A. there are no outgoing tree edges
B. there is one outgoing tree edge
C. there is more than one outgoing tree edge
D. there is no back edge returning to this vertex

13. Which of the following problems uses some edges with non-unit capacity when translated to a network flow problem?

A. Bipartite matching
B. Edge connectivity
C. Minimum vertex cover for bipartite graph
D. Vertex connectivity

14. Which string search method is potentially the most time-consuming?

A. Karp-Rabin
B. KMP with fail 1 links
C. KMP with fail 2 links
D. Z table

15. What is the maximum number of times that an edge can become critical in the Edmonds-Karp method?

A. Once
B. Twice
C. (V-2)/2
D. VE

16. Suppose you have a set of points in Euclidean 2-d space. Give an algorithm for finding a ρ-approximation for the minimum traveling salesperson path. Be sure to give the value of ρ. (5 points)

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1. Use the Gale-Shapley algorithm to determine the male-optimal solution for the following instance of the stable marriages problem. In addition, show the preference lists at termination. Note that the preference lists are given left-to-right. (15 points)

```
male preference lists are:

1:  1  2  3  4  5

2:  2  3  4  5  1

3:  3  4  2  5  1

4:  1  3  2  5  4

5:  3  5  4  2  1

female preference lists are:

1:  4  3  5  2  1

2:  2  3  5  4  1

3:  1  2  5  4  3

4:  3  2  4  5  1

5:  2  4  3  1  5
```

2. The *hitting set* problem gives a collection C of subsets of a set S and a positive integer k. We would like to know if there is a subset S' of S with |S'| ≤ k such that S' contains at least one element from each subset in C.

   Give a proof that *hitting set* is NP-complete by using the fact that vertex cover is NP-complete. (15 points)

3. List the lift and push operations for the preflow-push algorithm on the following network. In addition, give a minimum cut. (20 points)