CSE 5311 Name _____

Test 1 - Closed Book

Summer 2004 Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. Which of the following random permutation techniques is not uniform?
   A. PERMUTE-BY-SORTING
   B. PERMUTE-WITH-ALL
   C. RANDOMIZE-IN-PLACE
   D. Randomly choose one of the n! ranks and then apply unranking
2. Fibonacci trees are used in the analysis of which technique?
   A. AVL
   B. red-black
   C. splay
   D. treap
3. Which analysis of self-organizing linear search did not involve probabilities?
   A. Count vs. optimal fixed order
   B. Lab Assignment 1
   C. Markov analysis of move-to-front for Zipf's distribution
   D. Move-to-front (online) vs. optimal offline
4. The median of a set of n numbers may be found optimally in time:
   A. $\theta(\log n)$
   B. $\theta(n)$
   C. $\theta(n \log n)$
   D. $\theta(n^2)$
5. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:
   A. the sum of all keys in the entire tree
   B. the sum of all keys in the left subtree
   C. the sum of all keys that are no larger than the stored key
   D. the sum of all keys stored in the subtree rooted by this node
6. When using Brent's rehash, the number of previously inserted keys that may move is:
   A. 1
   B. 2
   C. $\frac{1}{\alpha}$
   D. H_m, where m is the number of stored keys
7. Assuming that a random n-permutation is provided, the expected number of hires for the hiring problem is:
   A. 2
   B. H_n
   C. $\frac{1}{n}$
   D. n
8. Which data structure is not used to implement a dictionary?
   A. AVL tree
   B. Red-black tree
   C. Self-organizing list
   D. Union-find

9. Which binary search tree method stores the same information in each node as an unbalanced binary search tree, yet performs retrieval, insertion, and deletion in O(log n) amortized time?
   A. AVL
   B. red-black
   C. splay
   D. treap
10. Which priority queue implementation generalizes binary heaps by increasing the branching?
   A. Binomial heaps
   B. d-heaps
   C. Fibonacci heaps
   D. Leftist heaps
11. Suppose you already have 15 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 15 you already have?
   A. 3
   B. 4
   C. 5
   D. 15
12. When is path compression used?
   A. After an insertion into any type of balanced binary search tree.
   B. After an insertion into a splay tree.
   C. After a FIND operation.
   D. After a UNION operation.

13. How many nodes does a B_5 tree in a binomial heap have? (2 points)

CSE 5311 Name _____

Test 1 - Open Book

Summer 2004 Student ID # _____

1. Give the range of possible heights for an AVL tree with 100 keys. (10 points)
2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (15 points)

a. $T(n) = 2T\left(\frac{n}{4}\right) + 1$

b. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

c. $T(n) = 2T\left(\frac{n}{4}\right) + n^2$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (15 points)

```
n=6;
q[0]=0.01;
key[1]=10;
p[1]=0.09;
q[1]=0.02;
key[2]=20;
p[2]=0.1;
q[2]=0.03;
key[3]=30;
p[3]=0.2;
q[3]=0.04;
key[4]=40;
p[4]=0.2;
q[4]=0.0;
key[5]=50;
p[5]=0.12;
q[5]=0.04;
key[6]=60;
p[6]=0.12;
q[6]=0.03;
w[0][0]=0.010000
w[0][1]=0.120000
w[0][2]=0.250000
w[0][3]=0.490000
w[0][4]=0.690000
w[0][5]=0.850000
w[0][6]=1.000000
w[1][1]=0.020000
w[1][2]=0.150000
w[1][3]=0.390000
w[1][4]=0.590000
w[1][5]=0.750000
w[1][6]=0.900000
```

```
w[2][2]=0.030000
w[2][3]=0.270000
w[2][4]=0.470000
w[2][5]=0.630000
w[2][6]=0.780000
w[3][3]=0.040000
w[3][4]=0.240000
w[3][5]=0.400000
w[3][6]=0.550000
w[4][4]=0.000000
w[4][5]=0.160000
w[4][6]=0.310000
w[5][5]=0.040000
w[5][6]=0.190000
w[6][6]=0.030000
Building c(0,2)
    using roots 1
    thru 2
Building c(1,3)
    using roots 2
    thru 3
Building c(2,4)
    using roots 3
    thru 4
```

```
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 2 thru 3
Building c(1,4) using roots 3 thru 3
Building c(2,5) using roots 3 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 3 thru 3
Building c(1,5) using roots 3 thru 4
Building c(2,6) using roots 4 thru 5
Building c(0,5) using roots 3 thru 4
Building c(1,6) using roots 4 thru 4
Building c(0,6) using roots ? thru ?
Counts - root trick 28 without root
    trick 50
Average probe length is ???
trees in parenthesized prefix
c(0,0) cost 0.000000
c(1,1) cost 0.000000
c(2,2) cost 0.000000
c(3,3) cost 0.000000
c(4,4) cost 0.000000
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.120000 10
```

```
c(1,2) cost 0.150000 20
c(2,3) cost 0.270000 30
c(3,4) cost 0.240000 40
c(4,5) cost 0.160000 50
c(5,6) cost 0.190000 60
c(0,2) cost 0.370000 20(10,)
c(1,3) cost 0.540000 30(20,)
c(2,4) cost 0.710000 30(,40)
c(3,5) cost 0.560000 40(,50)
c(4,6) cost 0.470000 60(50,)
c(0,3) cost 0.860000 30(20(10,),)
c(1,4) cost 0.980000 30(20,40)
c(2,5) cost 1.060000 40(30,50)
c(3,6) cost 0.980000 50(40,60)
c(0,4) cost 1.300000 30(20(10,),40)
c(1,5) cost 1.450000 40(30(20,),50)
c(2,6) cost 1.520000 40(30,60(50,))
c(0,5) cost 1.780000 30(20(10,),40(,50))
c(1,6) cost 1.910000 40(30(20,),60(50,))
c(0,6) cost ??? ??????????
```

4. Suppose all $2^k - 1$ nodes, along with the sentinel, in a red-black tree are colored black. Explain what will happen if any key is deleted. (10 points)

CSE 5311 Name _____

Test 2 - Closed Book

Summer 2004 Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following problems is not NP-complete? (Assume P ≠ NP)
   A. Testing if a graph is 3-colorable
   B. Testing if an undirected graph has a Hamiltonian cycle
   C. Testing if the number of colors needed to edge color a graph is the degree of the graph
   D. 2-satisfiability
2. How many times will -1 occur in the style 1 fail link table for the pattern `abaabaab`?
   A. 1
   B. 2
   C. 3
   D. 4
3. How many times will -1 occur in the style 2 fail link table for the pattern `abaabaab`?
   A. 1
   B. 2
   C. 3
   D. 4
4. Which of the following do Kruskal's and Boruvka's algorithms have in common?
   A. They can take advantage of Union-Find structures
   B. They do not work correctly unless all edge weights are unique
   C. They use min-heaps

D. They use the edges in ascending weight order

5. Under what condition does an instance of stable marriages have only one solution?
   A. Every male preference list is identical to some female preference list
   B. No female appears at the beginning of multiple male preference lists
   C. The male-optimal solution and female-optimal solution are the same
   D. There is only one rotation
6. Which of the following algorithms does not preprocess using a sort?
   A. Closest points in 2-d space
   B. Graham scan
   C. Jarvis march
   D. Kruskal
7. Which algorithm is defined using the notions of left-turn and right-turn?
   A. Closest points in 2-d space
   B. Graham scan
   C. Jarvis march
   D. Longest common subsequence
8. Which of the following is not a condition for performing a lift operation at u?
   A. all exiting saturated edges have heads with height ≥ height of u
   B. all exiting unsaturated edges have heads with height ≥ height of u
   C. u has excess > 0
9. Which of the following does not have a polynomial-time approximation algorithm?
   A. Bin packing
   B. Edge coloring
   C. Traveling salesperson with triangle inequality
   D. Vertex coloring
10. The length of a longest monotone increasing subsequence for `1 4 3 2 3 4 3 6 3 4` is:
    A. 4
    B. 5
    C. 6
    D. 7
11. Which longest common subsequence method is potentially the most space-consuming?
    A. Compact version of dynamic programming
    B. Method based on subsequence indices and longest strictly increasing subsequence
    C. Ordinary dynamic programming
12. Consider the technique for determining articulation points using depth-first search. If a vertex has no predecessors (first vertex discovered for a "restart"), it can be an articulation point only if
    A. there are no incident tree edges
    B. there is one incident tree edge
    C. there is more than one incident tree edge
    D. there is no back edge incident to this vertex
13. Which minimum spanning tree algorithm is the slowest?
    A. Boruvka
    B. Kruskal
    C. Prim
    D. Warshall
14. Which string search method is potentially the most time-consuming?
    A. Karp-Rabin
    B. KMP with fail 1 links
    C. KMP with fail 2 links

D. Z table

15. What is the minimum increase in the tail's distance from the source between the first and second times that an edge becomes critical in the Edmonds-Karp method?
    A. 1
    B. 2
    C. (V-2)/2
    D. VE

16. Explain how the Z Algorithm ("Fundamental String Preprocessing") may be used to find all occurences of string 1 within string 2. (5 points)

CSE 5311 Name _____

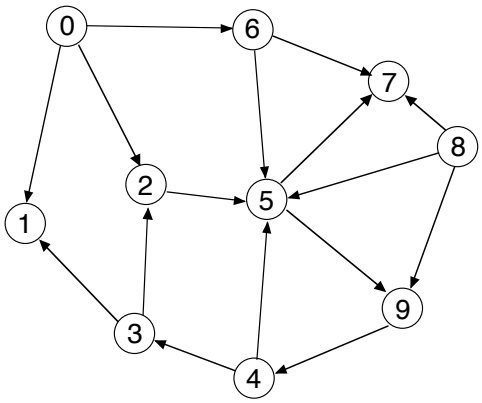
Test 2 - Open Book

Summer 2004 Student ID # _____

1. Use the Gale-Shapley algorithm to determine the male-optimal solution for the following instance of the stable marriages problem. In addition, show the preference lists at termination. Note that the preference lists are given left-to-right. (15 points)

```
male preference lists are:
1: 1 2 3 4 5
2: 3 2 1 5 4
3: 3 4 2 5 1
4: 1 3 2 5 4
5: 2 3 4 5 1
female preference lists are:
1: 4 5 3 2 1
2: 1 2 3 4 5
3: 2 3 4 5 1
4: 3 2 1 5 4
5: 4 5 2 1 3
```

2. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward. Assume that the adjacency lists are ***ordered***. Write your answer in the tables below. 15 points



Vertex	Start	Finish
0	1	____
1	____	____
2	____	____
3	____	____
4	____	____
5	____	____
6	____	____
7	____	____

Edge	Type
0 1	____
0 2	____
0 6	____
2 5	____
3 1	____
3 2	____
4 3	____
4 5	____

Edge	Type
6 5	____
6 7	____
8 5	____
8 7	____
8 9	____
9 4	____

8	____	____	5	7	____
9	____	____	5	9	____

3. ***Clearly*** list the lift and push operations for the preflow-push algorithm on the following network. In addition, give a minimum cut. (20 points)