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Test 1 - Closed Book
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Multiple Choice. Write your answer to the LEFT of each problem. 4 points each

1. Assuming that |E| > |V|, the time for Prim's algorithm using a Fibonacci heap is:
   A. O(|E|)
   B. O(|E| log |E|)
   C. O(|E| log |V|)
   D. O(|V| log |V| + |E|)
2. What is the worst-case number of rotations when performing deletion on an AVL tree?
   A. $\Theta(1)$
   B. $\Theta(\log n)$
   C. $\Theta(n)$
   D. No rotations are ever used
3. The maximum value of the potential function when comparing MTF and OPT on a list with *n* elements is:
   A. n
   B. $\frac{n^2-n}{2}$
   C. $\frac{n^2+n}{2}$
   D. n^2-n
4. When performing selection in worst-cast linear time, roughly what fraction of the set of n keys is kept (in the worst case) for the next round?
   A. 10%
   B. 20%
   C. 30%
   D. 70%
5. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:
   A. the sum of all keys in the entire tree
   B. the sum of all keys in the left subtree
   C. the sum of all keys that are no larger than the stored key
   D. the sum of all keys stored in the subtree rooted by this node
6. The worst-case placement of keys for an open-address method may be found using:
   A. Binary search on instances of unweighted bipartite matching
   B. Brent's method
   C. Perfect hashing
   D. Weighted bipartite matching by placing weight jP_i at $M_{iS_{ij}}$ for the probe sequence $S_{i1}, S_{i2}, \cdots, S_{im}$
7. Which of the following statements is true?
   A. A binary search tree may be assigned legal AVL balance factors if and only if it may be legally colored as a red-black tree.
   B. If a binary search tree may be assigned legal AVL balance factors, then it may be legally colored as a red-black tree.
   C. If a binary search tree may be legally colored as a red-black tree, then it may be assigned legal AVL balance factors.
   D. No binary search tree may be assigned both legal AVL balance factors and be legally colored as a red-black tree.
8. Path compression is used in which algorithm?
   A. DECREASE-KEY (e.g. CASCADING-CUT) for Fibonacci heaps
   B. FIND for disjoint sets
   C. UNION for disjoint sets
   D. UNION for Fibonacci heaps
9. Which priority queue is defined using the notion of null path length?
   A. Binary heap
   B. Binomial heap
   C. Fibonacci heap
   D. Leftist heap
10. Which situation is true regarding a cascade cut that produces c trees for a Fibonacci heap?
    A. Both the actual and amortized costs are O(1).

B. The actual cost is O(c). The amortized cost is O(1).
C. The actual cost is O(1). The amortized cost is O(c).
D. The potential can become negative.

11. Which of the following is not true regarding the amortized analysis of binary tree traversals?
A. INIT had an amortized cost of 0.
B. SUCC had an actual cost determined by the number of edges followed.
C. SUCC had an amortized cost of 2.
D. The potential was defined without regard for the type of traversal being performed.

12. Which of the following data structures offers similar capabilities and performance characteristics to skip lists?
A. AVL trees
B. Splay trees
C. Treap
D. Union-find with path compression

13. Draw an S_5 tree for a Fibonacci heap. (2 points)

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1. "During Disney's biggest celebration, find one of 50 character Wobblers inside specially-marked ***Kellogg's***® cereals!".

a. Assuming all Wobblers are equally likely to be the one that occurs in a box, what is the expected number of boxes to obtain all 50 Wobblers ? (3 points. You may leave your answer as an expression.)

b. Under the same assumption as a., what is the expected number of boxes to obtain just 25 different Wobblers (of the available 50)? (7 points. You may leave your answer as an expression.)

2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)

a. $T(n) = 8T\left(\frac{n}{2}\right) + n^4$

b. $T(n) = 8T\left(\frac{n}{2}\right) + n^2$

c. $T(n) = 8T\left(\frac{n}{2}\right) + n^3$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```
n=6;
q[0]=0.12;
key[1]=10;
p[1]=0.12;
q[1]=0.09;
key[2]=20;
p[2]=0.2;
q[2]=0.03;
key[3]=30;
p[3]=0.1;
q[3]=0.04;
key[4]=40;
p[4]=0.2;
q[4]=0.0;
key[5]=50;
p[5]=0.01;
q[5]=0.04;
key[6]=60;
p[6]=0.03;
q[6]=0.02;
w[0][0]=0.120000
w[0][1]=0.330000
w[0][2]=0.560000
w[0][3]=0.700000
w[0][4]=0.900000
w[0][5]=0.950000
w[0][6]=1.000000
w[1][1]=0.090000
w[1][2]=0.320000
w[1][3]=0.460000
w[1][4]=0.660000
w[1][5]=0.710000
w[1][6]=0.760000
w[2][2]=0.030000
w[2][3]=0.170000
w[2][4]=0.370000
w[2][5]=0.420000
w[2][6]=0.470000
w[3][3]=0.040000
w[3][4]=0.240000
w[3][5]=0.290000
w[3][6]=0.340000
w[4][4]=0.000000
w[4][5]=0.050000
w[4][6]=0.100000
w[5][5]=0.040000
w[5][6]=0.090000
w[6][6]=0.020000
Building c(0,2) using roots 1 thru 2
Building c(1,3) using roots 2 thru 3
Building c(2,4) using roots 3 thru 4
Building c(3,5) using roots 4 thru 5
Building c(4,6) using roots 5 thru 6
Building c(0,3) using roots 1 thru 2
Building c(1,4) using roots 2 thru 4
Building c(2,5) using roots 4 thru 4
Building c(3,6) using roots 4 thru 6
Building c(0,4) using roots 2 thru 2
Building c(1,5) using roots 2 thru 4
Building c(2,6) using roots 4 thru 4
Building c(0,5) using roots 2 thru 2
Building c(1,6) using roots 2 thru 4
Building c(0,6) using roots ? thru ?
Counts - root trick 31 without root trick
    50
Average probe length is ???
```

```
trees in
    parenthesized
    prefix
c(0,0) cost
0.000000
c(1,1) cost
0.000000
c(2,2) cost
0.000000
c(3,3) cost
0.000000
c(4,4) cost
0.000000
```

```
c(5,5) cost 0.000000
c(6,6) cost 0.000000
c(0,1) cost 0.330000 10
c(1,2) cost 0.320000 20
c(2,3) cost 0.170000 30
c(3,4) cost 0.240000 40
c(4,5) cost 0.050000 50
c(5,6) cost 0.090000 60
c(0,2) cost 0.880000 10(,20)
c(1,3) cost 0.630000 20(,30)
c(2,4) cost 0.540000 40(30,)
c(3,5) cost 0.340000 40(,50)
c(4,6) cost 0.150000 60(50,)
```

```
c(0,3) cost 1.200000 20(10,30)
c(1,4) cost 1.200000 20(,40(30,))
c(2,5) cost 0.640000 40(30,50)
c(3,6) cost 0.490000 40(,60(50,))
c(0,4) cost 1.770000 20(10,40(30,))
c(1,5) cost 1.350000 20(,40(30,50))
c(2,6) cost 0.790000 40(30,60(50,))
c(0,5) cost 1.920000 20(10,40(30,50))
c(1,6) cost 1.540000 40(20(,30),60(50,))
c(0,6) cost ??? ????????????????
```

4. The hash table below was created using double hashing with Brent's rehash. The initial slot ($h_1(key)$) and rehashing increment ($h_2(key)$) are given for each *key*. Show the result from inserting 1300 using Brent's rehash when $h_1(1300) = 5$ and $h_2(1300) = 3$. (10 points)

	key	$h_1(key)$	$h_2(key)$
0			
1	1000	6	2
2			
3	1200	3	3
4	500	3	1
5	12	5	1
6	27	6	3

5. Give an instance of deletion from a red-black tree that will require the maximum number of rotations. Also, explain how the rotations will occur using the four cases. (10 points)

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following problems is not NP-complete? (Assume P ≠ NP)
   A. 3-satisfiability
   B. Testing if a bipartite graph has a vertex cover with no more than k vertices
   C. Testing if a graph is 3-colorable
   D. Testing if the number of colors needed to edge color a graph is the degree of the graph
2. How many times will `-1` occur in the style 1 fail link table for the pattern `acaababc`?
   A. 1
   B. 2
   C. 3
   D. 4
3. How many times will `-1` occur in the style 2 fail link table for the pattern `acaababc`?
   A. 1
   B. 2
   C. 3
   D. 4
4. Radix sort is useful for:

A. Constructing a longest common subsequence using O(m + n) space
B. Constructing a suffix array
C. Constructing the polynomial for Karp-Rabin string search
D. Constructing the Z table for a sequence

5. Which of the following is a deficiency of the maximum capacity path technique?
   A. An augmenting path is blocked if it introduces a cycle of flow.
   B. Augmenting paths will be discovered in descending incremental flow increase order.
   C. Flow decomposition must be applied.
   D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.
6. Which of the following algorithms does not preprocess using a sort?
   A. Closest points in 2-d space - divide and conquer
   B. Closest points in 2-d space - sweepline
   C. Graham scan
   D. Jarvis march
7. Which algorithm is defined using the notions of left-turn and right-turn?
   A. Closest points in 2-d space
   B. Graham scan
   C. Jarvis march
   D. TSP approximation when vertex distances observe the triangle inequality
8. Which of the following is not a condition for performing a push operation from u to v?
   A. the height at u is one more than the height at v
   B. (u, v) has capacity > flow
   C. u has excess > 0
   D. v has a path to the sink in the residual graph
9. Which of the following does not have a polynomial-time approximation algorithm?
   A. Bin packing
   B. Edge coloring
   C. General traveling salesperson
   D. Vertex cover
10. The length of a longest strictly increasing subsequence for `1 4 3 2 3 4 3 6 3 4` is:
    A. 4
    B. 5
    C. 6
    D. 7
11. Which of the following is not true regarding the reduction from 3-sat to 3-colorability?
    A. The widget appears once for each literal.
    B. The widget will use three colors when corresponding to a satisfied clause.
    C. The widget will use four colors when corresponding to an unsatisfied clause.
    D. There is a triangle of vertices for establishing the colors of TRUE, FALSE, and RED.
12. Consider the technique for determining articulation points using depth-first search. If a vertex has no predecessors (first vertex discovered for a "restart"), it can be an articulation point only if
    A. there are no incident tree edges
    B. there is one incident tree edge
    C. there is more than one incident tree edge
    D. there is no back edge incident to this vertex
13. On an augmenting path, a critical edge is:
    A. an edge from the source
    B. an edge into the sink
    C. an edge that used to be saturated
    D. an edge with the minimum residual capacity
14. Which string search method computes signatures to avoid byte-to-byte comparisons?
    A. Karp-Rabin
    B. KMP with fail 1 links
    C. KMP with fail 2 links
    D. Z table
15. The most general approximation result that can be achieved for an NP-hard problem is:
    A. Approximation Algorithm
    B. Approximation Scheme

C. Fully Polynomial-time Approximation Scheme
D. Polynomial-time Approximation Scheme

16. Explain how the Z Algorithm ("Fundamental String Preprocessing") may be used to determine whether two strings are circularly equal. (5 points)

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1. Solve the following instance of Longest Common Subsequence using the method based on the Longest Strictly Increasing Subsequence problem. (15 points)

0	1	2	3	4	5	6	7
a	b	d	c	d	e	f	g
b	c	d	a	e	d	g	f

2. Use dynamic programming, either with a table or lists, to determine a subset that sums to 20. DO NOT SOLVE BY INSPECTION! (15 points)

2 3 5 7 11 13 17

3. List the remaining operations to complete this instance of network flows by push-relabel. In addition, give a minimum cut. (20 points)