CSE 5311 Name _____

Test 1 - Closed Book

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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which priority queue is defined using the notion of null path length?
   A. Binary heap B. Binomial heap C. Fibonacci heap D. Leftist heap
2. Dynamic optimality is a concept involving the comparison of
   A. a key-comparison based data structure to hashing.
   B. amortized complexity to actual complexity.
   C. an online data structure to a fixed, unchanging data structure.
   D. an online data structure to an offline data structure.
3. Suppose you already have 16 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 16 you already have?
   A. 3 B. 4 C. 5 D. 15
4. What is required when calling `union(i,j)` for maintaining disjoint subsets?
   A. `i` and `j` are in the same subset
   B. `i` and `j` are leaders for different subsets
   C. `i` and `j` are leaders for the same subset
   D. `i` is the ancestor of `j` in one of the trees
5. To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree stores the following at every node:
   A. the sum of all keys in the entire tree
   B. the sum of all keys in the left subtree
   C. the sum of all keys that are no larger than the stored key
   D. the sum of all keys stored in the subtree rooted by this node
6. Which Fibonacci heap operation has $\Theta(\log n)$ worst-case actual cost?
   A. FIB-HEAP-DECREASE-KEY B. FIB-HEAP-DELETE
   C. FIB-HEAP-EXTRACT-MIN D. FIB-HEAP-UNION
7. When are Fibonacci trees used?
   A. Constructing a priority queue with excellent amortized complexity for DECREASE-KEY.
   B. Defining the potential function for Fibonacci heaps.
   C. Demonstrating worst-case behaviors for AVL trees.
   D. Demonstrating worst-case behaviors for red-black trees.
8. To reduce the probability of having any collisions to < 0.5 when hashing n keys, the table should have at least this number of elements.
   A. n B. $n \ln n$ C. n^2 D. n^3
9. The minimum number of nodes in a tree in a Fibonacci heap where the root has 7 children is:
   A. 34 B. 61 C. 121 D. 128
10. Which property does not hold for binomial heaps?
    A. DECREASE-KEY takes $O(1)$ time.
    B. MINIMUM takes $O(\log n)$ time.
    C. Performing n INSERT operations into an empty heap will take $O(n)$ time.
    D. The number of trees is based on the binary representation of the number of stored items.
11. Which of the following is not true regarding the amortized analysis of binary tree traversals?
    A. INIT had an amortized cost of 0.
    B. SUCC had an actual cost determined by the number of edges followed.
    C. SUCC had an amortized cost of 2.
    D. The potential was defined without regard for the type of traversal being performed.

12. Which data structure is not used to implement a dictionary?
    A. AVL tree B. Red-black tree C. Self-organizing list D. Union-find
13. Which of the following is not a property of splay trees?
    A. Splaying an accessed node to the root can cause the potential to decrease
    B. The amortized cost of a splaying sequence is bounded logarithmically
    C. The zig rotation is only applied at the root
    D. They are a form of balanced tree
14. The two parts of using a Markov model to analyze a self-adjusting data structure are:
    A. define the potential function, compare to the optimal offline (OPT) method.
    B. determine the probability for each state, then compute the expected number of probes.
    C. simulate the data structure long enough to ensure convergence, the compute the expected number of probes.
    D. use Knuth's root trick to find the optimal subtrees in $O\left(n^2\right)$ time, backtrace to print the tree.
15. Assuming a random *n*-permutation is provided, the expected number of hires for the hiring problem is:
    A. 2 B. H_n C. $\sqrt{n}$ D. $\ln\ln n$
16. Suppose there are 20 coupon types for the coupon collecting problem. Each of the coupon types is identified by an integer in the range 1 to 20. If you are given two random cereal boxes, what is the probability (ignoring the order in which the coupons were obtained) that the two coupons have consecutive numbers, i.e. some *i* and *i* + 1? (5 points)

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1. Suppose you are given *n* values. Give a linear-time algorithm to build a binomial heap with the *n* values. 10 points.
2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)

a. $T(n) = 4T\left(\frac{n}{2}\right) + n^3$

b. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

c. $T(n) = 4T\left(\frac{n}{2}\right) + 1$

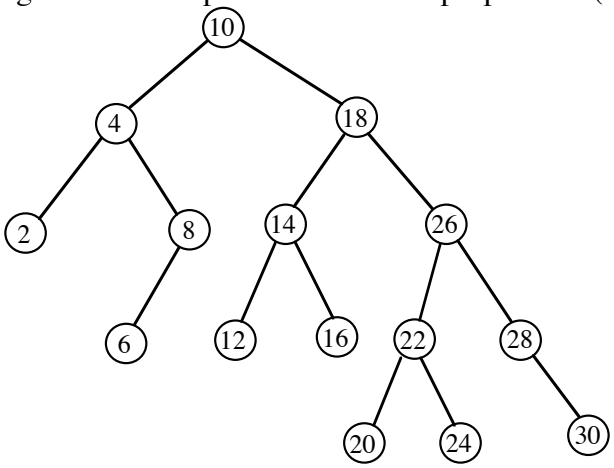
3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)
4. The hash table below was created using double hashing with Brent's rehash. The initial slot ($h_1(key)$) and rehashing increment ($h_2(key)$) are given for each *key*. Show the result from inserting 1300 using Brent's rehash when $h_1(1300) = 5$ and $h_2(1300) = 3$. (10 points)

	key	$h_1(key)$	$h_2(key)$
0			
1	1000	6	2
2			
3	1200	3	3

4	500	3	1
5	12	5	1
6	27	6	3

	key
0	
1	
2	
3	
4	
5	
6	

5. Insert 23 into the following AVL tree to preserve the AVL properties. (10 points)



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Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. Which of the following problems uses some edges with non-unit capacity when translated to a network flow problem?
   A. Bipartite matching
   B. Edge connectivity

C. Minimum vertex cover for bipartite graph
D. Vertex connectivity

2. Which is not true regarding the Karp-Rabin method?
   A. It always runs in $\Theta(m+n)$ time.
   B. It computes a signature for the pattern and each *m* contiguous text symbols.
   C. The signature is computed using a polynomial.
   D. When the signature for *m* contiguous text symbols matches the signature for the pattern, a `strcmp` must be performed.
3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?
   A. Convex hull
   B. Delaunay triangulation
   C. Euclidean minimum spanning tree
   D. Voronoi diagram
4. Constructing a suffix array for a sequence with *n* symbols by using an optimal key-comparison sort has this worst-case time:

   A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta\left(n^2\right)$ D. $\Theta\left(n^2 \log n\right)$

5. The algorithm for finding a maximum capacity path for network flows is most similar to which algorithm?
   A. Breadth-first search
   B. Decomposition of a flow into *E* augmenting paths
   C. Dijkstra
   D. Floyd-Warshall
6. Which of the following algorithms may take time longer than $O(n \log n)$?
   A. Closest points in 2-d space - divide and conquer
   B. Closest points in 2-d space - sweepline
   C. Graham scan
   D. Jarvis march
7. What data structure is used for the sweep-line status when computing the 2-d closest pair?
   A. BST of points with x-coordinates as the key
   B. BST of points with y-coordinates as the key
   C. Interval tree
   D. Sorted array by ascending x-coordinates
8. The length of a longest (monotone) increasing subsequence for `1 4 3 2 3 4 3 6 3 4` is:

   A. 4 B. 5 C. 6 D. 7

9. Which of the following is NOT required when showing that problem B is NP-complete by a reduction from problem A?
   A. Problem A is NP-complete.
   B. The reduction has an inverse that takes each instance of problem B to an instance of problem A.
   C. The reduction must be consistent for the decision results for each instance of problem A and and the corresponding instance of problem B.
   D. The reduction takes polynomial time.
10. Which minimum spanning tree algorithm is the slowest?

    A. Boruvka B. Kruskal C. Prim D. Warshall

11. When <u>combining</u> the left and right parts for the 2-d closest pair algorithm along the vertical dividing line, each left-side point "near" the line has its distance computed to no more than this number of right-side points.

    A. 6 B. 8 C. $\lg n$ D. $n/2$

12. Which of the following problems is not NP-complete? (Assume P ≠ NP)

A. Testing if a graph is bipartite
B. Testing if an undirected graph has a Hamiltonian cycle
C. Testing if the number of colors needed to edge color a graph is the degree of the graph
D. 3-satisfiability

13. Which of the following is a deficiency of the maximum capacity path technique?
A. An augmenting path is blocked if it introduces a cycle of flow.
B. Augmenting paths will be discovered in descending incremental flow increase order.
C. Flow decomposition must be applied.
D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.

14. When performing bin packing using the First-Fit Decreasing technique, the total number of items placed in the bins past the optimal bins (1 .. OPT) is no more than:
A. $1 + \varepsilon$ B. 2 C. OPT - 1 D. OPT

15. Which of the following does not have a polynomial-time approximation algorithm?
A. Bin packing
B. Edge coloring
C. Traveling salesperson with triangle inequality
D. Vertex coloring

16. Give the result of the Z algorithm for the following sequence. (5 points)

```
0 a
1 c
2 a
3 b
4 a
5 c
6 a
7 c
8 a
9 b
10 a
11 c
12 a
13 b
14 a
15 c
16 a
17 c
18 a
19 b
20 a
```

21 c

22 a

23 c

24 a

25 b

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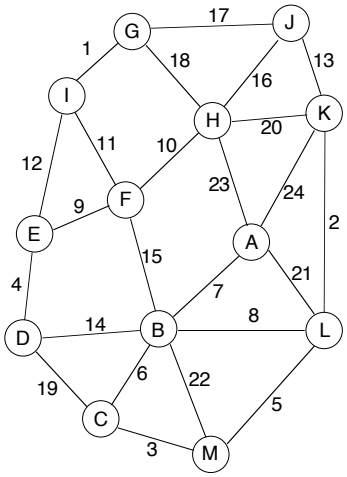
1. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (10 points)

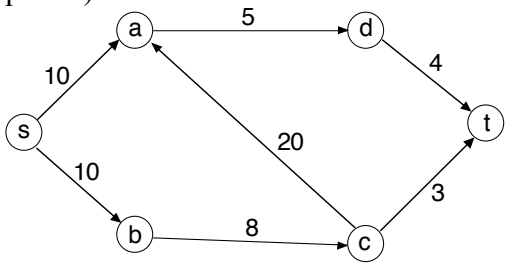


2. Solve the following instance of Longest Common Subsequence using the method based on the Longest Strictly Increasing Subsequence problem. (10 points)

0	1	2	3	4	5	6	7
a	b	d	c	d	e	f	g

b	c	d	f	e	d	g	f

3. List the lift and push operations to solve for the maximum flow. In addition, give a minimum cut. (15 points)



4. Prove that the **SET PACKING** problem is NP-complete (15 points).

Hint 1: There is a straightforward reduction from ***K*-CLIQUE**.

Hint 2: It is often helpful to give an example of the reduction used.

INSTANCE: Collection C of finite sets, positive integer $K \leq |C|$.

QUESTION: Does C contain at least K mutually (i.e. pairwise) disjoint sets?