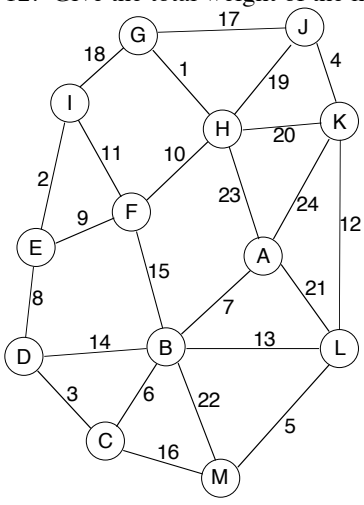
CSE 5311 Name _____

Test 1 - Closed Book

Summer 2011 Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:
   A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$
2. What is the worst-case number of rotations when performing insertion on an AVL tree?
   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. No rotations are ever used
3. Suppose you already have 10 different coupons when there are 20 coupon types. What is the expected number of boxes for obtaining a coupon different from the 10 you already have?
   A. 2 B. 3 C. 4 D. 5
4. Which of the following is not true regarding Bloom filters?
   A. An indication that a candidate element is not in the set is always correct.
   B. They are an especially compact dictionary representation.
   C. Several hash functions are used.
   D. The optimal bit array size depends on the number of items and the false positive probability.
5. If the universe size at the root of a van Emde Boas tree is u, then the number of children is:
   A. 2 B. $\log u$ C. $\log \log u$ D. $\sqrt{u}$
6. The summary structures are critical for implementing which operation efficiently for van Emde Boas trees?
   A. Member B. Minimum C. Successor D. all of these
7. How many inversions are there for the lists 1, 2, 5, 4, 3 and 2, 5, 4, 3, 1?
   A. 2 B. 3 C. 4 D. 5
8. Suppose there are n entries in a Fibonacci heap. The maximum number of trees is:
   A. $\theta(\log \log n)$ B. $\theta(\log n)$ C. $\theta(n)$ D. $\theta(n \log n)$
9. Which property does not hold for binomial heaps?
   A. DECREASE-KEY takes $O(1)$ time.
   B. MINIMUM takes $O(\log n)$ time.
   C. Performing n INSERT operations into an empty heap will take $O(n)$ time.
   D. The number of trees is based on the binary representation of the number of stored items.
10. What is the main contribution of leftist heaps?
    A. The minimum is found in $O(1)$ time.
    B. The amortized complexity of DECREASE-KEY is $O(1)$.
    C. The height of the tree is $O(\log n)$.
    D. The UNION is computed in $O(\log n)$ time.
11. Dynamic optimality is a concept involving the comparison of
    A. a key-comparison based data structure to hashing.
    B. amortized complexity to actual complexity.
    C. an online data structure to a fixed, unchanging data structure.
    D. an online data structure to an offline data structure.
12. Give the total weight of the minimum spanning tree for this graph. (5 points)



13. Give an example of a binary search tree that can be colored as a legal red-black tree, but does not satisfy the AVL balance conditions. (6 points)
14. Give the binomial (min) heap that results from inserting 1, 2, 3, 4, 5, 6, 7, 8 (in that order) into an empty heap. (6 points)

CSE 5311 Name _____

Test 1 - Open Book

Summer 2011 Student ID # _____

1. The hash table below was created using double hashing with Brent's rehash. The initial slot ($h_1(key)$) and rehashing increment ($h_2(key)$) are given for each *key*. Show the result from inserting 7095 using Brent's rehash when $h_1(7095) = 4$ and $h_2(7095) = 4$. (10 points)

	key	$h_1(key)$	$h_2(key)$
0	7000	0	5
1	7081	4	2
2	7002	2	1
3			
4	7004	4	3
5			
6	7006	6	5

	key
0	
1	
2	
3	
4	
5	
6	

2. Evaluate the following recurrences using the master method. (10 points)

a. $T(n) = T(0.7n) + n$

b. $T(n) = T(0.7n) + 1$

c. $T(n) = 16T\left(\frac{n}{2}\right) + n^3$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

```
n=6;
q[0]=0.01;
key[1]=10;
p[1]=0.19;
q[1]=0.02;
key[2]=20;
p[2]=0.1;
q[2]=0.03;
key[3]=30;
p[3]=0.2;
q[3]=0.04;
key[4]=40;
p[4]=0.2;
q[4]=0.0;
key[5]=50;
p[5]=0.02;
q[5]=0.04;
key[6]=60;
p[6]=0.12;
q[6]=0.03;
```

```
w[0][0]=0.010000
w[0][1]=0.220000
w[0][2]=0.350000
w[0][3]=0.590000
w[0][4]=0.790000
w[0][5]=0.850000
w[0][6]=1.000000
w[1][1]=0.020000
w[1][2]=0.150000
w[1][3]=0.390000
w[1][4]=0.590000
w[1][5]=0.650000
w[1][6]=0.800000
w[2][2]=0.030000
w[2][3]=0.270000
w[2][4]=0.470000
w[2][5]=0.530000
w[2][6]=0.680000
w[3][3]=0.040000
w[3][4]=0.240000
```

```
w[3][5]=0.300000
w[3][6]=0.450000
w[4][4]=0.000000
w[4][5]=0.060000
w[4][6]=0.210000
w[5][5]=0.040000
w[5][6]=0.190000
w[6][6]=0.030000
Building c(0,2) using roots 1
   thru 2
Building c(1,3) using roots 2
   thru 3
Building c(2,4) using roots 3
   thru 4
Building c(3,5) using roots 4
   thru 5
Building c(4,6) using roots 5
   thru 6
Building c(0,3) using roots 1
   thru 3
```

```
Building c(1,4)   Building c(1,6) using roots 3     c(1,3) cost 0.540000 30(20,)
   using roots       thru 4                         c(2,4) cost 0.710000 30(,40)
   3 thru 3       Building c(0,6) using roots ?     c(3,5) cost 0.360000 40(,50)
Building c(2,5)      thru ?                         c(4,6) cost 0.270000 60(50,)
   using roots    Counts - root trick 29 without    c(0,3) cost 1.080000 20(10,30)
   3 thru 4          root trick 50                  c(1,4) cost 0.980000 30(20,40)
Building c(3,6)   Average probe length is ????     c(2,5) cost 0.860000 40(30,50)
   using roots    trees in parenthesized prefix     c(3,6) cost 0.720000 40(,60(50,))
   4 thru 6       c(0,0) cost 0.000000              c(0,4) cost 1.530000
Building c(0,4)   c(1,1) cost 0.000000                 30(10(,20),40)
   using roots    c(2,2) cost 0.000000              c(1,5) cost 1.160000
   2 thru 3       c(3,3) cost 0.000000                 30(20,40(,50))
Building c(1,5)   c(4,4) cost 0.000000              c(2,6) cost 1.220000
   using roots    c(5,5) cost 0.000000                 40(30,60(50,))
   3 thru 4       c(6,6) cost 0.000000              c(0,5) cost 1.710000
Building c(2,6)   c(0,1) cost 0.220000 10              30(10(,20),40(,50))
   using roots    c(1,2) cost 0.150000 20           c(1,6) cost 1.610000
   4 thru 4       c(2,3) cost 0.270000 30              40(30(20,),60(50,))
Building c(0,5)   c(3,4) cost 0.240000 40           c(0,6) cost ????????
   using roots    c(4,5) cost 0.060000 50              ????????????????
   3 thru 3       c(5,6) cost 0.190000 60
                  c(0,2) cost 0.500000 10(,20)
```

4. Fill in the `min` and `max` blanks for the following instance of a van Emde Boas tree for the set {1, 8, 9, 10, 12, 13, 14}. You should give these as values in the local universe (`0..u-1`). Instead of using the symbol "/" for NIL, use the symbol "∅". (10 points)

```
root (base 0) u 16 min _______ max _______
    summary (base 0) u 4 min _______ max _______
        summary (base 0) u 2 min _______ max _______
        cluster[0] (base 0) u 2 min _______ max _______
        cluster[1] (base 2) u 2 min _______ max _______
    cluster[0] (base 0) u 4 min _______ max _______
        summary (base 0) u 2 min _______ max _______
        cluster[0] (base 0) u 2 min _______ max _______
        cluster[1] (base 2) u 2 min _______ max _______
    cluster[1] (base 4) u 4 min _______ max _______
        summary (base 0) u 2 min _______ max _______
        cluster[0] (base 4) u 2 min _______ max _______
        cluster[1] (base 6) u 2 min _______ max _______
    cluster[2] (base 8) u 4 min _______ max _______
        summary (base 0) u 2 min _______ max _______
        cluster[0] (base 8) u 2 min _______ max _______
        cluster[1] (base 10) u 2 min _______ max _______
    cluster[3] (base 12) u 4 min _______ max _______
        summary (base 0) u 2 min _______ max _______
        cluster[0] (base 12) u 2 min _______ max _______
        cluster[1] (base 14) u 2 min _______ max _______
```

Give the range of possible heights for a red-black tree with 200 keys. Your answer should be two natural numbers giving the minimum and maximum heights. (A tree with one node has height 0.) Show your work! (10 points)

CSE 5311 Name _____

Test 2 - Closed Book

Summer 2011 Last 4 Digits of Student ID # _____

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. When performing selection in worst-cast linear time for n numbers, roughly how many column medians are computed in the first round?

   A. $W\left(\frac{n}{5}\right)$

   B. m, the median-of-medians

   C. $.7n$

D. $\frac{n}{5}$

2. What is the nature of the signature function for the Karp-Rabin method?
   A. the remainder by discarding the overflow for a polynomial
   B. it is similar to the KMP failure links
   C. similar to a double hash function for string keys
   D. a polynomial of arbitrary precision implemented using a bignum package
3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?
   A. Euclidean minimum spanning tree
   B. Delaunay triangulation
   C. Convex hull
   D. Voronoi diagram
4. Which of the following is not a goal for suffix array construction methods?
   A. Fast in practice
   B. Based on radix sort
   C. Lightweight
   D. Linear-time in worse case
5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:
   A. V
   B. $\mathrm{O}(VE)$
   C. f
   D. E
6. Which algorithm is defined using the notions of left-turn and right-turn?
   A. Graham scan
   B. Closest points in 2-d space
   C. Suffix array construction
   D. Jarvis march
7. What data structure is used for the sweep-line status when computing the 2-d closest pair?
   A. BST of points with y-coordinates as the key
   B. Interval tree
   C. BST of points with x-coordinates as the key
   D. Sorted array by ascending x-coordinates
8. The four russians' concept is to:
   A. Pack bits into an efficient storage unit
   B. Trade-off between enumerating situations and referencing these situations
   C. Implement longest common subsequences using linear space
   D. Trade-off between scalar additions and multiplications
9. When coloring the edges of a graph, a dc-path gets inverted because:
   A. All edges in the fan will be colored with d or c.
   B. d is a free color for all fan vertices.
   C. d is the free color for two fan vertices.
   D. We are trying to minimize the number of colors used by the path.
10. How many times will `-1` occur in the style 2 fail link table for the pattern `abacabac`?
    A. 1
    B. 2
    C. 3
    D. 4
11. How many times will `-1` occur in the style 1 fail link table for the pattern `abacabac`?
    A. 1
    B. 2
    C. 3
    D. 4
12. Which of the following problems is NP-complete? (Assume P ≠ NP)
    A. 3-satisfiability
    B. Testing if a graph is 2-colorable
    C. Testing if a table is in sorted order
    D. Verifying a solution to traveling salesperson
13. Which of the following is a deficiency of the maximum capacity path technique?
    A. Augmenting paths will be discovered in descending incremental flow increase order.

B. Flow decomposition must be applied.
C. An augmenting path is blocked if it introduces a cycle of flow.
D. The maximum number of potential augmenting paths depends on the achievable flow, in addition to the number of vertices and edges.

14. Constructing a suffix array for a sequence with n symbols by using an optimal key-comparison sort has this worst-case time:

A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^2 \log n)$

15. Which of the following does not have a polynomial-time approximation algorithm?
A. Bin packing
B. Edge coloring
C. Traveling salesperson with triangle inequality
D. Vertex coloring

16. Determine a monotone longest increasing subsequence for the sequence below. (5 points)

```
1  4  3  2  3  4  3  6  3  4
```

CSE 5311 Name _____

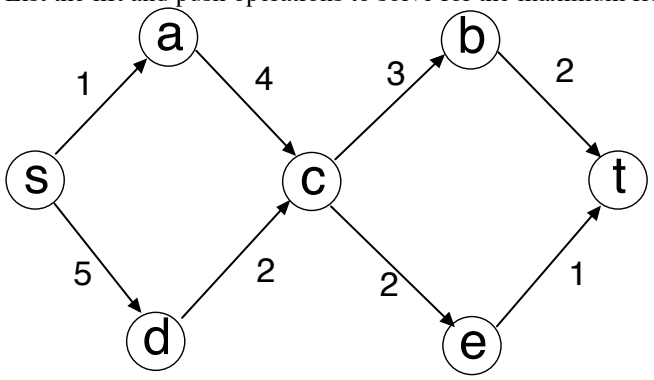
Test 2 - Open Book

Summer 2011 Last 4 Digits of Student ID # _____

1. Fill in the blanks in the following instance of a suffix array with lcp values and ranks. As usual, s[21] is NULL ('\0'). (15 points)

i	sa	suffix	lcp	s	rank	lcp[rank]
0	21		-1	0	__	__
1	20	0	0	1	__	__
2	7	00100101001010	1	0	__	__
3	15	001010	4	0	__	__
4	2	0010100100101001010	6	1	20	4
5	10	00101001010	9	0	7	3
6	18	010	1	1	15	2
7	5	0100100101001010	3	0	2	__
8	13	01001010	6	0	10	__
9	0	010010100100101001010	8	1	18	__
10	___	0100101001010	11	0	5	9
11	16	01010	3	0	13	8
12	3	0101001001010	5	1	21	7
13	11	__101001010	8	0	8	6
14	19	__0	0	1	16	5
15	6	100100101001010	2	0	3	4
16	14	1001010	5	0	11	3
17	1	10010100100101001010	7	1	19	2
18	9	100101001010	10	0	6	1
19	___	1010	2	1	14	0
20	4	10100100101001010	4	0	1	0
21	12	101001010	7		0	-1

2. List the lift and push operations to solve for the maximum flow. In addition, give a minimum cut. (15 points)



3. Give the result of the Z algorithm and both KMP methods for the following sequence. (20 points)

```
0 a
```

1 c

2 a

3 b

4 a

5 c

6 a

7 c

8 a

9 b

10 a

11 c

12 a

13 b

14 a

15 c

16 a

17 c

18 a

19 b

20 a

21 c

22 a

23 c

24 a

25 b