CSE 5311

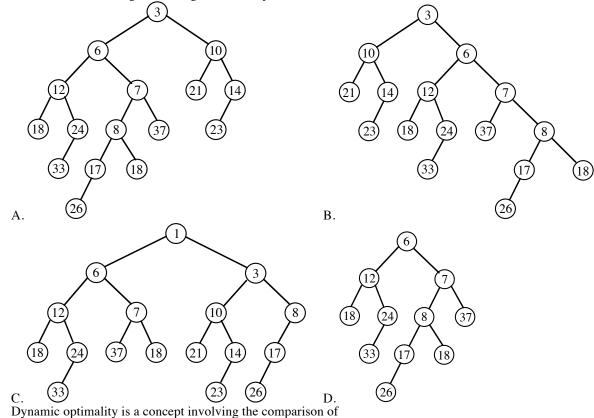
## Name \_

Test 1 - Closed Book Summer 2012

## Last 4 Digits of Student ID #

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

- 1. When are Fibonacci trees used?
  - A. Defining the potential function for Fibonacci heaps.
  - B. Constructing a priority queue with excellent amortized complexity for DECREASE-KEY.
  - C. Demonstrating worst-case behaviors for red-black trees.
  - D. Demonstrating worst-case behaviors for AVL trees.
- 2. Which of the following is not a legal leftist heap?



3.

- A. a key-comparison based data structure to hashing.
- B. amortized complexity to actual complexity.
- C. an online data structure to an offline data structure.
- D. an online data structure to a fixed, unchanging data structure.
- Suppose you already have 16 different coupons when there are 20 coupon types. What is the expected number of boxes 4. for obtaining a coupon different from the 16 you already have? A. 4 D. 10

5. If the universe size at the root of a van Emde Boas tree is u, then the number of children is:

A. lg 
$$u$$
 B. log  $u$  C. log log  $u$  D.  $\sqrt{u}$ 

- To support computing prefix sums of all keys that are no larger than some query key, an augmented binary search tree 6. stores the following at every node:
  - A. the sum of all keys in the entire tree
  - B. the sum of all keys in the left subtree
  - C. the sum of all keys that are no larger than the stored key
  - D. the sum of all keys stored in the subtree rooted by this node
- Which Fibonacci heap operation has  $\Theta(\log n)$  worst-case actual cost? 7. **B.** FIB-HEAP-UNION
  - A. FIB-HEAP-EXTRACT-MIN C. FIB-HEAP-DECREASE-KEY

- 8. What is the main contribution of leftist heaps?
  - A. The minimum is found in O(1) time.
  - B. The amortized complexity of DECREASE-KEY is O(1).
  - C. The height of the tree is  $O(\log n)$ .
  - D. The UNION is computed in  $O(\log n)$  time.
- 9. The number of potential probe sequences when using linear probing with a table with *m* entries (*m* is prime) is:

A. $O(\log m)$ B. $m$ C. $m(m-1)$ D. $m!$ 10. Which of the following is not true regarding the amortized analysis of binary tree traversals?					
A. INIT had an amortized cost of 1.					
B. SUCC had an actual cost determined by the number of edges followed.					
C. SUCC had an amortized cost of 2.					
D. The potential was defined with regard to the type of traversal being performed.					
11. Which data structure is not used to implement a dictionary?					
A. vEB tree B. Red-black tree C. Self-organizing list D. Splay tree 12. The reason for marking nodes in a Fibonacci heap is:					
A. to indicate nodes that have lost a child since becoming a child themselves.					
B. to allow computing the value of the potential function.					
C. to assure that the structure is a Fibonacci heap rather than a binomial heap.					
D. to improve the performance of CONSOLIDATE.					
13. If a Fibonacci tree appears as a subtree of an AVL tree, which nodes would be assigned a balance factor of 0?					
A. none of them B. only the root C. only the leaves D. the leaves and the root					
14. The main difference between MTF and OPT for self-organizing linear lists is:					
A. MTF is given the entire request sequence in advance, while OPT receives the requests one-at-a-time					
B. OPT counts inversions					
C. OPT is given the entire request sequence in advance, while MTF receives the requests one-at-a-time					
D. MTF can do transpositions 15. To reduce the probability of having any collisions to $< 0.5$ when hashing <i>n</i> keys, the table should have at least this number					
15. To reduce the probability of having any constons to $< 0.5$ when hashing <i>n</i> keys, the table should have at least this humber of elements.					
A. $n$ B. $n \ln n$ C. $n^2$ D. $n^3$					
16. Suppose you roll three standard six-sided dice. What is the probability that the sum of the three values rolled does not					
exceed 11? Show your work. (5 points)					
CSE 5311 Name					
Test 1 - Open Book       Summer 2012       Last 4 Digits of Student ID #					
1 Fill in the min and max blanks for the following instance of a van Ende Boas tree for the set $\{1, 2, 3, 8, 9, 10, 14, 15\}$					
1. Fill in the min and max blanks for the following instance of a van Emde Boas tree for the set $\{1, 2, 3, 8, 9, 10, 14, 15\}$ . You should give these as values in the local universe $(0, -u-1)$ . Instead of using the symbol "/" for NU, use the symbol					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol					
You should give these as values in the local universe $(0 \cdot u - 1)$ . Instead of using the symbol "/" for NIL, use the symbol " $\emptyset$ ". (10 points)					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol " $\emptyset$ ". (10 points) root (base 0) u 16 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 4 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 4 min max summary (base 0) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 4 min max cluster[0] (base 0) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 4 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 4 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max summary (base 0) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 4 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max summary (base 0) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 2) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 4) u 4 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 4 min max cluster[0] (base 4) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 6) u 2 min max					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 6) u 2 min max					
<pre>You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 6) u 2 min max cluster[2] (base 8) u 4 min max summary (base 0) u 2 min max</pre>					
You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 6) u 2 min max cluster[1] (base 6) u 2 min max cluster[2] (base 8) u 4 min max cluster[2] (base 8) u 4 min max cluster[2] (base 8) u 2 min max					
<pre>You should give these as values in the local universe (0u-1). Instead of using the symbol "/" for NIL, use the symbol "Ø". (10 points) root (base 0) u 16 min max summary (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[0] (base 0) u 4 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[0] (base 0) u 2 min max cluster[1] (base 2) u 2 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 4 min max cluster[1] (base 4) u 2 min max cluster[1] (base 4) u 2 min max cluster[1] (base 6) u 2 min max cluster[2] (base 8) u 4 min max summary (base 0) u 2 min max</pre>					

2

summary (base 0) u 2 min \_\_\_\_ max \_\_\_\_ cluster[0] (base 12) u 2 min \_\_\_\_ max \_\_\_\_ cluster[1] (base 14) u 2 min \_\_\_\_ max \_\_\_\_ 2. Evaluate the following recurrences using the master method. Indicate the case that is used for each. (10 points)

a. 
$$T(n) = 4T\left(\frac{n}{2}\right) + n^3$$

b. 
$$T(n) = 4T(\frac{n}{2}) + n^2$$

c. 
$$T(n) = 4T\left(\frac{n}{2}\right) + 1$$

3. Construct the final optimal binary search tree (using Knuth's root trick) and give its cost. SHOW YOUR WORK. (10 points)

n=6;	w[1][5]=0.650000	Building c(1,6) using roots 3
q[0]=0.01;	w[1][6]=0.800000	thru 4
key[1]=10;	w[2][2]=0.030000	Building c(0,6) using roots ?
p[1]=0.19;	w[2][3]=0.270000	thru ?
q[1]=0.02;	w[2][4]=0.470000	Counts - root trick 29 without
key[2]=20;	w[2][5]=0.530000	root trick 50
p[2]=0.1;	w[2][6]=0.680000	Average probe length is ????
q[2]=0.03;	w[3][3]=0.040000	trees in parenthesized prefix
key[3]=30;	w[3][4]=0.240000	c(0,0) cost 0.000000
p[3]=0.2;	w[3][5]=0.300000	c(1,1) cost 0.000000
q[3]=0.04;	w[3][6]=0.450000	c(2,2) cost 0.000000
key[4]=40;	w[4][4]=0.000000	c(3,3) cost 0.000000
p[4]=0.2;	w[4][5]=0.060000	c(4,4) cost 0.000000
q[4]=0.0;	w[4][6]=0.210000	c(5,5) cost 0.000000
key[5]=50;	w[5][5]=0.040000	c(6,6) cost 0.000000
p[5]=0.02;	w[5][6]=0.190000	c(0,1) cost 0.220000 10
q[5]=0.04;	w[6][6]=0.030000	c(1,2) cost 0.150000 20
key[6]=60;	Building c(0,2) using roots 1	c(2,3) cost 0.270000 30
p[6]=0.12;	thru 2	c(3,4) cost 0.240000 40
q[6]=0.03;	Building c(1,3) using roots 2	c(4,5) cost 0.060000 50
w[0][0]=0.01000	thru 3	c(5,6) cost 0.190000 60
0	Building c(2,4) using roots 3	c(0,2) cost 0.500000 10(,20)
w[0][1]=0.22000	thru 4	c(1,3) cost 0.540000 30(20,)
0	Building c(3,5) using roots 4	c(2,4) cost 0.710000 30(,40)
w[0][2]=0.35000	thru 5	c(3,5) cost 0.360000 40(,50)
0	Building c(4,6) using roots 5	c(4,6) cost 0.270000 60(50,)
w[0][3]=0.59000	thru 6	c(0,3) cost 1.080000 20(10,30)
0	Building c(0,3) using roots 1	c(1,4) cost 0.980000 30(20,40)
w[0][4]=0.79000	thru 3	c(2,5) cost 0.860000 40(30,50)
0	Building c(1,4) using roots 3	c(3,6) cost 0.720000 40(,60(50,))
w[0][5]=0.85000	thru 3	c(0,4) cost 1.530000
0	Building c(2,5) using roots 3	30(10(,20),40)
w[0][6]=1.00000	thru 4	c(1,5) cost 1.160000
0	Building c(3,6) using roots 4	30(20,40(,50))
w[1][1]=0.02000	thru 6	c(2,6) cost 1.220000
0	Building c(0,4) using roots 2	40(30,60(50,))
w[1][2]=0.15000	thru 3	c(0,5) cost 1.710000
0	Building c(1,5) using roots 3	30(10(,20),40(,50))
w[1][3]=0.39000	thru 4	c(1,6) cost 1.610000
0	Building c(2,6) using roots 4	40(30(20,),60(50,))
w[1][4]=0.59000	thru 4	c(0,6) cost ???????
0	Building c(0,5) using roots 3	???????????????????????????????????????
	thru 3	

4. The hash table below was created using double hashing with Brent's rehash. The initial slot ( $h_1(key)$ ) and rehashing increment  $(h_2(key))$  are given for each key. Show the result from inserting 5559 using Brent's rehash when

$h_1(55)$	(559) = 1 and	$h_2(5559) =$	4. (10 points)
	key	$h_1(key)$	$h_2(key)$
0			
1	2199	1	4
2	3299	2	6
3	5500	5	5
4	4400	4	3
5	1111	5	2

6

6

Give the precise range of possible heights for an AVL tree with 300 keys. (10 points)

CSE 5311 Test 2 - Closed Book Summer 2012

key

Student ID #	

Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

Constructing a suffix array for a sequence with n symbols by using the original Manber-Myers radix sort construction has 1. this worst-case time:

Name \_\_\_

A. 
$$\Theta(n)$$
 B.  $\Theta(n \log n)$  C.  $\Theta(n^2)$  D.  $\Theta(n^2 \log n)$ 

What is the nature of the signature function for the Karp-Rabin method? 2.

- A. it is similar to the KMP failure links
  - B. the remainder by discarding the overflow for a polynomial
- C. a polynomial of arbitrary precision implemented using a bignum package
- D. similar to a double hash function for string keys
- 3. Which of the following is helpful if you wish to know the farthest pair in a set of points in 2D?

  - A. Voronoi diagramB. Convex hullC. Delaunay triangulation
  - D. Euclidean minimum spanning tree
- 4. While constructing a suffix array for a sequence with n symbols by using radix sorts, sequence symbols are used in which of the radix sorts?
  - A. The last one
  - B. The first one
  - C. None of them
  - D. All of them
- 5. In a maximum flow problem, the number of augmenting paths in a flow decomposition is bounded by:
  - A. the number of edges in the graph
  - B. the number of saturated edges
  - C. the number of unsaturated edges
  - D. the number of vertices
- Which algorithm is defined using the notions of left-turn and right-turn? 6
  - A. Suffix array construction
  - B. Graham scan
  - C. Closest points in 2-d space
  - D. Jarvis march
- 7. What data structure is used for the sweep-line status when computing the 2-d closest pair?
  - A. BST of points with y-coordinates as the key
  - B. Sorted array by ascending x-coordinates
  - C. BST of points with x-coordinates as the key
  - D. Interval tree
- 8. The four russians' concept is to:
  - A. Trade-off between generating situations and referencing these situations
  - B. Trade-off between scalar additions and multiplications
  - C. Implement longest common subsequences using linear space

- D. Pack bits into an efficient storage unit
- When coloring the edges of a graph, a dc-path gets inverted because:
- A. d is the free color for two fan vertices.
  - B. We are trying to minimize the number of colors used by the path.
- C. All edges in the fan will be colored with d or c.
- D. d is a free color for all fan vertices.
- 10. What is the nature of the linear-space method for the longest common subsequence problem?
  - A. Build a suffix array and lcp array for the concatenated input sequences B. Recursive divide-and-conquer

  - C. Radix sort
  - D. Use a polynomial for the signature function
- 11. The reduction from 3-sat to 3-colorability will give a graph requiring four colors when:
  - A. Removing one clause from the 3-sat instance will leave a satisifiable set of clauses.
    - The 3-sat instance is unsatisfiable. B
    - C. The 3-sat instance is satisfiable.
    - D. The 3-sat instance is a tautology.
- 12. How many times will -1 occur in the style 1 fail link table for the pattern abcabd?
  - A. 1

9.

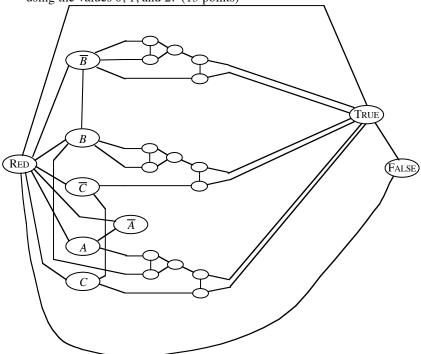
- B. 2 C. 3
- 4 D.

13. How many times will -1 occur in the style 2 fail link table for the pattern abcabd?

- A. 1
- B. 2
- C. 3
- D. 4
- 14. Vizing's theorem is the basis for:
  - A. Approximate bin packing.
  - B. Traveling salesperson approximation when the triangle inequality applies.
  - C. Approximate edge coloring.
  - D. Approximate a vertex coloring.
- 15. The technique for approximating a subset cover proceeds by:
  - A. Choosing the subset with the smallest fraction of its elements uncovered
  - B. Choosing the subset with the smallest number of uncovered elements
  - C. Choosing the subset with the largest fraction of its elements uncovered
  - D. Choosing the subset with the largest number of uncovered elements
- 16. Describe the initialization for push/relabel techniques. You may give an example. (5 points) Name
- CSE 5311
- Test 2 Open Book Summer 2012

Student ID #

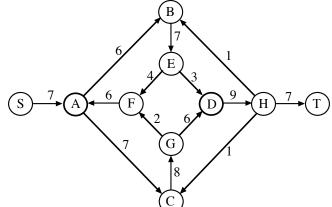
Give the set of clauses for the instance below of the reduction from 3-sat to 3-colorability and an appropriate coloring 1. using the values 0, 1, and 2. (15 points)



- 2. Give both types of KMP failure link tables for the following string: (10 points)
- 0 C
- 1 A
- 2 В
- 3 A
- 4 В
- 5 C
- 6 C
- 7 A
- 8 B
- 9 C
- 10 A
- 11 В
- 12 A
- 13 B

С

- 14
- 3. Give a flow decomposition for the network flow below. (10 points)



4. Demonstrate Boruvka's algorithm on the following graph. Be sure to indicate the edges that are inserted into the MST in each phase. (15 points)

