



Figure 2.19 A sequence of triangulations (a) and the corresponding search-directed acyclic graph (b).

have reached a node in the path (i.e., we have located  $z$  in the corresponding triangle) we test  $z$  for inclusion in all of its descendants; since  $z$  is included in exactly one descendant, this advances the path one more arc. We may also view the search as the successive location of  $z$  in triangulations  $S_{h(N)}$ ,  $S_{h(N)-1}$ ,  $\dots$ ; since  $S_{i-1}$  is a refinement of  $S_i$ , this justifies the name of the technique.

Less informally, we assume that all descendants of a node  $v$  of  $T$  be arranged in a list  $\Gamma(v)$ , and let  $\text{TRIANGLE}(v)$  denote the triangle corresponding to node  $v$ . We then have the following search algorithm:

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procedure POINT-LOCATION
begin if ( $z \notin \text{TRIANGLE}(\text{root})$ ) then print "z belongs to unbounded region"
else begin  $v := \text{root}$ ;
while ( $\Gamma(v) \neq \emptyset$ ) do
  for each  $u \in \Gamma(v)$  do if ( $z \in \text{TRIANGLE}(u)$ ) then  $v := u$ ;
  print  $v$ 
end
end.

```

As mentioned earlier, the choice of the set of triangulation vertices to be removed in constructing  $S_i$  from  $S_{i-1}$  is crucial to the performance of the technique. Suppose we are able to choose this set so that, denoting by  $N_i$  the number of vertices of  $S_i$ , the following properties hold: