Segment noisy 3D point clouds by utilizing persistent homology theory.

Introduction

Segmentation algorithms aim to divide a point cloud into constituent clusters that are perceptually meaningful and serve as a vital preprocessing step in robotic systems. The performance of high level tasks such as object localization, feature extraction, and classification are dependent upon the quality of the segmented data. Large datasets produced by low-cost RGB-D sensors have attracted the attention of researchers towards developing efficient algorithms and data structures for point cloud processing. The key contributions of our work are:

- The introduction of persistent homology to the area of point cloud processing for 3D perception
- A novel approach for segmenting 3D point clouds based on topological persistence

Mathematical Background

The discrete space that we work in uses simplices as building blocks:

- A d-simplex \( \sigma \) is the convex hull of \( d+1 \) affinely independent vertices \( v_0, \ldots, v_d \in \mathbb{R}^n \). We denote \( \sigma = \text{conv}\{v_0, \ldots, v_d\} \) where the dimension of \( \sigma \) is \( d \).
- A face of \( \sigma \) is \( \text{conv} S \) where \( S \subset \{v_0, \ldots, v_d\} \) is a subset of the \( d+1 \) vertices.
- Simplices can be joined to form simplicial complexes:
  - A simplicial complex \( K \) is a finite collection of simplices such that if \( \sigma \in K \) and \( \tau \) is a face of \( \sigma \), then \( \tau \in K \) and if \( \sigma, \sigma' \in K \) then \( \sigma \cap \sigma' \) is either empty or a face of both \( \sigma \) and \( \sigma' \).
  - Given a set of points \( X = \{x_0, \ldots, x_n\} \in \mathbb{R}^n \) in Euclidean \( n \)-space and a fixed radius \( \epsilon \), the Vietoris-Rips complex of \( X \) is an abstract simplicial complex whose \( d \)-simplices correspond to \( (d+1) \)-tuples of points that are pairwise within \( \epsilon \) distance of each other, Figure 1.

Problem Statement

Let \( X \) be a topological space where \( X = \{x_0, \ldots, x_n\} \in \mathbb{R}^n \) and \( x_0, \ldots, x_n \) are the points in a point cloud captured by an RGB-D sensor. To find the persistent homology of \( X \) we first represent the topology of the space with a Vietoris-Rips complex. Next, we compute the zero-dimensional homology group \( \ker \partial_0 / \text{im} \partial_1 \) of \( X \) which corresponds to the number of connected components. Finally, we extract the connected component clusters.

Simplicial Complex Construction

Given an input point cloud we form the Vietoris-Rips complex, \( K = \{\sigma \subset \{x_0, \ldots, x_n\} | \text{dist}(x_i, x_j) \leq \epsilon, \forall x_i, x_j \in \sigma \} \) where dist is the Euclidean metric and the vertices of \( \sigma \) are pairwise within distance \( \epsilon \). We observe that the 1-skeleton of the Vietoris-Rips complex is sufficient to compute the zeroth homology group of the space.

Computing Persistent Homology

We construct a filtration that stores the complexes across the entire range of possible values of a scale parameter. By allowing us to exclude short lived topological features, we can control how long a topological feature has to exist in the filtration before we consider it significant.

Connected Component Extraction

A disjoint-set data structure is initialized by making each 0-simplex its own set (all points are born at time zero). While performing the filtration, connected 0-simplices are merged within the data structure. At the end, we find the connected components based on the sets of points that are joined to 0-dimensional simplices of infinite persistence.

Experimental Results

The experiments are conducted using the Object Segmentation Database [3]. For each point cloud, we show the filtered representation prior to segmentation followed by the color coded clusters extracted after segmentation, Figure 2. The filtration is performed for 10 steps up to the maximum distance value set for the complex. The barcodes show the lifespan of the generators of homology.

For Further Information

For the details of our work:

- A video visualizing the topological segmentation process can be seen on the Center for Distributed Robotics YouTube channel: https://www.youtube.com/user/distrob

References


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