CSE 2315 - Discrete Structures

Homework 4- Fall 2010

Due Date: Nov. 11 2010, 3:30 pm

Relations

1. Indicate which of the listed ordered pairs are part of the given relations \( \rho \) on \( \mathbb{N} \)
   
   a) \( xy \iff y = x^2 + x : (1, 3), (1, 2), (3, 10), (4, 20), (4, 15), (5, 30) \)
   
   b) \( xy \iff x^2 + y = 27 : (2, 5), (3, 3), (4, 7), (4, 11), (5, 2), (5, 6) \)
   
   c) \( xy \iff x \text{ is even } \land x^3 \leq y : (1, 1), (1, 2), (2, 3), (2, 8), (2, 11), (3, 30) \)
   
   d) \( xy \iff y^2 = 25 : (2, 4), (2, 5), (3, 7), (3, 9), (4, 5), (7, 5) \)

   Rewrite the following relations on the set \( S = \{2, 5, 6, 9\} \) as a set of ordered pairs.

   e) \( xy \iff x - y = 3 \)
   
   f) \( xy \iff x \ast y > 10 \)
   
   g) \( xy \iff (x + y) \text{ is odd } \land x > y \)
   
   h) \( xy \iff (x \ast y) \text{ is even } \land x \text{ is a prime number} \)

2. Indicate for each of the following binary relations on the positive integers if they are one-to-one, one-to-many, many-to-one, or many-to-many. Also determine if the relations are reflexive, symmetric, antisymmetric, or transitive.

   a) \( \rho = \{(1, 2), (2, 3), (2, 4), (3, 6), (4, 5)\} \)
   
   b) \( xy \iff x \ast y = 36 \)
   
   c) \( \rho = \{(1, 1), (1, 3), (1, 6), (2, 2), (3, 3), (3, 6), (6, 6)\} \)
   
   d) \( xy \iff x \text{ is a prime number} \)
   
   e) \( \rho = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 5), (4, 4), (5, 3), (5, 5)\} \)
   
   f) \( xy \iff x + y \text{ is even} \)

3. Form the desired closures of the given relations over the set \( S = \{1, 2, 3, 4, 5\} \).

   a) Reflexive closure of \( \rho = \{(2, 3), (3, 3), (3, 5), (4, 2), (4, 4), (4, 5)\} \)
   
   b) Transitive closure of \( \rho = \{(1, 1), (1, 2), (2, 3), (3, 1), (3, 4), (5, 5)\} \)
   
   c) Symmetric closure of \( \rho = \{(2, 2), (2, 3), (2, 4), (3, 1), (3, 4), (4, 3)\} \)
d) Symmetric and transitive closure of $\rho = \{(1, 2), (2, 4), (3, 1), (3, 3), (4, 5), (5, 1)\}$

e) Perform the closures required to transform the relation $\rho = \{(1, 1), (1, 2), (2, 3), (2, 4), (3, 5)\}$ into a partial ordering.

f) Perform the closures required to transform the relation $\rho = \{(1, 4), (2, 5), (3, 1), (3, 4), (4, 1), (5, 5)\}$ into an equivalence relation.

4. Draw the Hasse diagrams for the following partial orderings on $S = \{1, 2, 3, 4, 5, 6\}$. Also list the least, greatest, minimal, and maximal elements.

$$\begin{align*}
a) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (2, 4), (1, 4), (4, 6), (2, 6), (3, 5), (1, 6)\} \\
b) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (2, 3), (3, 5), (2, 5), (3, 5)\} \\
c) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (4, 2), (6, 4), (5, 3), (6, 2), (3, 1), (5, 1), (2, 4)\} \\
d) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (2, 4), (1, 4), (2, 6), (4, 6), (3, 6), (1, 6), (5, 6), (1, 3)\}
\end{align*}$$

List all the equivalent classes of the following equivalence relations on the set $S = \{1, 2, 3, 4, 5\}$.

$$\begin{align*}
e) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3), (2, 5), (5, 1), (1, 3), (5, 3), (3, 2), (3, 1), (1, 5), (2, 1), (3, 5), (5, 2)\} \\
f) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (3, 5), (2, 4), (5, 3), (4, 2)\} \\
g) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 3), (3, 2), (4, 5), (2, 1), (3, 1), (2, 3), (5, 4), (1, 2)\} \\
h) & \quad \rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (2, 4), (4, 2)\}
\end{align*}$$

**Functions**

5. Indicate if the following functions are one-to one, onto, or bijective. If a function is a bijection also find its inverse.

$$\begin{align*}
a) & \quad f : \mathbb{Z} \to \mathbb{R} \ , \ f(x) = x^2 + 5 \\
b) & \quad f : \mathbb{N} \to \mathbb{Z} \ , \ f(x) = (-1)^x \ast \left\lfloor \frac{x}{2} \right\rfloor \\
c) & \quad f : \{1, 2, 3, 5\} \to \{4, 6, 8, 9\} \ , \ f = \{(1, 4), (2, 6), (3, 8), (5, 9)\} \\
d) & \quad f : \{a, b, c, d, e\} \to \{\text{vowel, consonant}\} \ , \ f = \{(a, \text{vowel}), (b, \text{consonant}), (c, \text{consonant}), (d, \text{consonant}), (e, \text{vowel})\}
\end{align*}$$

e) $f : \mathbb{R} \to \mathbb{R} \ , \ f(x) = x - 7$

f) $f : (\mathbb{N} \cup \{0\}) \times \{0, 1\} \to \mathbb{Z} \ , \ f((x, y)) = x \ast (y - 1)$

6. Rewrite the following permutations on $S = \{1, 2, 3, 4, 5, 6\}$ in cycle notation.

$$a) \quad f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 1 & 4 & 6 \end{pmatrix}$$
b) $f = \{(1, 2), (2, 4), (3, 3), (4, 1), (5, 6), (6, 5)\}$

c) $f = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
5 & 2 & 4 & 6 & 1 & 3
\end{pmatrix}$

d) $f = \{(1, 1), (2, 6), (3, 3), (4, 2), (5, 5), (6, 4)\}$

e) $f = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
3 & 5 & 1 & 6 & 2 & 4
\end{pmatrix}$

7. Prove by giving appropriate constants that the following functions have the indicated orders of magnitude.

a) $f(x) = \sin(x^2 - \pi)$, $f = \Theta(1)$

b) $f(x) = 2 * x^3 - x + 100$, $f = O(x^3)$

c) $f(x) = \frac{x^2 - 7x + 5}{4x^2 - 3}$, $f = \Theta(x)$

d) $f(x) = 4 * x^3 - 7 * x + 2^x$, $f = O(2^x)$

e) $f(x) = \log(x + x^2)$, $f = O(\log(x))$

f) $f(x) = \log(3^x + x^2)$, $f = \Theta(x)$