Reasoning with Uncertainty

Probability
Bayesian probabilities summarize the effects of uncertainty on the state of knowledge

- Probabilities represent the values of statistics
  \[ P(o) = \frac{\text{(# of times of outcome } o)}{\text{( # of outcomes)}} \]

- All types of uncertainty are incorporated into a single number
  
  \[ P(H \mid E) \]

- Probabilities follow a set of strict axioms
Probability

- Random variables define the entities of probability theory
  - Propositional random variables:
    - E.g.: IsRed, Earthquake
  - Multivalued random variables:
    - E.g.: Color, Weather
  - Potentially Real-Valued
    - E.g.: Height, Weight
Axioms of Probability

- Probability follows a fixed set of rules
  - Propositional random variables:
    - \( P(A) \in [0..1] \)
    - \( P(T) = 1 \), \( P(F) = 0 \)
    - \( P(A \lor B) = P(A) + P(B) - P(A \land B) \)
    - \( P(A \land B) = P(A) \cdot P(B|A) \)
    - \( \sum_{x \in \text{Values}(X)} P(X=x) = 1 \)
Unconditional or prior probabilities represent the state of knowledge before new observations or evidence

- $P(H)$

A probability distribution gives values for all possible assignments to a random variable

A joint probability distribution gives values for all possible assignments to all random variables
Conditional Probability

- Conditional probabilities represent the probability after certain observations or facts have been considered.
  - $P(H|E)$ is the posterior probability of $H$ after evidence $E$ is taken into account.
  - Bayes rule allows to derive posterior probabilities from prior probabilities.
    - $P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$
Conditional Probability

- Probability calculations can be conditioned by conditioning all terms
  - Often it is easier to find conditional probabilities
- Conditions can be removed by marginalization
  - \( P(H) = \sum_E P(H|E) P(E) \)
Joint Distributions

- A joint distribution defines the probability values for all possible assignments to all random variables
  - Exponential in the number of random variables
  - Conditional probabilities can be computed from a joint probability distribution
    - \[ P(A/B) = P(A \cap B)/P(B) \]
Inference

- Inference in probabilistic representation involves the computation of (conditional) probabilities from the available information.
  - Most frequently the computation of a posterior probability $P(H|E)$ form a prior probability $P(H)$ and new evidence $E$. 
Probabilistic Inference

- **Benefits:**
  - Precise quantitative measure of uncertainty
  - Inference can be automated

- **Problems:**
  - *Worst case time complexity:*
    - $O(d^n)$: $d = \text{arity}, \ n = \#\text{of random variables}$
  - *Worst case space complexity:*
    - $O(d^n)$