1. Insert the following numbers into an initially empty red-black tree: 8 2 4 7 5 3 1 6.

2. Design a “worst-case” red-black tree with 10 nodes, i.e., a red-black tree with the longest possible path from the root to a leaf.

3. Design an algorithm to check whether an undirected graph G = (V, E) has an Euler cycle (find out what Euler cycle means). What is the complexity of your algorithm?

4. Suppose you are given a graph that is “almost disconnected”, i.e., the removal of a single edge will disconnect the graph. Design an algorithm to efficiently find this edge.

5. Design an example of a graph where the shortest path tree is longer than the minimum spanning tree. In the worst case, how much longer can the shortest path tree be than the minimum spanning tree?

6. Incremental tree calculations:
   a. Suppose you have constructed a minimum spanning tree of a graph. A new edge is now inserted into the graph. Design an algorithm that will efficiently compute the new MST.
   b. Suppose you have constructed a shortest path tree from a given vertex of a graph. A new edge is now inserted into the graph. Design an algorithm that will efficiently construct the new shortest path tree.